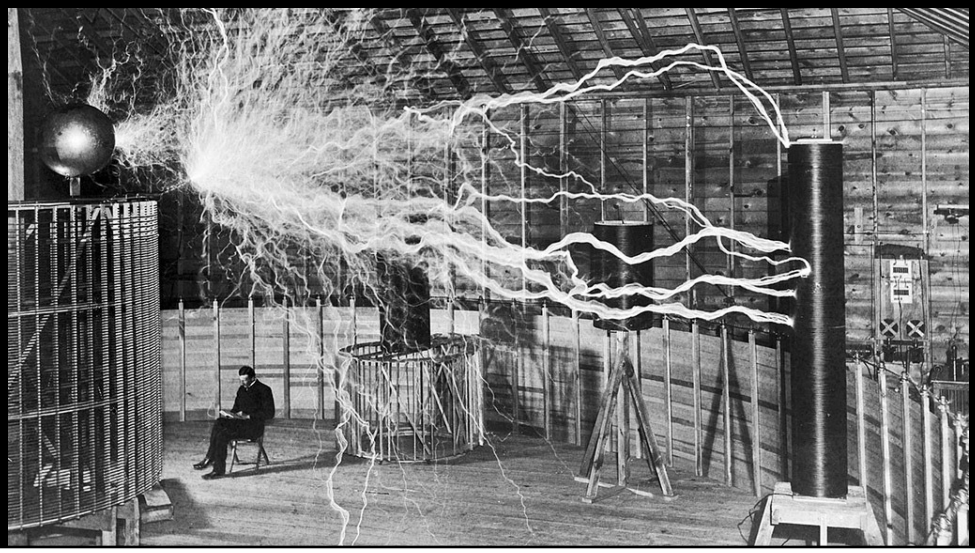
# Electromagnetism

## lecture summaries



1

## Recap of Lecture 1

- $\nabla \cdot \underline{E} = \frac{\rho}{\varepsilon_0}$
- $\nabla \cdot \underline{B} = 0$
- $\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$
- $\nabla \times \underline{B} = \mu_0 \underline{J} + \mu_0 \varepsilon_0 \frac{\partial \underline{E}}{\partial t}$

- $E_{//}$ continuous
- $E_{\perp}$ discontinuous (surface charges)
- $V$ continuous
- $B_{\perp}$ continuous
- $B_{//}$ discontinuous (surface currents)

2

## Recap of Lecture 2

$\nabla^2 V = 0$ (Laplace)

$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ (Poisson)

Separation of variables $V(x,y,z) = X(x)Y(y)Z(z)$

Fourier series

TWO TRICKS FOR SOLVING PROBLEMS

3

## Recap of Lecture 3

- $\nabla^2 V = 0$ in spherical polars

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = 0$$

$$V(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)P_\ell(\cos\theta)$$ [azimuthal symmetry]

- $\nabla^2 V = 0$ in cylindrical polars

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\theta^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

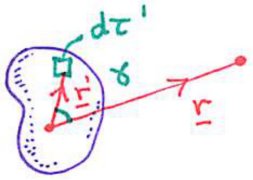
$$V(r,\theta) = \sum_{n=1}^{\infty}\left(A_n\sin n\theta + B_n\cos n\theta\right)\left(C_n r^n + D_n r^{-n}\right) + C_0\ln r + D_0$$ [no z-dependence]

4

## Recap of Lecture 4

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \Rightarrow V(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\underline{r}')}{|\underline{r}-\underline{r}'|}\,d\tau'$$

$$\nabla^2 \underline{A} = -\mu_0 \underline{J} \Rightarrow \underline{A}(\underline{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\underline{J}(\underline{r}')}{|\underline{r}-\underline{r}'|}\,d\tau'$$




$$V(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{l=0}^{\infty}\frac{1}{r^{l+1}}\int_V (r')^l P_l(\cos\gamma)\,\rho(\underline{r}')\,d\tau'$$

$$= \frac{Q}{4\pi\varepsilon_0 r} + \frac{\underline{p}\cdot\underline{r}}{4\pi\varepsilon_0 r^3} + \cdots$$

$$Q = \int \rho(\underline{r}')\,d\tau'$$

$$\underline{p} = \int \underline{r}'\,\rho(\underline{r}')\,d\tau'$$

MULTIPOLE EXPANSION

5

## Recap of Lecture 5

Polarization: $\underline{P} = n\underline{p}$

Bound charges: $\rho_b = -\nabla\cdot\underline{P}$ $\quad \sigma_b = \underline{P}\cdot\hat{\underline{n}}$

Electric displacement: $\underline{D} = \varepsilon_0\underline{E} + \underline{P}$

Maxwell: $\nabla\cdot\underline{D} = \rho_f$

Linear dielectrics: $P = \varepsilon_0 \underbrace{\chi_e}_{\varepsilon_r - 1} E$

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## Recap of Lecture 6

$\underline{D} = \varepsilon_0 \underline{E} + \underline{P}$ $\quad \nabla \cdot \underline{D} = \rho_f$

$E_{//}$ continuous $\quad \nabla \times \underline{D} \neq 0$

$D_{\perp}$ continuous

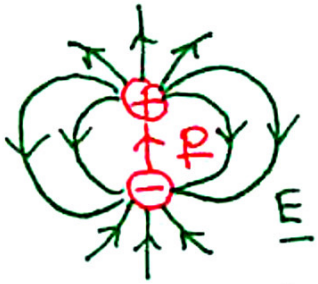
e.g. dielectric sphere

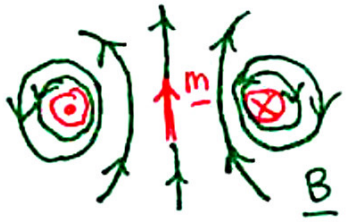
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## Recap of Lecture 7

Current loop $\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi r^2} \underline{m} \times \hat{\underline{r}}$

Magnet moment $\underline{m} = I \underline{S}$ ← vector area of loop




Magnetization $\underline{M} = n\underline{m}$

diamagnetism, paramagnetism, ferromagnetism

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## Recap of Lecture 8

Bound currents $\underline{J}_b = \nabla \times \underline{M}$ (bulk) $\quad \underline{K}_b = \underline{M} \times \hat{\underline{n}}$ (surface)

$\boxed{\underline{B} = \mu_0 (\underline{H} + \underline{M})}$ $\quad \nabla \times \underline{H} = \underline{J}_f \quad \nabla \cdot \underline{H} = -\nabla \cdot \underline{M}$

Linear materials $\underline{M} = \chi_m \underline{H} \quad \underline{B} = \mu_0 \mu_r \underline{H}$

$\mu_r = 1 + \chi_m$

Boundary conditions $E''$, $D^{\perp}$, $H''$, $B^{\perp}$ continuous

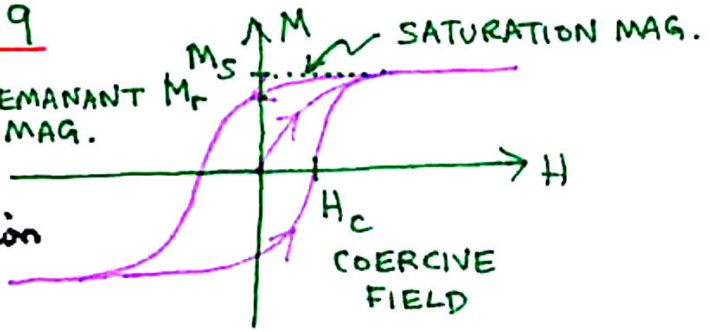
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## Recap of Lecture 9

$\underline{B} = \mu_0 (\underline{H} + \underline{M})$

FERROMAGNETS

- exchange interaction
- domains
- hard vs. soft magnetic materials




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## Recap of Lecture 10

$$\nabla \times \underline{H} = \underline{J}_f + \frac{\partial \underline{D}}{\partial t}$$ ← additional term

$$\text{or} \quad \nabla \times \underline{B} = \mu_0 (\underline{J}_f + \underline{J}_b + \underline{J}_p) + \mu_0 \varepsilon_0 \frac{\partial \underline{E}}{\partial t}$$

$\underline{J}_p$ ← POLARIZATION CURRENT

$\underline{J}_b = \nabla \times \underline{M}$ $\quad \underline{J}_p = \partial \underline{P}/\partial t$

Insulating dielectrics $\quad v = \frac{c}{n}$, $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, $n = \sqrt{\mu_r \varepsilon_r}$

transverse waves

Impedance $\quad Z = \left|\frac{E_0}{H_0}\right| = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0}}$ $\qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$ (FREE SPACE)

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## Recap of Lecture 11

CONDUCTORS $\quad \underline{J} = \sigma \underline{E}$

Free charge decays in time $\sim \frac{\varepsilon_r \varepsilon_0}{\sigma}$

Skin depth $\quad \delta = \sqrt{\frac{2}{\mu \sigma \omega}}$ $\qquad \tilde{k} = k + i\kappa$

Good conductor $\quad k = \kappa = \frac{1}{\delta}$

Poor conductor $\quad k \simeq \sqrt{\mu \varepsilon}\,\omega$ $\qquad \kappa = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$

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## Recap of Lecture 12

Poynting vector $\underline{S} = \underline{E} \times \underline{H}$

rate of flow of energy / unit area

Radiation pressure $P_{rad} = \frac{\langle S \rangle}{c}$

perfect adsorber

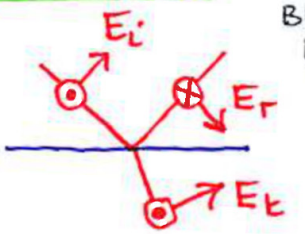
$P_{rad} = 2\frac{\langle S \rangle}{c}$

perfect reflector

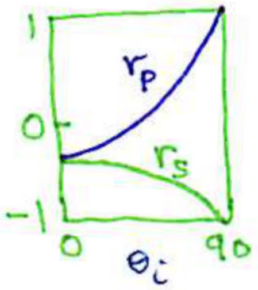
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## 2 conventions for p-polarization

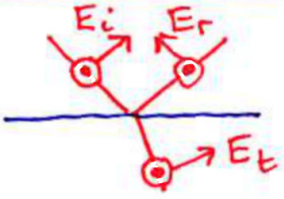
OUR CONVENTION



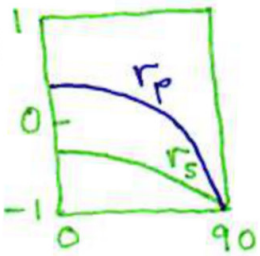

Griffiths
Bleaney & Bleaney
(EM)




"HECHT" CONVENTION




Hecht
wikipedia
(Optics)




14

## Recap of Lecture 13

Reflection and refraction

$$\theta_i = \theta_r$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} = \frac{n_1}{n_2}$$

Fresnel equations for $\underline{E}$ // or ⊥ to the plane of incidence.

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## Recap of Lecture 14

Fresnel equations

$$r_p = \frac{\alpha-\beta}{\alpha+\beta} \qquad r_s = \frac{1-\alpha\beta}{1+\alpha\beta}$$

$$t_p = \frac{2}{\alpha+\beta} \qquad t_s = \frac{2}{1+\alpha\beta}$$

$$\beta = \frac{n_2}{n_1}$$

$$\alpha = \frac{\cos\theta_t}{\cos\theta_i}$$

Total internal reflection when $\beta < 1$

$$\sin\theta_c = \beta$$

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## Recap of Lecture 15

$\beta = \frac{n_2}{n_1}$

Critical angle $\sin\theta_c = \beta$

Brewster angle $\tan\theta_B = \beta$ $[r_p = 0]$

Producing partially polarized light by reflection

Reflection from metals $|r|^2 = 1 - \frac{4\pi\delta}{\lambda}$

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## Recap of Lecture 16

PLASMA = neutral gas of ionised particles

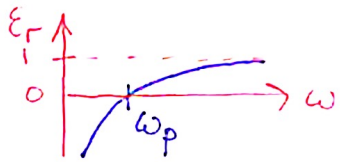
Longitudinal oscillations at $\omega_p$ (PLASMA FREQUENCY)

$\omega_p^2 = \frac{ne^2}{\varepsilon_0 m}$ $\quad \varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$

Propagating modes for $\omega > \omega_p$

$\omega^2 = c^2k^2 + \omega_p^2$

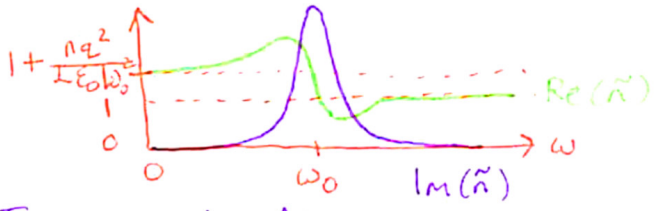
No propagating modes for $\omega < \omega_p$



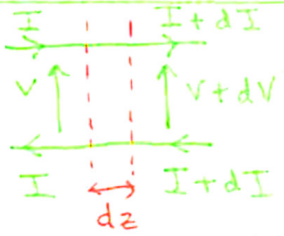

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## Recap of Lecture 17

**Dispersion** $m\ddot{x} + m\gamma\dot{x} + m\omega_0^2 x = qE_0 e^{-i\omega t}$




**Transmission lines**




$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2}$$

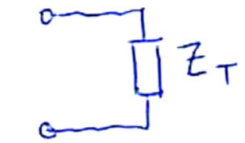
$$v = \frac{1}{\sqrt{LC}} \qquad Z_c = \pm\sqrt{\frac{L}{C}}$$

e.g. coaxial transmission line

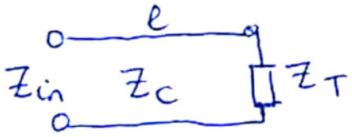
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## Recap of Lecture 18

**Transmission lines** $\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2}$




$$r = \frac{Z_T - Z_c}{Z_T + Z_c} \qquad t = \frac{2Z_T}{Z_T + Z_c}$$



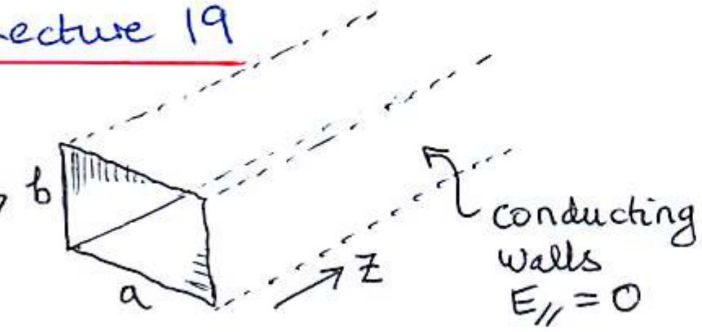

$$Z_{in} = Z_c\left(\frac{Z_T\cos k\ell + iZ_c\sin k\ell}{Z_T\cos k\ell + iZ_T\sin k\ell}\right)$$

20

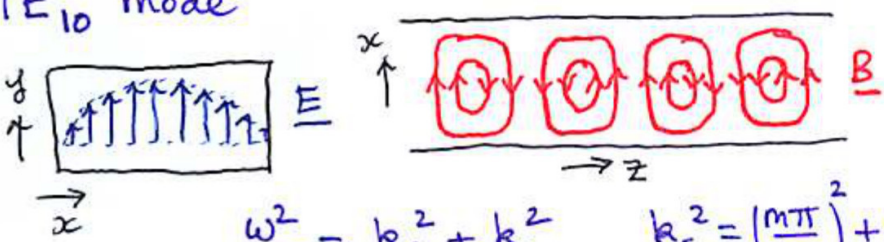
## Recap of Lecture 19

### Waveguides

rectangular waveguide




### $TE_{10}$ mode




$$\frac{\omega^2}{c^2} = k_g^2 + k_c^2 \qquad k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

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## Recap of Lecture 20

Fourier transform $\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt$

gives the frequency spectrum.

$\delta(t) \rightarrow \tilde{\delta}(\omega) = 1$

Dirac comb → Dirac comb

Sampling: if V(t) is band limited to $f_{Nyquist}$, it can be perfectly sampled if sampled at a frequency $\geq 2 f_{Nyquist}$. (the Nyquist rate).

At a lower sampling frequency, you can get aliasing.

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## Recap of Lecture 21

Autocorrelation: $\gamma(t) = \int_{-\infty}^{\infty} f(\tau) f^*(\tau - t)\, d\tau$

$\tilde{\gamma}(\omega) = |\tilde{f}(\omega)|^2$

Parseval: $\int_{-\infty}^{\infty} |f(t)|^2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\tilde{f}(\omega)|^2\, d\omega$

Johnson noise: $\langle V^2 \rangle = 4 k_B T R \Delta f$

Shot noise: $\langle I^2 \rangle = 2 q \langle I \rangle \Delta f$

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## Recap of Lecture 22

- Other sources of noise (all add in quadrature)
- Impulse response [related to convolution theorem]
- Filters [best understood in the frequency domain]
- The lock-in amplifier

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## Recap of Lecture 23

Magnetism is a RELATIVISTIC effect

$J^\mu = (c\rho, \underline{J})$ $\quad \partial_\mu J^\mu = 0$

$A^\mu = \left(\frac{V}{c}, \underline{A}\right)$ ← FOUR VECTORS

The fields are encoded in

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ $\quad \partial_\mu F^{\mu\nu} = \mu_0 J^\nu$ (Maxwell)

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## Plan of the lectures

1. **Fields and potentials** (Lectures 1–4): *Maxwell's equations, boundary conditions, Laplace and Poisson equations, Laplace equation in spherical polars and cylindrical coordinates, multipole expansion*
2. **Electric polarization** (Lectures 5–6): *Bound charges, polarization $P$, the electric displacement $D$, linear dielectrics, boundary conditions*
3. **Magnetization** (Lectures 7–9): *Field from a current loop, magnetizable materials, diamagnetism, paramagnetism, ferromagnetism, magnetization $M$, $B$ and $H$ fields, hysteresis loops*
4. **Electromagnetic waves in materials** (Lectures 10–17): *Displacement currents, Maxwell's equations in insulating dielectrics and in conductors, Poynting vector, radiation pressure, refraction, reflection, Fresnel equations, Brewster angle, plasmas, dispersion*
5. **Confined electromagnetic waves** (Lectures 17–19): *Transmission lines, waveguides*
6. **Signals and noise** (Lectures 20–22): *Fourier transforms of signals, sampling, Johnson noise, shot noise*
7. **Special relativity and electromagnetism** (Lectures 23): *Lorentz transformation of $E$ and $B$ fields*
8. **Examples and summary** (Lecture 24)

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