Notes 4: Experimental evidence for electroweak

We have seen how the EM and weak forces can be arranged in a 'unified' way, i.e. all described under a single combined set of mathematics. To summarise, the main steps were (1) 'Rebundle' the $\frac{1}{2}(1-\gamma_5)$ from the interaction to be part of the spinors instead and treat the left and right chiral parts of a particle as separate entities (even though in reality they always stay together). (2) Introduce a W^0 using symmetry, guided by the weak theory of the W^+ and W^-. (3) Cook up an additional field B^0 which is really the 'bits of the EM interaction which we didn't let in when we added the W^0'. (4) Allow the W^0 and B^0 to mix so that one component reproduces the known EM force. The part left over (which is completely constrained) is the neutral current. We now look over some points concerning the electroweak theory.

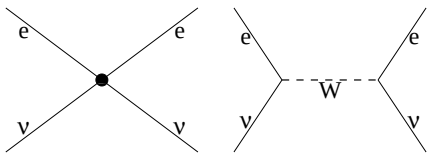
1. Electroweak theory gives us the unification condition linking the coupling constants for the weak interaction g and g' to the electromagnetic coupling e. $e = \sqrt{4\pi\alpha} = g\sin\theta_W = g'\cos\theta_W$ We can eliminate θ_W between these two equalities to get

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} \tag{49}$$

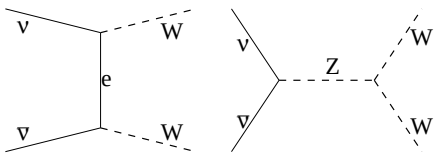
2. Comparing the weak charged current at low energy ($q^2 << M^2$) [see Burcham+Jobes p501] we get a relation between g and G_F from the old theory $G_F/\sqrt{2} = g^2/(8M_W^2)$. Doing a similar comparison for neutral current processes (e.g. $\nu_\mu e \to \nu_\mu e$) we obtain $G_F/\sqrt{2} = g^2/(8M_Z^2\cos^2\theta_W)$. Putting these two relations together, we obtain

$$\frac{M_W}{M_Z} = \cos\theta_W \tag{50}$$

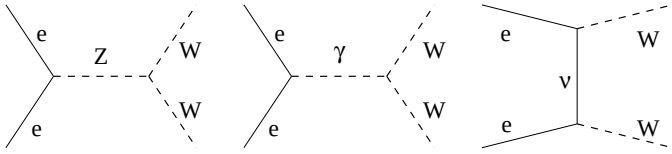
3. The original Fermi 4-point theory had a problem, the cross section $\sigma \propto G_F E^2$ where E is the centre of mass energy. This is fine at low energy, but at 300 GeV the scattering probability becomes bigger than one. This is known as unitarity violation and is bad news for a theory. By introducing the W boson and moving away from a four point interaction, a propagator term $1/(M_W^2 - q^2)$ is introduced and the cross section stops increasing.



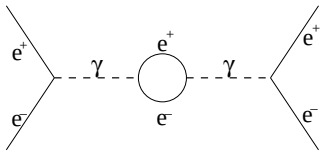
4. Another similar problem occurs even with W-bosons in the theory. The process $\nu\overline{\nu} \to W^+W^-$ in the theory before electroweak unification (diagram on left) is divergent (i.e. becomes very large at high energy). Electroweak theory predicts neutral currents and in particular the diagram on the right involving $Z \to WW$ which cancels the divergence.



5. Also the two diagrams below on the right need cancellation from the left diagram which comes from the combined electroweak theory.



6. **Renormalisation** We have not gone into full detail about calculating Feynman diagrams. Higher order diagrams contain internal loops. The Feynman rules require that we integrate over the particle momentum in each loop. The diagram



is a problem in that the integral (which we want to evaluate with an upper limit of infinity) is

$$\int^{\infty} \frac{1}{q^4} q^3 dq = \ln|q| \tag{51}$$

i.e. it diverges. The solution to this took a long time to develop and prove that it works. The result is renormalisation theory. 't Hooft proved that all local gauge invariant theories are renormalisable.

7. Our theory of the EM interaction (QED) is locally gauge invariant and we can use 't Hooft's theorem to be assured that it is renormalisable. Good. Same for QCD.

8. For the weak interaction however we have a problem. The bosons are **massive** and enter in a different (non locally gauge invariant) way to the EM interaction in the Dirac equation. So we can't apply 't Hooft's theorem and don't know whether our theory is renormalisable. The solution to this is the Higgs.

We now turn to experimental evidence for the electroweak theory, the neutral currents were first discovered in neutrino interactions and subsequently studied there. We defer our discussion of the discovery of the W and Z bosons to Trinity term, and move straight to the most accurate measurements made on the Z, which were at LEP.

Discovery of neutral currents

The neutral current was first discovered in the Gargamelle bubble chamber at CERN in the reaction $\overline{\nu}_\mu e \to \overline{\nu}_\mu e$ i.e. elastic scattering off atomic electrons. The scattering reaction of either ν_μ or $\overline{\nu}_\mu$ off electrons is an unambiguous signal for neutral currents. Scattering of either ν_e or $\overline{\nu}_e$ is ambiguous as there is a charged current diagram for each of these reactions. About 100 events were observed in total and by measuring the cross section, a value of $\sin^2\theta_W = 0.24 \pm 0.04$ was obtained. The amplitude for the process from the electroweak lecture is

$$M = \frac{g^2}{8M_Z^2 \cos^2\theta_W} \left[\overline{u}(\overline{\nu}_\mu)\gamma_\mu(c_V^{(\nu)} - c_A^{(\nu)}\gamma_5)u(\overline{\nu}_\mu)\right] \left[\overline{u}(e)\gamma^\mu(c_V^{(e)} - c_A^{(e)}\gamma^5)u(e)\right] \tag{52}$$

We can look up the couplings from the lecture: $c_V^{(\nu)} = c_A^{(\nu)} = \frac{1}{2}$, $c_V^{(e)} = -\frac{1}{2}$ and $c_A^{(e)} = -\frac{1}{2} + 2\sin^2\theta_W$. The calculation of the cross section involves averaging over spins, since

the electrons are unpolarized. It is a long calculation with several time saving tricks. The result for $d\sigma(\overline{\nu}_\mu e \to \overline{\nu}_\mu e)/dE_e$ is

$$\frac{d\sigma}{dE_e} = \frac{G_F^2 m_e}{2\pi}\left[(c_V^{(e)} - c_A^{(e)})^2 + (c_V^{(e)} - c_A^{(e)})\left(1 - \frac{E_e}{E_\nu}\right)^2 - \frac{m_e E_e}{E_\nu^2}((c_V^{(e)})^2 - (c_A^{(e)})^2)\right] \quad (53)$$

This can be integrated to get the cross section $\sigma = \int \frac{d\sigma}{dE_e} dE_e = E_\nu 10^{-46}\mathrm{m}^2$ where E_ν is in GeV. i.e. it is tiny.

Neutral currents in hadronic neutrino interactions

The neutral current is also observable in neutrino-nucleon collisions. The cross sections are still small, but somewhat larger than for scattering off electrons. Neutrino detectors are therefore made as massive and as cheap per kg as possible. MINOS for example is made from a sandwich of steel sheets with scintillator. SuperKamiokande is a large tank of water and has phototubes surrounding to detect Cerenkov light. SNO is similar, but uses heavy water (and is consequently smaller). Both charged current and neutral current interactions are seen. Charged current (CC) interactions appear as a lepton (corresponding to the neutrino flavour) and a clump of hadronic particles. Neutral currents (NC) appear as just the clump of hadronic particles as the outgoing neutrino is not detected. We can use the ratio of cross sections $\sigma_{\mathrm{NC}}/\sigma_{\mathrm{CC}}$ to obtain the measurement $\sin\theta_W = 0.230 \pm 0.004$. The cross section calculation proceeds as for the electron scattering with a few extra details: The quarks are inside nucleons so we need form factors; Cabibbo rotation must be applied in the charged current case; for neutral currents, it is equivalent to use either Cabibbo rotated or unrotated states — the result is the same.

Tests of Electroweak theory at LEP

e^+e^- machines

As a prelude to studying phtsics at colliders in more detail in Trinity term, we now take a detailled look at the LEP accelerator and experiments. This will lead on to the more experimentally challenging hadron colliders (short reason they are more difficult: more backgrounds). All the details on LEP are collected here, but depending on how much time is available in the lectures, some of it may be discussed in the trinity term lectures.

e^+e^- machines are the place of choice to study Z^0s — the e^+ and e^- are point like so, unlike what we will see in a hadron collider, there is no underlying event (from the other quarks in the particles which collide) and we can assume that the centre of mass of the collision is fixed. The antiparticle e^+ is also much easier to produce than antiprotons. The big disadvantage of e^+e^- collisions, however, is that at high energy, Bremsstrahlung is much worse for electrons — the ring circumference of the SPS (used to discover the W and Z is 5 km) while for LEP it is 27 km! It is also possible to make a linear collider (at SLAC) — all in a straight line where the beams collide only once. Since the luminosity

$$L = \frac{f N_1 N_2}{A} \quad (54)$$

it is important focus the beams down to the smallest possible area A at the points where they collide.

Detailed look at the detectors

The detectors are almost '4π' (steradians) i.e. they are sensitive to particles going in any direction from the interaction point (no cracks except for the two beam holes), This is very important for some of the analysis techniques we are going to discuss later.

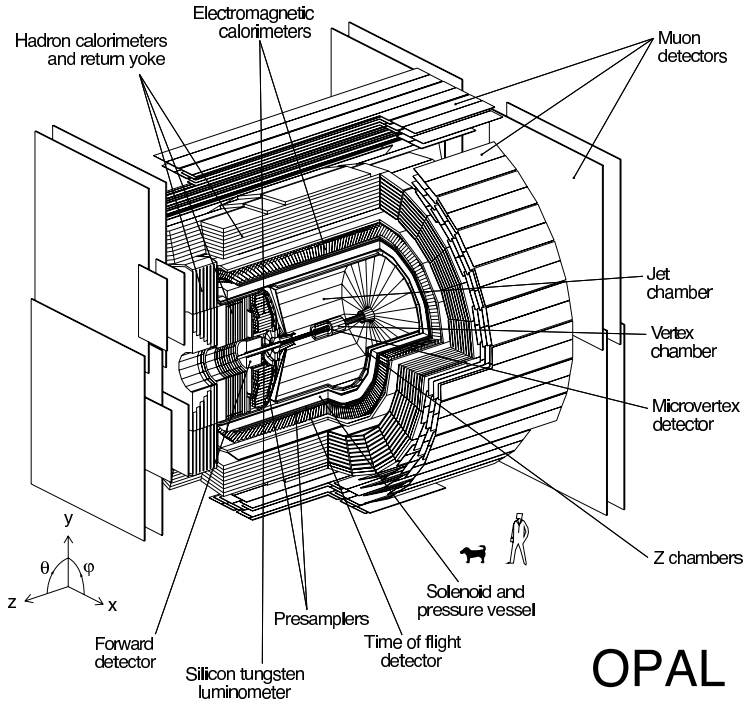


Figure 1: The Opal detector

The detectors are layered like rings on an onion with tracking detectors in the middle, then particle ID detectors (e.g. Cerenkov detection), then electromagnetic calorimeters, then hadronic calorimeters and finally muon chambers on the outside. There is usually a magnetic field so the momentum of the particles is measured as they traverse the tracking region. There were 4 LEP detectors — all different (different ways of detecting and identifying particles). An important part is the silicon vertex detectors which were used at all four LEP experiments - tracking detectors with high position resolution close to the vertex — used to distinguish b-jets.

We now go through the different types of events seen at LEP when running at the Z^0 resonance as a review of what particles do as they pass through material and on what the detectors see (see event displays in lecture).

$Z \to e^+e^-$	2 back-to-back showers in EM calorimeter with tracks leading towards them. Nothing in hadron calorimeter or muon detectors.
$Z \to \mu^+\mu^-$	2 back-to-back tracks leading all the way out to the muon detectors.
$Z \to \tau^+\tau^-$	A variety of topologies which have low multiplicities and missing momentum (from the neutrino).
$Z \to \nu\overline{\nu}$	Nothing
$Z \to$2-jets	$Z \to q\overline{q}$ the quarks form 'jets' — a series of mesons and baryons which appear along the same direction as the initial quark direction. Generally it is not possible to tell what flavour the original quark had, however ...
b-jets	with b (and c) quarks, using the silicon vertex detectors, it is possible to detect whether the particles all extrapolate back to the primary vertex or whether there is a secondary vertex where the b-quark decayed.
$Z \to$3-jets	$Z \to q\overline{q}$ with a gluon 'Bremsstrahlung' emitted from one of the quarks.

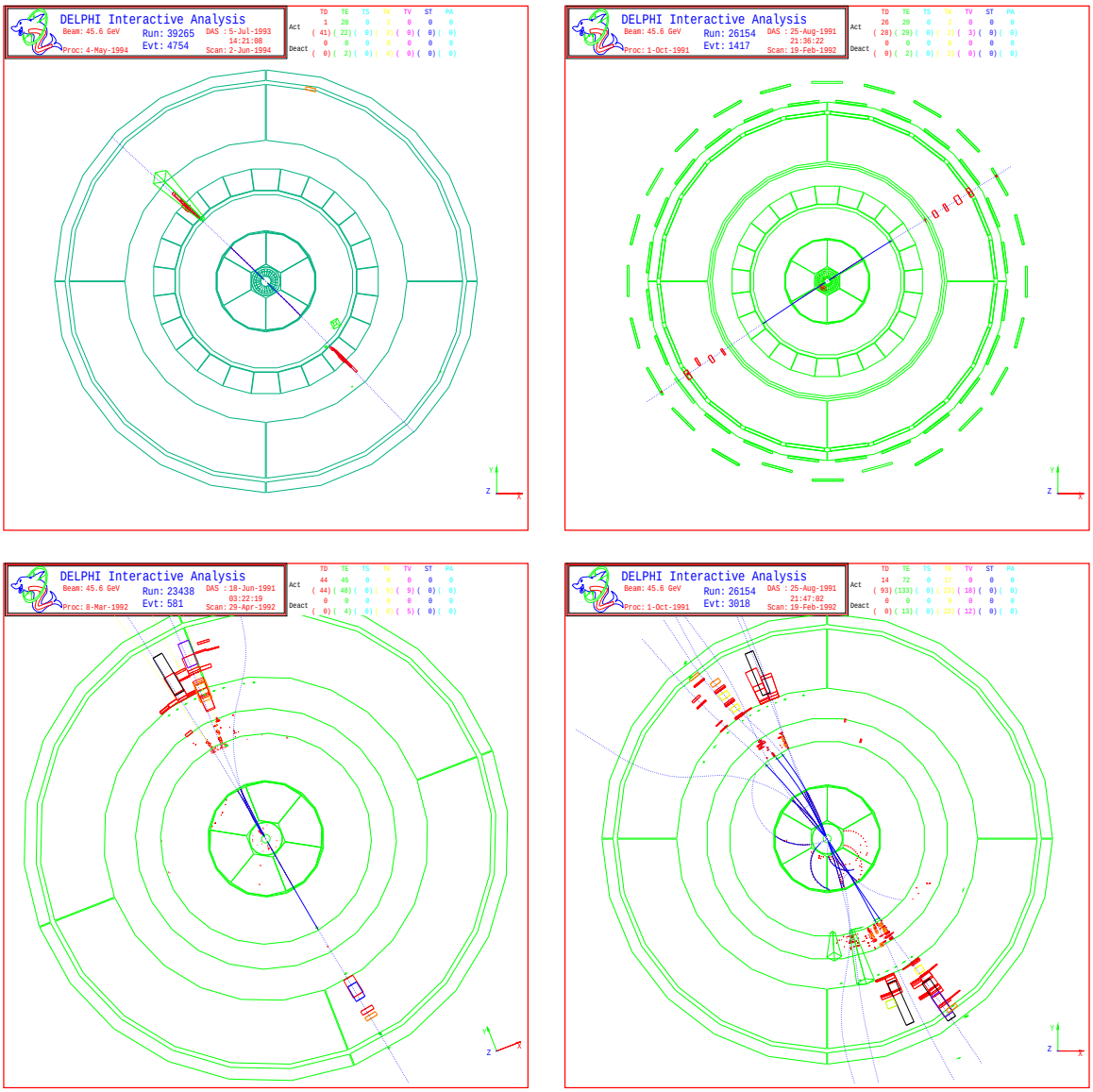


Figure 2: Four event types (a) e^+e^-, (b) $\mu^+\mu^-$, (c) $\tau^+\tau^-$, (d) quark–antiquark pair.

Aspects of a physics analysis

What is said here is true for any analysis of data but I describe it in the context of a LEP experiment. An analysis generally goes along the following lines: the experimenters first choose selection criteria (**cuts**) which select the events which are needed for studying such as requiring n tracks which look like muons, or tracks above a certain energy etc. This may involve reconstructing the mass of a combination of particles from the measured four-momenta of the tracks and showers. Quite often there is **background** in the sample which is selected — where a different physical process produces events which pass the same cuts (and is therefore indistinguishable from the signal). **Triggering** is an important concept in a high energy physics experiment — the electronics is monitoring the signals on all the electronics channels all the time, but just like with a digital camera, someone has to choose at what moment to store the information which is going on in the detector. This involves some fast-thinking electronics computing various quantities (number of tracks, energy etc.) 'online' and choosing the moments in time when something interesting is happening for the **data acquisition** to then record everything as an **event**.

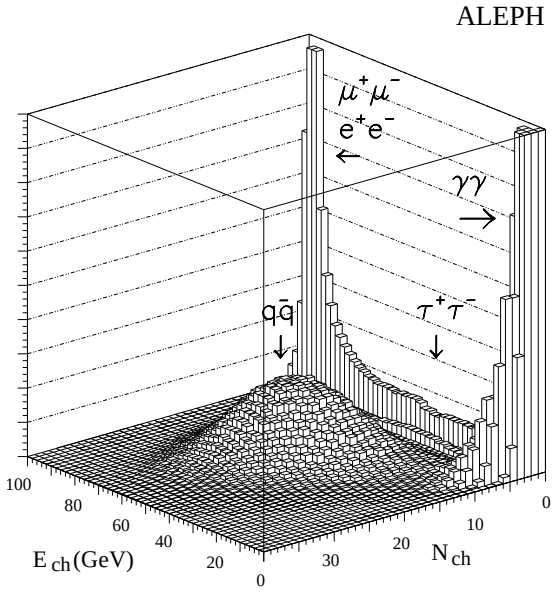


Figure 3: Main cuts used to separate the different classes of e^+e^- events.

Calibration concerns subtracting pedestals and measuring the gains and linearities of each channel (important particularly in the calorimeters). If the calibration is not done then the energy resolution of the calorimeters suffers. The **acceptance** for the process we are studying has to be measured. Acceptance concerns what is the probability of the particles in an event to hit a certain part of the detector. Usually in a 4π LEP detector it is almost 100% however it may be reduced if cuts are made around any dead channels. **Accidentals** or pile-up concerns the problem that two events occur at the same time (within detectable resolution) and the resulting combination gets into the data sample — accidentals can also be a problem if a preceding event causes electronics to momentarily become dead before ready to measure a new pulse or if the preceding event causes a movement in the pedestals which causes the energy to be measured slightly wrongly. Accidental problems are not big in LEP experiments where the rate of events is low.

A big tool for this analysis is a Monte–Carlo[10] simulation computer program. The idea is to produce simulated events, using a random number generator at each point where a choice in what happens must be made. So for example in generating a simulated $e^+e^- \to Z^0 \to \tau^+\tau^-$, the choices begin with (1) pick the direction along which the τ^+ decays along and use momentum conservation to work out the direction of the τ^-. (2) choose the length of time each τ lives by picking from an exponential probability distribution e^{-t/τ_τ} where τ_τ is the mean decay time of the τ-lepton and work out where it decays. (3) choose from a table of measured branching ratios what daughter particles each τ will decay into. ... and so on. The random numbers can also be used to choose how each particle proceeds as it passes through various bits of material in the detector and can produce realistic looking electromagnetic and hadronic showers. The technique works well and by generating lots of these simulated events we are effectively performing an integral over all the possible outcomes of what might happen by randomly sampling the integrand. The Monte–Carlo technique is a widely used one and can be used to compute trigger efficiencies, acceptances and accidental effects in an analysis.

[10]Named after the city in Monaco (tax-haven adjacent to France) where there is a big casino.

Physics at LEP

LEP produced a huge number of Z^0s, $\sim 4.5 \times 10^6$ per detector. Z^0s decay into almost every type of particle we know, so many things can be studied. There were also a number of individual measurements of high importance which we will now discuss, the mass of the Z^0 boson, the measurements of the number of neutrinos, and production cross section and decay parameters of the Z^0 which are used to obtain the couplings $c_V^{(f)}$ and $c_A^{(f)}$ to compare with the electroweak theory prediction. We will discuss measurements made when LEP was run above M_Z in a later lecture.

So the list of what to do (i.e. what was done) is:

Take runs at different values of the beam energy around the Z^0 peak.
Record everything whenever there is a trigger ($\equiv$ an event).
Reconstruct events offline.
Classify as ee, $\mu\mu$, $\tau\tau$, $q\overline{q}$, Luminosity Bhabha.
Measure cross section as a function of energy $\sqrt{s}$ and sub cross sections.

The cross section as a function of $\sqrt{s}$ (i.e. beam energy) displays a nice peak at the Z-resonance.

The cross sections as a function of s for $e^+e^- \to Z^0 \to f\overline{f}$ (where f is one of e, μ, τ or quarks) is given by:

$$\sigma_f(s) = \sigma_f^0(s)\frac{s\Gamma_Z^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \otimes (\text{QEDcorr.}) \otimes (\text{QCDcorr.}) \tag{55}$$

where

$$\sigma_f^0(s) = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2} \tag{56}$$

and the electroweak theory from last time can be used to compute Γ_f to give

$$\Gamma_f = \frac{G_F\sqrt{2}M_Z^3}{12\pi}\left[(c_V^{(f)})^2 + (c_A^{(f)})^2\right] N_{\text{colours}} \otimes (\text{QCDcorr.}) \tag{57}$$

Equations (55) and (56) when put together make the usual Breit-Wigner formula. In the lecture, there will be a quick review of the electroweak theory. Here we just note the formulae for the final steps: when we inspect the electromagnetic part of the expression for the EW interaction, we get the unification conditions

$$e = g\sin\theta_W = g'\cos\theta_W; \qquad \text{also} \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \tag{58}$$

and when we look at the neutral current part of the EW interaction we get

$$g_z J_\mu^Z = \frac{g}{\cos\theta_W}\overline{u}\gamma_\mu\frac{1}{2}\left[c_V^{(f)} - c_A^{(f)}\gamma_5\right]u \tag{59}$$

where $c_V^{(f)} = I_3(f) - 2Q(f)\sin^2\theta_W$ and $c_A^{(f)} = I_3(f)$ for each fermion type f (e, μ, τ or quark).

LEP Measurement 1: Mass of the Z

This is done by simply taking the curve of the cross section as a function of beam energy and fitting the expression given in equations (55) to (57) to find the best value for M_Z. In practice there are various complications to take care of including the QED and QCD corrections indicated in the formulae. Another important point is to know exactly what the beam energy is.

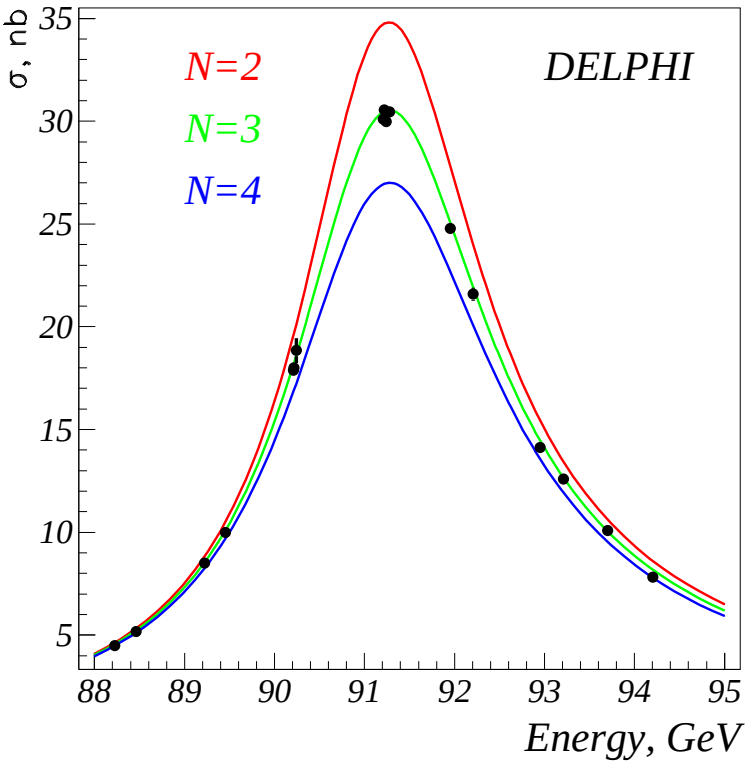
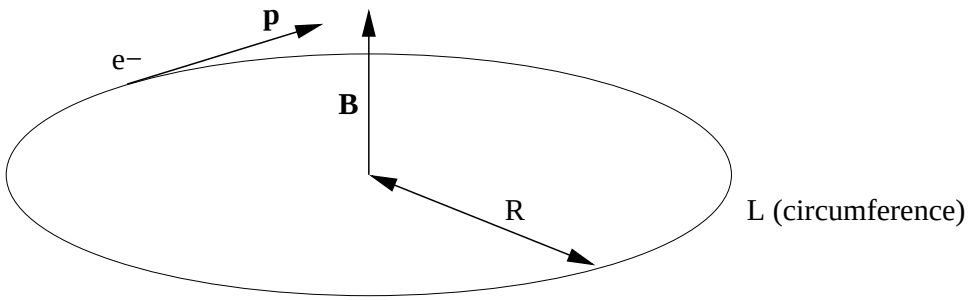


Figure 4: N_ν and mass peak plot

LEP beam energy measurement (detail is not on syllabus)

When we talk about the LEP precision measurement of M_Z in a moment, we will need to know the actual energy of the beam as precisely as possible. This was a very careful study. How do you measure the beam energy ? A technique called resonant depolarisation involving the anomalous magnetic moment of the electron (g-2) was employed (recall the experiment to measure accurately the anomalous magnetic moment of the muon as one of the most stringent tests of QED). Approximate LEP by a circle immersed in a uniform vertical B field (providing the bending).



$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -e\mathbf{v} \times \mathbf{B}; \qquad |p| = eBR = \frac{e}{2\pi}BL \tag{60}$$

Also the orbit angular frequency is $\omega_c = eB/\gamma m_e$. Electrons 'naturally' become polarised over about 5 hours by going around in LEP. The polarisation can be measured with

backscattered light. The spin precession angular frequency is given by

$$\omega_s = \frac{eB}{\gamma m_e}\left[1+\gamma\left(\frac{g-2}{2}\right)\right] \tag{61}$$

and so we can compute the number of precessions per turn in LEP ν_s

$$\nu_s = \frac{\omega_s - \omega_c}{\omega_c} = \gamma\left(\frac{g-2}{2}\right) = \frac{E_{\rm beam}}{m_e}\left(\frac{g-2}{2}\right) \tag{62}$$

$(g-2)/2$ is known to an accuracy of 4×10^{-9} and the electron mass m_e to a precision of 3×10^{-7} so if we can measure ν_s we get the energy of the beam.

The technique for measuring ν_s (resonant depolarisation) proceeds by adding a small magnet with field in the x direction (horizontal, transverse to the beam) with a field which varies as $\sin\nu t$. When $\nu=\nu_s$ the contribution will accumulate each turn and cause the beam to depolarise (as measured in the backscattered light). The precision obtained in the beam energy was 2 MeV (out of 45 GeV). It involved understanding various effects including the tidal pull of the moon (which changes L slightly), movements in the water table and even ground currents caused when the fast TGV trains to Paris passed by (discovered on a day when there was a rail strike !).

Parameters of the electroweak theory

Recalling again formulae from the electroweak theory:

$$e = g\sin\theta_W = g'\cos\theta_W; \qquad M_\gamma = 0; \qquad M_W = M_Z\cos\theta_W \tag{63}$$

we see that there are three parameters left, g, $\sin^\theta_W$ and M_W.

In practice, these three parameters are expressed in terms of the best measured quantities:

$$\alpha_{\rm QED}(0) = \frac{e^2}{4\pi} = \frac{1}{137.0359895(61)} \tag{64}$$

$$(\mu-\text{decay}) \quad G_F = 1.16689(2)\times10^{-5}\text{GeV}^{-2} \tag{65}$$

$$(\text{LEP}) \quad M_Z = 91.1872\pm0.0021\text{GeV} \tag{66}$$

then we can check the other parameters by measuring them and checking with

$$\sin^2\theta_W\cos^2\theta_W = \frac{\pi\alpha_{\rm QED}}{\sqrt{2}G_F M_Z^2} \tag{67}$$

$$M_W = M_Z\cos\theta_W \tag{68}$$

$$g^2 = \frac{4\pi\alpha_{\rm QED}}{\sin^2\theta_W} \tag{69}$$

LEP Measurement 2: The Z width; Number of neutrinos

The Z^0 can decay into a pair of neutrinos $Z^0\to\nu\overline{\nu}$. How many generations of leptons are there ? We know of three (e, μ and τ). Provided the mass of the associated neutrino is less than half the Z^0 mass, there is a way to detect if there are any more. The total width of the Z^0, Γ_Z is made up of the sum of the partial width of all it's decay modes, i.e.

$$\Gamma_Z = \Gamma_{\rm HAD} + \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + N_\nu\Gamma_\nu \tag{70}$$

$$= \Gamma_{\rm HAD} + 3\Gamma_{l\bar{l}} + N_\nu\Gamma_\nu \tag{71}$$

where lepton universality has been assumed for the second step. The partial widths can be predicted from the electroweak model (equation 57) and the branching ratios can be used to check. Therefore, by measuring the total width Γ_Z (again by fitting the shape of the cross section as a function of beam energy using equations 55 to 57, the number of neutrinos N_ν can be measured. However, since

$$\sigma_0 \propto \frac{1}{\Gamma_Z^2} \tag{72}$$

(equation 56) it is most sensitive to simply measure the cross section at the very top of the peak. We find the result is

$$N_\nu = 2.9841 \pm 0.0083. \tag{73}$$

LEP Measurement 3: σ, $A_{\rm FB}$

We are now going to show how the couplings c_V and c_A for each fermion type can be extracted from measurements of the cross section and the forward-backward asymmetry $A_{\rm FB}$ (to be defined shortly). Starting with equation (59), we can show (complicated)

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 M_Z^4}{32\pi^2\Gamma_Z^2}\left[\left((c_V^{(e)})^2+(c_A^{(e)})^2\right)\left((c_V^{(f)})^2+(c_A^{(f)})^2\right)(1+\cos^2\theta)+8c_V^{(e)}c_A^{(e)}c_V^{(f)}c_A^{(f)}\cos\theta\right] \tag{74}$$

where θ is the angle between the incoming electron direction and the outgoing lepton or quark (or between the incoming positron and outgoing anti-lepton or anti-quark). Integrating this expression, we find

$$\sigma(e^+e^-\to f\overline{f}) \propto \left[(c_V^{(e)})^2+(c_A^{(e)})^2\right]\left[(c_V^{(f)})^2+(c_A^{(f)})^2\right] \tag{75}$$

Defining the forward backward asymmetry $A_{\rm FB}$ as

$$A_{\rm FB}^f = \frac{N_F - N_B}{N_F + N_B} \tag{76}$$

where N_F is the number of events in which $\theta < 90^\circ$ (forward) and N_B is the number of events in which $\theta > 90^\circ$ (backward). Integrating again from equation (74)

$$A_{\rm FB} = \frac{3}{4}\frac{2(c_V^{(e)})^2(c_A^{(e)})^2}{(c_V^{(e)})^2+(c_A^{(e)})^2}\frac{2(c_V^{(f)})^2(c_A^{(f)})^2}{(c_V^{(f)})^2+(c_A^{(f)})^2} \tag{77}$$

$$= \frac{3}{4}A_eA_f \qquad \text{where} \quad A_e = \frac{2(c_V^{(e)})^2(c_A^{(e)})^2}{(c_V^{(e)})^2+(c_A^{(e)})^2} \qquad \text{and} \quad A_f = \frac{2(c_V^{(f)})^2(c_A^{(f)})^2}{(c_V^{(f)})^2+(c_A^{(f)})^2} \tag{78}$$

So by measuring σ and $A_{\rm FB}$ for a particular final state f, we get two equations involving $c_V^{(f)}$ and $c_A^{(f)}$ which can be solved to get $c_V^{(f)}$ and $c_A^{(f)}$ separately. Measuring σ correctly requires accurate knowledge of the luminosity which is described in the section below. Meanwhile, lets look at the electroweak information we can obtain by making these measurements. On a plot of c_V vs c_A, $\sigma \propto c_V^2 + c_A^2$ is a circle and $A_{\rm FB}$ is a line. [See finals question 2001-7)]. This can be done for each final state f. One subtlety is that σ and $A_{\rm FB}$ still need the electron constants $c_V^{(e)}$ and $c_A^{(e)}$ to be completely unraveled. This can be done with a separate measurement on the τ decay products.

We have derived the $A_{\rm FB}$ formula assuming all the interactions are mediated by the Z only, but since the photon is involved as well, $A_{\rm FB}$ varies as a function of beam energy as shown in the plots.

So in summary, from the measured quantities σ_{ee}, $\sigma_{\mu\mu}$, $\sigma_{\tau\tau}$, $A_{\rm FB}^e$, $A_{\rm FB}^\mu$, $A_{\rm FB}^\tau$ and some information from τ decay, we obtain $c_V^{(e)}$, $c_V^{(\mu)}$, $c_V^{(\tau)}$, $c_A^{(e)}$, $c_A^{(\mu)}$ and $c_A^{(\tau)}$. From data, we see:

1. Universality: $c_V^{(e)} = c_V^{(\mu)} = c_V^{(\tau)}$ and also $c_A^{(e)} = c_A^{(\mu)} = c_A^{(\tau)}$ to high precision. All three types of lepton couple to the Z^0 with the same strength.

2. $\sin^2\theta_W = 0.23189 \pm 0.00024$ (this is the combination from all LEP techniques).

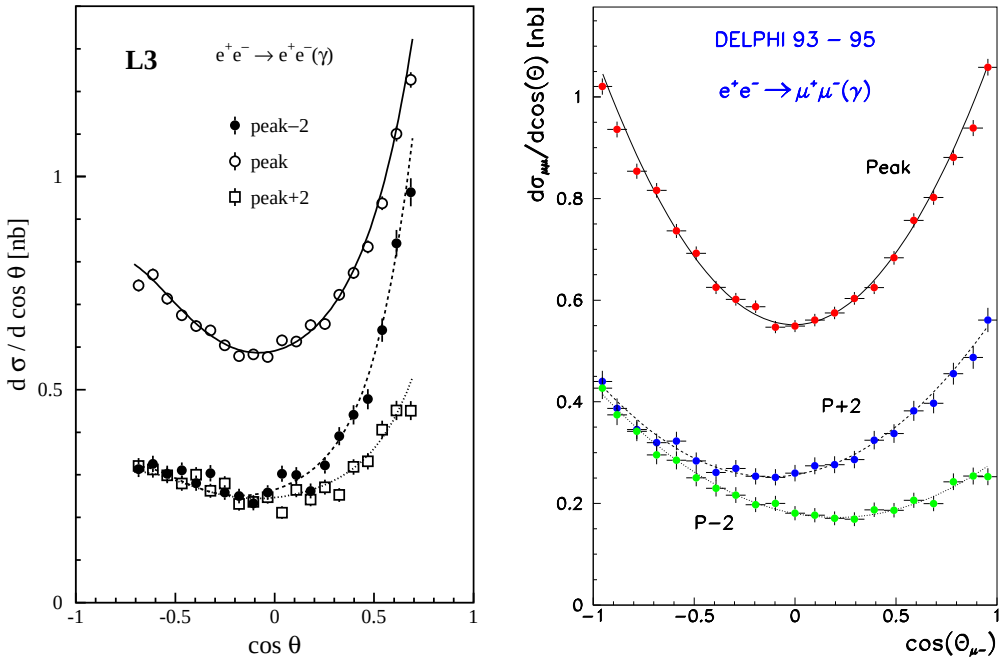


Figure 5: Cross section as a function of θ, the angle between the incoming electron and outgoing lepton (the two plots are the results from two different LEP detectors, the three curves show the three main beam energies at and near the Z-pole where LEP was run.

LEP Luminosity Measurement (detail is not on syllabus)

It is important to be able to measure the LEP luminosity to be able to do the cross section measurements. Recall that luminosity L is defined by the following expression:

$$N_i = L\sigma_i \tag{79}$$

where N_i is the number of events from a given process i which occur in the detector (once detector effects like trigger efficiency and acceptance have been corrected for) and σ_i is the cross section. L is the same for all processes - it depends on the features of the accelerator and how well it is working (e.g. how well the two beams are steered into each other at a particular interaction region). The way we measure the luminosity is with a process j for which we know how to calculate the cross section theoretically. We count the numbers of events N_i and N_j, then using the formulae $L = N_j/\sigma_j$ and $\sigma_i = N_i/L$ we obtain the cross section we are interested in. For LEP, the channel j used to measure the luminosity is low angle electron-positron scattering (Bhabha scattering) for which the QED single photon exchange diagram dominates. There are only very small contributions to this from diagrams with Z^0 or involving annihilation and the theoretical uncertainty is below 0.1%. The Bhabha scatters are measured with little calorimeters situated on each side of the detectors (several tens of metres each side of the interaction point) and events

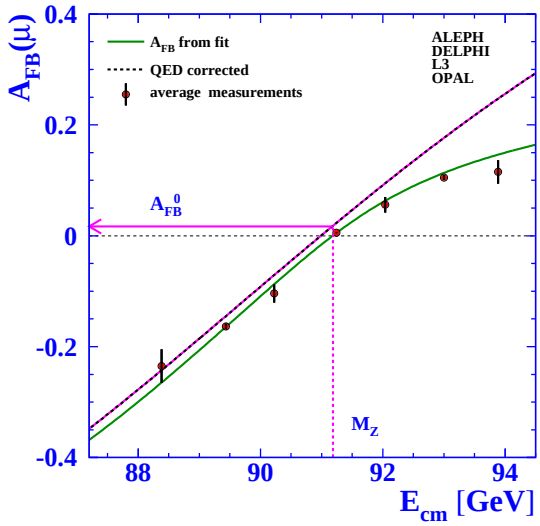
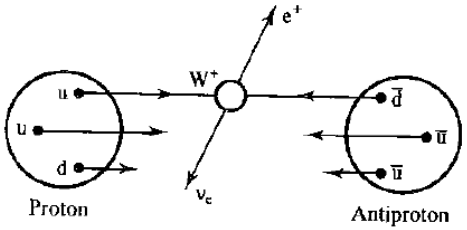
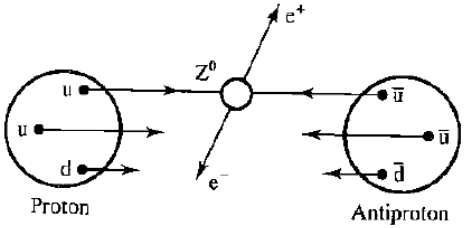
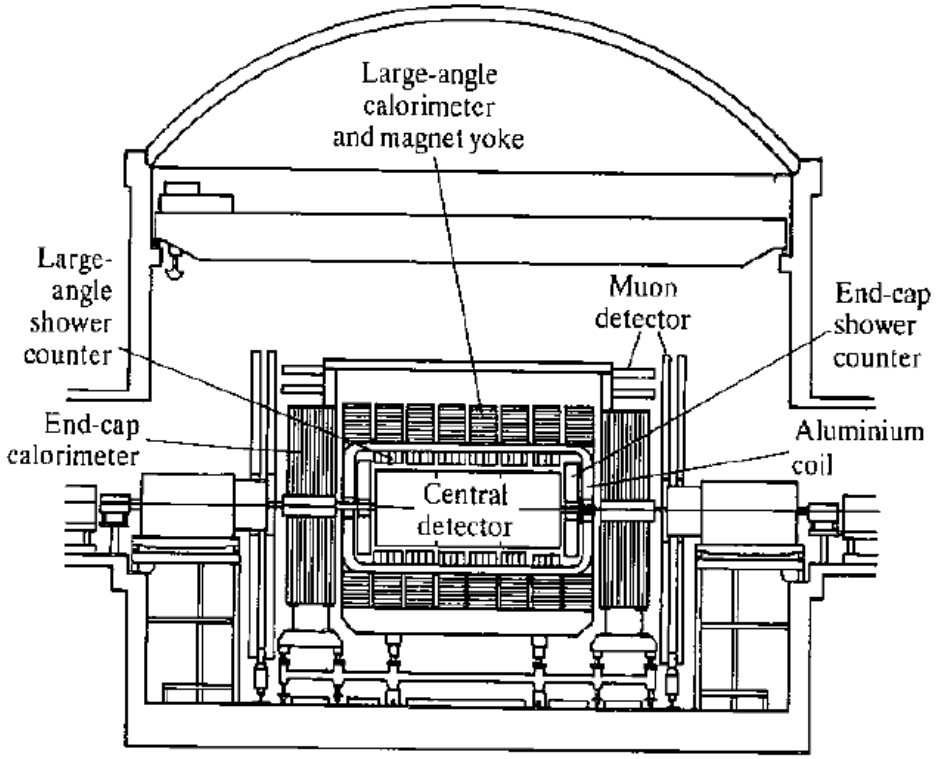


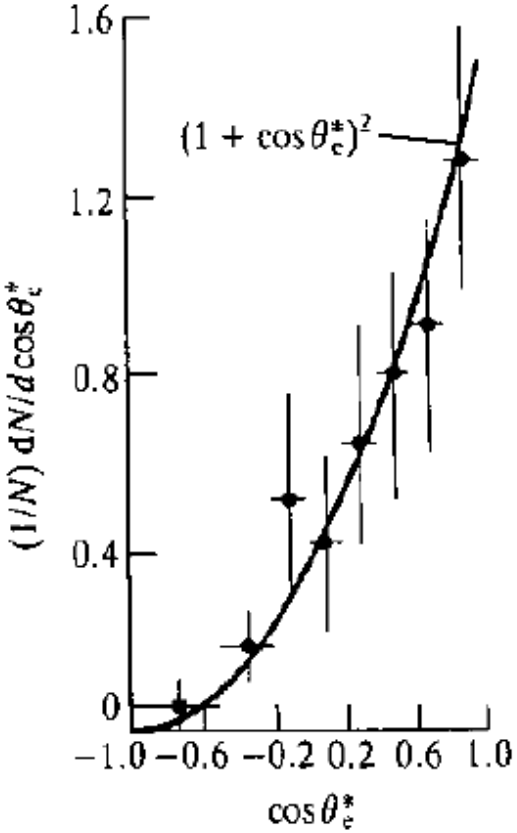
Figure 6: $A_{\rm FB}$ as a function of LEP beam energy. From LEP EWWG.

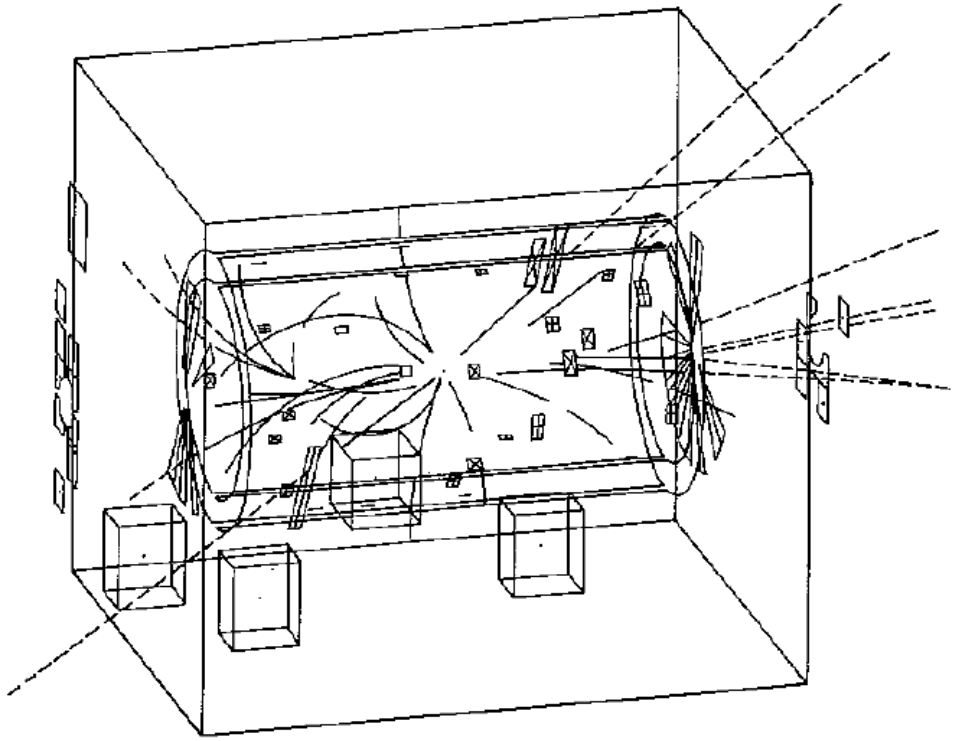
are counted when two showers with the appropriate energy are seen within a radius region of (6 cm $< R <$ 15 cm). This cut is made on one side only to reduce sensitivity to the location of the interaction point moving around.

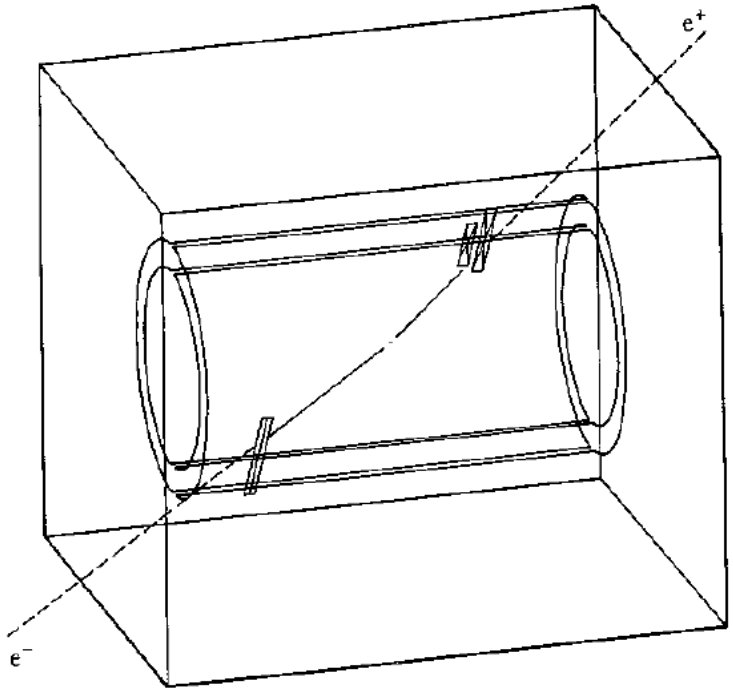

u
u
d
W+
e+
ν_e
$\bar{d}$
$\bar{u}$
$\bar{u}$
Proton
Antiproton

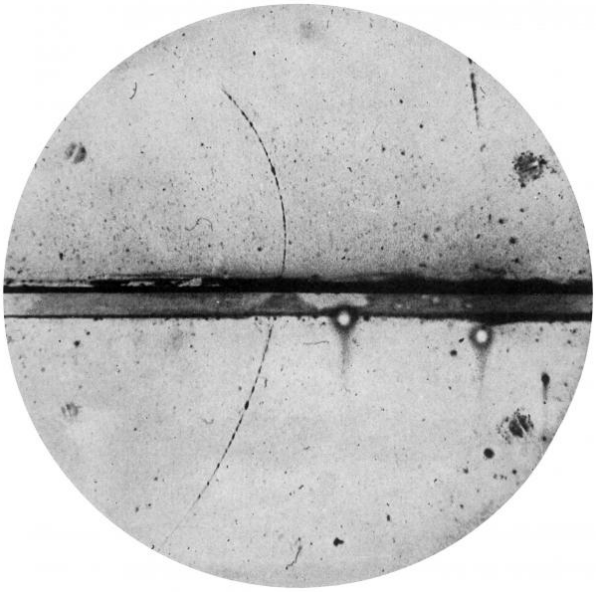

u
u
d
Z^0
e+
e−
$\bar{u}$
$\bar{u}$
$\bar{d}$
Proton
Antiproton


Large-angle calorimeter and magnet yoke
Large-angle shower counter
Muon detector
End-cap shower counter
End-cap calorimeter
Aluminium coil
Central detector


1.6
1.2
0.8
0.4
0
$(1 + \cos\theta_e^*)^2$
$(1/N)\, dN/d\cos\theta_e^*$
−1.0 −0.6 −0.2 0.2 0.6 1.0
$\cos\theta_e^*$




e+
e−



The Oxford MPhys Course

C4: Particle Physics

Problem Set Six

1. Neutral currents were first observed in the process $\overline{\nu}_\mu e^- \to \overline{\nu}_\mu e^-$, where the $\overline{\nu}$ were generated from $\pi^- \to \mu^- \overline{\nu}_\mu$ decays and the e^- were atomic electrons in a freon bubble chamber.

(a) Why is this evidence for neutral currents whereas the process $\overline{\nu}_e e^- \to \overline{\nu}_e e^-$ is not?
Show that, if the recoil electron has energy E_e ($\gg m_e c^2$) then its angle to the neutrino beam direction must be less than

$$\sqrt{\frac{2m_e}{E_e}}$$

whatever the energy of the incident neutrino.

(b) The neutral current consists of vector and axial vector components, but unlike the charged current, not in equal amounts. The differential cross sections can be written as

$$\frac{d\sigma(\overline{\nu})}{dy} = \frac{G^2 s}{\pi}\left[g_L^2(1-y)^2 + g_R^2\right]$$

and

$$\frac{d\sigma(\nu)}{dy} = \frac{G^2 s}{\pi}\left[g_L^2 + g_R^2(1-y)^2\right],$$

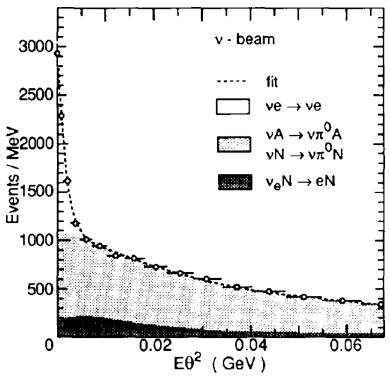
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where $y = E_e/E_{\nu(\overline{\nu})}$, $\sqrt{s}$ is the CMS energy and g_L and g_R are the couplings to LH and RH electrons respectively. If the incident ν ($\overline{\nu}$) are LH (RH), justify the distributions in y by angular momentum considerations.

(c) We can also write the y-dependence in terms of V and A couplings $g_L = \frac{1}{2}(c_V + c_A)$ and $g_R = \frac{1}{2}(c_V - c_A)$. In the standard model, $c_A = -\frac{1}{2}$ and $c_V = -\frac{1}{2} + 2\sin^2\theta_W$, where θ_W is the electroweak mixing angle. The CHARM II experiment at CERN measured neutral current scattering $\nu_\mu e^- \to \nu_\mu e^-$ and $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ (PLB 335 (1994) 246). The results are summarised in the table below. The integrated fluences for the ν_μ and $\bar{\nu}_\mu$ were equal. Use this data to estimate a value and uncertainty for $\sin^2\theta_W$.

Reaction	Number of events	Background fraction
$\nu_\mu e^- \to \nu_\mu e^-$	2105	0.176
$\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$	2215	0.221

(d) The figure shows the distribution of $E\theta^2$ from the CHARM II experiment where E is the energy of the scattered electron and θ is the angle between the electron and the incident neutrino. With reference to your answer to part (a), explain how the signal can be extracted from the background. Explain the origin of the main sources of backgrounds to the neutral current signal.



2. Consider the process $e^+e^- \to Z \to f\overline{f}$ at a centre of mass energy equal to the mass of the Z boson.

(a) Assuming helicity conservation at the vertices, explain why there are only 4 allowed spin combinations. Illustrate these allowed spin configurations for the incoming and outgoing fermions and on the same diagrams show the direction of the incoming and outgoing fermions. Let $\theta^\star$ be the angle between the directions of the outgoing fermion with respect to the incoming e^- in the centre of mass frame. Determine the distributions as a function of the angle $\theta^\star$ for the different spin configurations.

(b) Let g_L^e and g_R^e be the Standard Model couplings for left and right-handed electrons to the Z boson. In the Standard Model the coupling constants for electrons are

$$\begin{aligned} g_L^e &= \sin^2\theta_W \\ g_R^e &= -\frac{1}{2} + \sin^2\theta_W \end{aligned}$$

where θ_W is the weak mixing angle. Determine the angular distribution for this process when the outgoing fermion, f, is a muon, assuming that helicity is the same as chirality. (hint: assume lepton universality).

(c) The forward-backward asymmetry is defined as

$$A_{FB} = \frac{N_F - N_B}{N_F + N_B}$$

where $N_F(N_B)$ is the number of events with $\cos\theta^\star > 0$ ($\cos\theta^\star < 0$). Determine the value of A_{FB} for this case as a function of the weak mixing angle, θ_W. Explain why a non-zero value of A_{FB} is evidence for parity violation and hence explain the value of A_{FB} for the case that $g_L^e = g_R^e$. The measured value from the L3 experiment at LEP is $A_{FB} = 0{\cdot}0188 \pm 0{\cdot}0033$.

3.

(a) Draw Feynman diagrams for high-energy fermion interactions with gauge-boson vertices allowed by the Standard Model, including helicity dependence. Which are relevant for $e^+e^- \to \mu^+\mu^-$?

(b) For E^+e^- collisions at a CMS of energy $\sqrt{s} \ll M_Z$ where M_Z is the mass of the Z, the ratio of cross sections for pure photon exchange to pure Z exchange is

$$R = \frac{\sigma_\gamma}{\sigma_Z} = \left(\frac{\sqrt{s}}{M_Z}\right)^{-4}$$

Give a simple qualitative explanation for the form of this expression. Make an estimate of this ratio for a CMS energy $\sqrt{s} = 35$ GeV. Explain why the interference term is important and why this results in a forward-backward asymmetry for the outgoing μ^+.

(c) By considering rotation matrices and relevant couplings, or otherwise, show that the angular distribution for $e^+e^- \to \mu^+\mu^-$ is not symmetric at centre-of-mass energies equal to the Z^0 mass.

[At these energies the equivalence of chirality and helicity may be assumed.]

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4. The Z^0 boson decays into particle-antiparticle pairs of neutrinos, charged leptons and quarks $u,\ d,\ s,\ c$ and b. In the Standard Model, the partial width of Z^0 decay into fermions f of negligible mass is given by

$$\Gamma_f = \frac{G_F\sqrt{2}}{12\pi} M_Z^3[(c_V^f)^2 + (c_A^f)^2)]\,,$$

where G_F is the Fermi coupling constant, M_Z is the Z^0 mass and the vector and axial vector coupling constants c_V^f and c_A^f are given in the table:

	$\nu\bar{\nu}$	$\ell^+\ell^-$	$\bar{u}u,\ \bar{c}c$	$\bar{d}d,\ \bar{s}s,\ \bar{b}b$
$2c_V^f$	1	1	1	1
$2c_A^f$	1	$1-4\sin^2\theta_W$	$1-\frac{8}{3}\sin^2\theta_W$	$1-\frac{4}{3}\sin^2\theta_W$

(a) Explain:

- (a) why G_F appears to the first power in the expression for Γ_i;
- (b) why there are two sets of coupling parameters a_i and b_i and identify these parameters in the notation of Q7 of problem set 3 (from MT);
- (c) why the b_i for $\nu\bar{\nu}$ channels is different from the b_i for other channels and explain the appearance of the parameter $\sin^2\theta_W$;
- (d) why transitions such as $Z^0 \to s\bar{d}$ are forbidden.

(b) Calculate the total decay width Γ of the Z^0 and the quantities $R_b = \Gamma_b/\Gamma_h$ and $R_c = \Gamma_c/\Gamma_h$ (where h stands for all hadrons).

(c) Compare your results with measured values[1]

$$\Gamma_Z = 2{\cdot}4952 \pm 0{\cdot}0023\,\text{GeV}, \quad R_b = 0{\cdot}21629 \pm 0{\cdot}00066 \ \text{ and } \ R_c = 0{\cdot}1721 \pm 0{\cdot}0030.$$

Why is R_c less well measured than R_b? Suggest what might account for the difference between your estimates and the measured values.

5. The process

$$e^+ + e^- \to W^+ + W^-$$

has been studied at energies close to threshold.

(a) Draw the first-order diagrams of processes which are expected to contribute.

(b) Estimate the branching ratios for W decay, and hence calculate the fractions of W pair events containing 0, 1 and 2 leptonic decays of the Ws. Discuss how these events could be identified in a typical LEP experiment.

(c) Show that the cross section $\sigma(e^+e^- \to W^+W^-)$ close to threshold is proportional to the centre-of-mass velocity β of the W. Assume that the variation in the matrix element with energy is negligible in this energy range, so the calculation just needs to consider the phase space factor. (See PS1).

[1]The values are taken from the PDG 2008 and are the result of combining high precision e^+e^- measurements from the LEP experiments and from the SLAC Linear Collider Detector.

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(d) The cross section is given by

$$\sigma(e^+e^- \to W^+W-) \simeq \frac{2G_F^2 M_W^4}{\pi s}\sqrt{1-\frac{4M_W^2}{s}},$$

where G_F is the Fermi coupling constant, M_W the W mass, and s the centre-of-mass energy squared, all in natural units. Express σ in terms of s and β. With e^+ and e^- beams of energy 80·65 GeV, a cross section for all-hadronic decay modes of 1·7 pb is obtained. Hence calculate a value of the W mass.

6. Consider an e^+e^- collider experiment running at the Z^0 resonance centre of mass energy ($\sqrt{s} \approx 90$ GeV).

(a) Give a brief account of detectors that are used for tracking and for identification of e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$ and hadronic events.

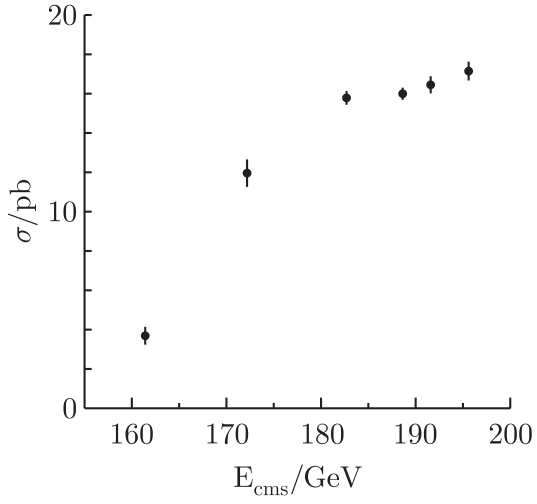
(b) How is luminosity measured?

(c) In an experimental run at the Z^0 peak energy, the integrated luminosity was 23·955 pb^{-1}. After corrections for background and detector efficiencies, a total of 993 797 hadronic events and 47 838 $\mu^+\mu^-$ events were obtained. Stating your assumptions, calculate the number of neutrino flavours including an estimate of its uncertainty. Comment on your result.

7. past paper question.
Pairs of W particles may be produced in electron-positron collisions.

(a) Draw Feynman diagrams illustrating possible production processes. [3]

(b)



The figure shows data for the cross-section for the production of W pairs in e^+e^- collisions. Obtain a rough estimate of the W mass. [3]

(c) List the possible decay modes of the W boson and give approximate branching ratios for (a) purely hadronic decays and (b) various leptonic modes. [4]

(d) Draw sketches to illustrate the appearance of W-pair events in a typical tracking detector for the cases where: [6]

(i) both W particles decay hadronically;

(ii) both W particles decay leptonically;

(iii) one decays leptonically and the other hadronically.

(e) With the LEP collider running with the beams each at 94·5 GeV a given W-pair event results in two jets and a muon. The vectors of their 3-momenta $\mathbf{p}$ are as follows (all components are in GeV/c):

	p_x	p_y	p_z
Jet 1	9·5	−64·4	6·2
Jet 2	−22·8	18·2	4·1
Muon	14·8	−21·0	−7·6

Using the above information determine the mass of the unobserved particle in this reaction. Assuming that the masses of jet 1, jet 2 and the muon can be neglected use the above data to produce two further estimates of the W mass. [9]

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