Geometric Optics

Problems labeled (*) are essentially standard derivations. These come up regularly on Prelims papers and are well worth learning.

A Standard Questions

1. *Describe Huygens's principle and use it to derive Snell's law of refraction.

2. Show how the critical angle θ_c depends on n_1 and n_2 and calculate values for light leaving ordinary glass ($n_1 \approx 1.5$), lead crystal ($n_1 \approx 1.8$), and diamond ($n_1 \approx 2.4$) to enter air ($n_2 \approx 1$). Comment on the relevance of these results to the design of costume jewelery.

3. An optic fibre can be modeled as a cylindrical core, refractive index n_c, surrounded by material with refractive index n_s. The ends of the fibre are plane surfaces at right angles. A ray of light from an external medium of refractive index n_1 enters one end of the fibre at a point on the axis and at an angle of incidence θ_i. Show how the maximum possible value of $n_1 \sin\theta_i$ (the *numerical aperture* of the fibre) depends on n_c and n_s.

4. A heron is standing in a stream of water ($n = 4/3$) observing a fish swimming almost beneath it at a depth of 40 cm. At what depth will the fish appear to be?

5. *A thin symmetric biconvex lens with surfaces of radius of curvature R_1 and R_2 is made of glass with refractive index n_2 and is used in a medium of refractive index $n_1 < n_2$. Show that an object at a distance u from the lens will produce a real image at a distance v, given by $1/u + 1/v = 1/f$ where
$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
(the lens-maker's formula) making clear any assumptions or approximations.

6. If such a lens has $n = 1.60$, by what factor does the focal length of the lens change when using it in water ($n = 1.33$) instead of air?

7. Consider a light source placed at a fixed distance s from a screen, such that a lens of focal length f can be placed between the source and the screen. Show that as long as $f < f_{max}$ then there are two positions where the lens can be placed so that an image is formed on the screen, and find a value for f_{max}.

8. A convex lens has a radius of 20 mm and a focal length of 100 mm. A point source of light is placed 500 mm behind the lens and a screen is placed 120 mm in front of it. Calculate the size of the illuminated spot on the screen. (Neglect effects due to diffraction at this stage).

9. The human eyeball is approximately spherical with a diameter of around 25 mm. At the back of the eye is the retina. At the front is a compound lens comprising the cornea (of fixed focal length) and the lens itself (with variable focal length), which can together be approximated as a thin lens at a distance of 17 mm from the retina. The eye is capable of focusing on objects

at distances varying from the near point (about 25 cm) to infinity. Estimate the range of lens powers which can be achieved by the lens and cornea in combination. A short-sighted optics lecturer can only focus on objects up to 50 cm away. What type and power of contact lens does he need to correct his sight?

10. *Explain how a simple magnifying glass can be used to create an enlarged virtual image of an object, and calculate the magnification of the image.

11. Leeuwenhoek constructed simple microscopes by using small glass spheres as extremely powerful magnifying glasses. Assuming the thin lens formula can be applied in this case find how the magnification depends on the radius of the sphere, and estimate the magnification achievable with a sphere of radius 1 mm. (Take $n = 3/2$.)

12. *Derive the mirror equation $1/u + 1/v = 1/f = 2/r$ for the formation of an image by a spherical mirror of radius r.

13. The Meade ETX80 refracting telescope has an 80 mm diameter objective lens with a focal length of 400 mm. It is sold with two eyepieces of focal lengths 9.7 mm and 26 mm. Calculate the magnifications achievable and the size of the exit pupil in each case.

14. A compound microscope has an objective lens with focal length f_o and an eyepiece with focal lens f_e separated by a fixed length L. Show that the total magnification is approximately given by
$$M = \frac{LD}{f_o f_e} \tag{1}$$
stating any approximations you make. For the case L=200 mm, f_o=10 mm and f_e=20 mm compare this magnification with the exact magnification actually achieved if the final virtual image is placed at infinity, and calculate the distance between the object and the objective lens. (You may assume that all lenses are thin.)

B Slightly more interesting questions

1. Total internal reflection inside glass prisms is used in many optical instruments as an alternative to using mirrors. Using a diagram show that a symmetric right angle glass prism in air will reflect a beam of light entering the prism at right angles to the hypotenuse. What happens to light entering at a small angle θ from the normal?

2. Consider a beam of light traveling through air (take $n = 1$) above an optical table which strikes a cylinder of refractive index μ sitting on the table with its long axis perpendicular to the table. Viewed from above the light strikes the cylinder at point A, making an angle of α to the surface normal, is refracted and is then partially reflected off the back of the cylinder at point B, and finally leaves the front of the cylinder at point C, making an angle γ to the normal. Use simple geometry to show that $\gamma = \alpha$, and find the angle δ between the incoming and outgoing beams.

3. Why does a diving mask help you see more clearly underwater? How do prescription swimming goggles work in both air and water?

4. Use Fermat's principle to show that a thin lens with thickness given by $w = w_0 - \alpha y^2$, where y is the distance from the axis, will focus paraxial rays from an object on the axis, and find the focal length as a function of α and the refractive index n. [Hint: the best way to interpret "paraxial" is to assume that (a) values of y are small compared with distances measured along the axis, and (b) light rays can therefore be taken to be parallel to the axis when they are within the thin lens.] Relate this result to the lens maker's formula for a biconvex lens with radii r_1 and r_2.

5. Starting from $1/u + 1/v = 1/f$ derive Newton's formula $x_o x_i = f^2$, where x_o and x_i are the distances of the object and the image respectively from the corresponding focal points, and find a formula for the magnification of a lens in terms of f and x_o. Does Newton's formula work for virtual images? Does it work for diverging lenses?

6. The Hubble Space Telescope is a Cassegrain design reflecting telescope. The primary mirror has a diameter of 2.4 m and a radius of curvature of 11.04 m while the secondary mirror has a radius of curvature of -1.36 m. The two mirrors are separated by 4.91 m. Calculate (a) the focal lengths of the mirrors; (b) the diameter of the secondary mirror; (c) the position of the focus point; (d) the effective focal length of the telescope. (Assume that only perfectly axial rays are observed.)

(Problems reproduced courtesy of Jonathan A. Jones)

Wave Optics 2016

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A Standard Questions

1. *In a Young's slits experiment a screen with two narrow slits of separation d is illuminated normally by coherent light of wavelength λ. The resulting pattern is viewed on a screen at a distance $D \gg d$. Derive an expression for the intensity distribution on the screen.

2. In a Young's slit experiment as described above, green light ($\lambda = 550\,\text{nm}$) is used. With $D = 0.7\,\text{m}$ the distance between the zeroth (central) maximum and the fifth maximum is found to be 7 mm. Calculate the separation of the slits.

3. A thin sheet of glass (0.2 mm thick, $n = 1.66$) is placed over one of the slits, on the directly illuminated side. What happens to the fringe pattern?

4. The sheet of glass is now removed and the green light source is replaced by a source producing three sharp lines at 450 nm (blue), 550 nm (green) and 650 nm (red) with equal intensities. Sketch the three different fringe patterns and hence describe the overall appearance of the fringes.

5. *A parallel beam of monochromatic light of wavelength λ is directed normally on to a reflection grating with line spacing d. Show that maxima in the intensity of the diffracted light occur at angles θ to the normal such that $n\lambda = d\sin\theta$ where n is an integer. How does the resulting pattern differ from that in a Young's slits experiment?

6. *Describe a practical arrangement for using a diffraction grating as a spectrometer.

7. A grating is used in normal incidence and third order to observe the red line ($\lambda = 656.3\,\text{nm}$) from a hydrogen discharge at a diffraction angle of 36.2°. What is the ruling spacing (in lines per mm) of this grating? What angle would the first order line occur at? The same grating is used to observe the sodium D lines ($\lambda = 589.0\,\text{nm}$ and $\lambda = 589.6\,\text{nm}$). Calculate the angular separation of the two lines at first order and at the largest order which can be observed.

8. *Derive the angular intensity distribution in the diffraction pattern when a slit of width w is illuminated normally with light of wavelength λ, and hence find the Rayleigh (diffraction) limit for a single slit.

9. *Why does diffraction limit the image formed by a lens? State the Rayleigh resolution limit for a circular lens.

B Slightly more interesting questions

1. A convex lens has a radius of 20 mm and a focal length of 100 mm. A point source of light is placed 500 mm behind the lens and a screen is placed 120 mm in front of it. Calculate the size of the illuminated spot on the screen. (Neglect effects due to diffraction at this stage). Now calculate the distance at which the screen should be placed to get the best possible image, and estimate the spot size in this case.

2. The pupil of the human eye is normally around 5 mm in diameter. A person with 20/20 vision should be able to read letters 9 mm high at a distance of 20 m, while in Chapter two of *The Two Towers* Legolas can resolve objects of around 10 cm size at a distance of five leagues (about 24 km). Comment.

3. Satellite TV is transmitted at a frequency of around 15 GHz and received using dish antennae with a diameter of around 0.7 m. Estimate the accuracy with which it is necessary to align a satellite dish.

4. The Hubble space telescope (a reflecting telescope with a main mirror diameter of 2.4 m) was recently used to view the Apollo landing sites on the moon using near UV light ($\lambda \approx 300$ nm). The pictures did not show any evidence of the lunar modules. Should we conclude that the landings were a hoax?

5. We can calculate the diffraction pattern from two finite-width slits by brute force, using the same technique we used for a single slit, except that the integral is now run over *both* slits. Suppose that the two slits are of width b and positioned at distances $\pm d/2$ (with $b < d$ of course). Find the light amplitude and intensity as a function of diffraction angle (do the integral in two pieces) and comment on the form of your result.

(Problems reproduced courtesy of Jonathan A. Jones)