Measurement of the Rydberg Constant from the Hydrogen Balmer Spectrum

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Abstract

The visible emission spectrum of atomic hydrogen was investigated to determine the Rydberg constant and verify the quantization of atomic energy levels. A Czerny-Turner grating monochromator was first empirically calibrated using the known vacuum wavelengths of a mercury discharge lamp, employing a quadratic fit to accurately account for mechanical non-idealities in the instrument's sine-bar drive. Using this calibration, the wavelengths of the four visible Balmer series lines (H_α, H_β, H_γ, H_δ) emitted by a hydrogen discharge tube were measured. The Rydberg constant was calculated from a weighted mean of these transitions, yielding $R = 1.09828(22) \times 10^7$ m^{-1}. While this demonstrates good agreement with quantum mechanical predictions, the experimental value exhibits a slight systematic overestimation of 0.14% compared to the theoretical finite-mass constant, R_H, placing it numerically closer to R_∞. This minor discrepancy is attributed to systematic uncertainties, predominantly a uniform zero-offset shift and residual mechanical backlash in the micrometer during the exchange of the light sources.

1 Introduction

As quantum theory developed, energy levels of atoms were found to be quantized, and the emission spectra of atoms were found to consist of discrete lines. By measuring the wavelengths of these lines, the energy levels of atoms could be determined, and we can verify the predictions of quantum theory. In this experiment, we will use a mercury lamp as a source of light to calibrate a Czerny-Turner monochromator, and then measure the emission spectrum of a hydrogen lamp, and determine the wavelengths each line corresponds to.

2 Theory

In the Czerny-Turner monochromator, concave mirrors are used to collimate the beam of light falling on the grating and to focus those beams diffracted from the grating. The grating is mounted on a sine-bar mechanism, which allows the angle of the grating to be varied by turning a micrometer. By adjusting the angle of the grating, different wavelengths of light can be diffracted into the exit aperture, allowing us to select a specific wavelength for measurement.

According to [1], the wavelength λ of the light diffracted by the grating is given by

$$\lambda = z\left(\frac{2d\cos\gamma}{pl}\right), \tag{1}$$

where z is the sine-bar micrometer reading, d is the grating spacing, γ is the Czerny–Turner angle, p is the diffraction order, and l is the sine-bar length.

For the grating used in [1], the spacing is $d \approx 1/(1200\ \text{mm}^{-1}) = 833$ nm, which is of the same order as visible wavelengths ($\lambda \sim 400$–700 nm). This confirms that first-order diffraction ($p = 1$) produces the observed visible lines.

According to the Bohr model of the hydrogen atom, the wavelength of the light emitted when an electron transitions from a higher energy level n_i to a lower energy level n_f is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), \tag{2}$$

where R is the Rydberg constant. By measuring the wavelengths of the spectral lines of hydrogen, we can determine the value of the Rydberg constant and compare it with the theoretical value.

In this experiment, we will measure the wavelengths of the four visible Balmer lines of the hydrogen spectrum, which correspond to transitions from $n_i = 3, 4, 5, 6$ to $n_f = 2$. By substituting these values into equation (2), we can calculate the Rydberg constant R for each line and take a weighted mean to obtain a more accurate value.

3 Experimental Setup

3.1 Calibration

The monochromator was calibrated using the known vacuum wavelengths of the mercury emission spectrum. Although the sine-bar mechanism provides an approximately linear relation between micrometer reading z and wavelength λ, mechanical imperfections and backlash in the drive mean that an empirical calibration is required.

With the mercury lamp aligned as described in figure 1, each visible spectral line was brought into coincidence with the cross-hair in the exit aperture. For each line, the micrometer was always approached from the same direction to eliminate backlash errors in the sine-bar mechanism.

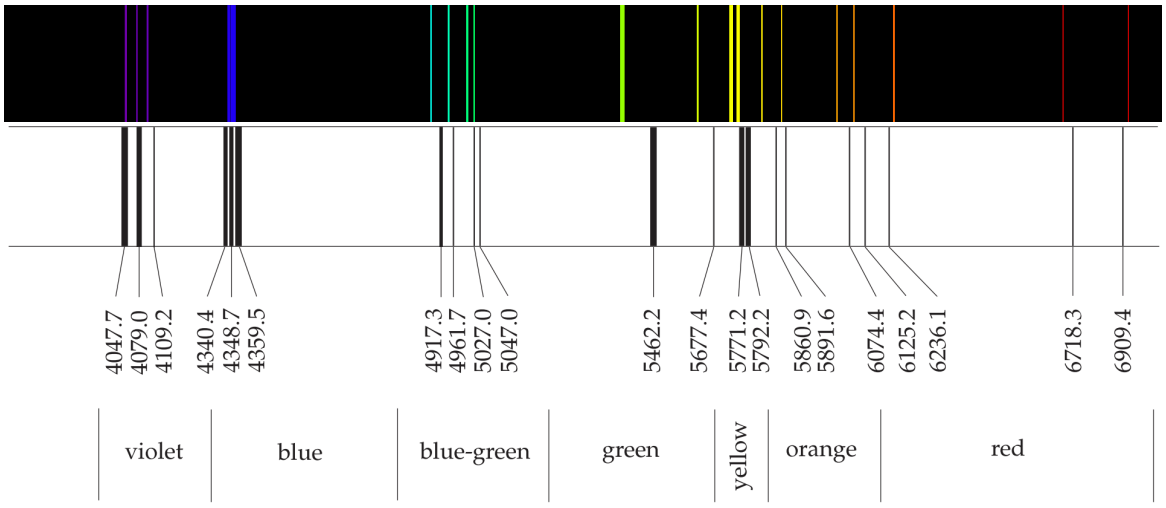


Figure 1: Corresponding mercury spectrum [1]

Three readings were taken for each spectral line and averaged. The data were recorded in table 3. The slit width was kept fixed throughout the calibration procedure and was not altered subsequently, in order to avoid systematic shifts in the apparent line centre.

The averaged micrometer readings were then paired with the corresponding known vacuum wavelengths to establish the calibration relation between z and λ.

3.2 Measurement of the Hydrogen Spectrum

After calibration, the mercury lamp was removed and replaced with the hydrogen discharge tube. The condensing lens and slit settings were left unchanged to ensure consistency with the calibration procedure.

The hydrogen source was aligned so that a uniform patch of light fell centrally on the grating. The cross-hair position was not altered after calibration, in order to preserve the established wavelength reference.

The four visible lines were identified and brought successively into coincidence with the cross-hair. As in the calibration procedure, all micrometer readings were approached from the same direction to minimise backlash effects.

For each spectral line, multiple readings were taken and averaged. The data were recorded in table 4.

4 Analysis

4.1 Calibration

We firstly performed a linear regression on the calibration data in table 3 using Python. The relationship is given by $z = a\lambda + b$, where a and b are the regression coefficients. The fitted line is shown in figure 2. The residual plot is shown in figure 3.

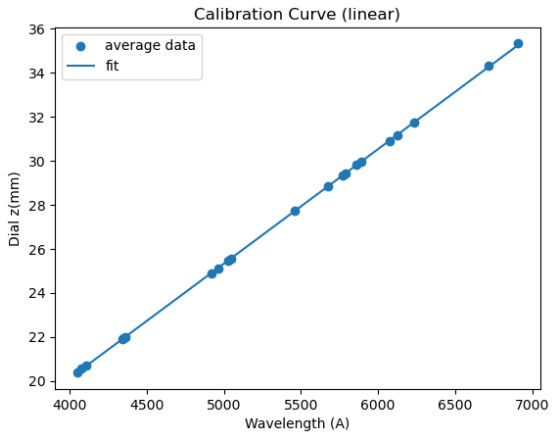


Figure 2: Calibration plot of the linear regression for calibration.

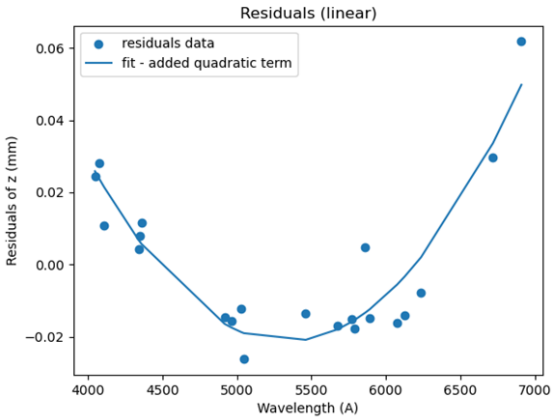


Figure 3: Residual plot of the linear regression for calibration.

It is obvious that the linear model fits the calibration data well, but the residuals show a quadratic pattern. This suggests that we should use a quadratic model to fit the calibration data. We then performed a quadratic regression on the calibration data using Python. The relationship is given by $z = a\lambda^2 + b\lambda + c$, where a, b and c are the regression coefficients. The fitted line is shown in figure 4. The residual plot is shown in figure 5. The residuals are randomly distributed around zero, indicating that the quadratic model is appropriate for the calibration data.

From figure 5, we cannot see any pattern in the residuals, which suggests that the quadratic model fits the calibration data correctly. The residuals in z are typically within $\pm 0.01mm$, comparable to the micrometer reading resolution; when propagated through the calibration this corresponds to a wavelength uncertainty of order a few Å, consistent with the monochromator resolution. Therefore, we can conclude that the quadratic model is appropriate for the calibration data. $R^2 = 0.999$ also indicates the goodness of fit of the quadratic model.

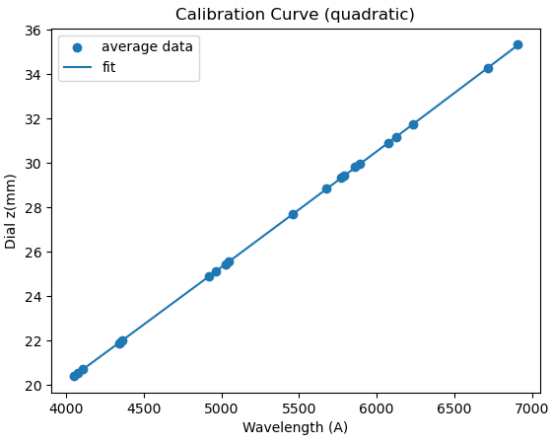


Figure 4: Calibration plot of the quadratic regression for calibration.

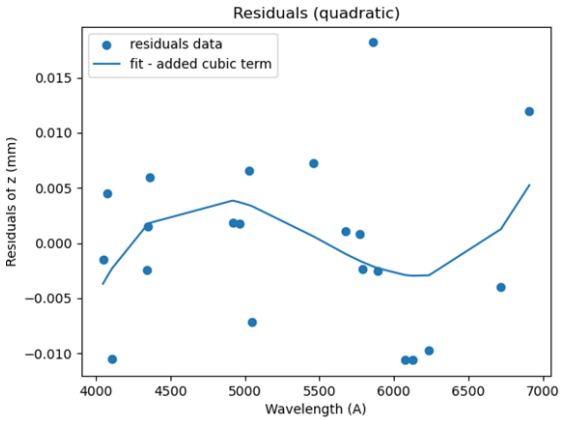


Figure 5: Residual plot of the quadratic regression for calibration.

From the model we can obtain the three coefficients: $a = 2.856(251) \times 10^{-8}$, $b = 4.900(27) \times 10^{-5}$ and $c = 1.017(704) \times 10^{-1}$. These coefficients can be used to convert the micrometer readings of the hydrogen spectrum to wavelengths.

4.2 Hydrogen wavelengths

Since the calibration fit was performed in the form $a\lambda^2 + b\lambda + (c - z) = 0$, the hydrogen wavelengths were obtained by analytically inverting this quadratic relation for $\lambda(z)$, and the positive root was chosen since the negative root gives unphysical wavelengths. Using the quadratic model obtained from the calibration, we can convert the micrometer readings of the hydrogen spectrum to wavelengths. The results are shown in table 1.

Table 1: Determination of the Rydberg constant from hydrogen spectra

Line	n	z (mm)	λ (Å)	R ($10^7\,\mathrm{m}^{-1}$)	σ_R ($10^3\,\mathrm{m}^{-1}$)
H_α	3	33.49(1)	6560(2)	1.09756	3.35
H_β	4	24.56(1)	4852(2)	1.09920	4.53
H_γ	5	21.90(1)	4338(2)	1.09772	5.06
H_δ	6	20.64(1)	4093(2)	1.09944	5.37

The wavelength uncertainty was taken as the quadrature sum of the reading uncertainty in z propagated through the calibration curve, $\sigma_{\lambda,z} = |d\lambda/dz|\sigma_z$, and the scatter of the mercury calibration about the fitted curve, $\sigma_{\lambda,\mathrm{cal}}$. In practice this is comparable to the instrument resolution, giving $\sigma_\lambda \approx 2$ Å. Since we obtained σ_R for each spectral line, a weighted mean of R was calculated. The resulting value is

$$\boxed{R = 1.09828(22) \times 10^7\ \mathrm{m}^{-1}}.$$

The uncertainty has been rounded to two significant figures, and the central value has been expressed to the corresponding precision.

4.3 Theoretical prediction

From values recommended in [2],

$$R_\infty = 1.0973732 \times 10^7\ \mathrm{m}^{-1}, \qquad R_H = 1.0967758 \times 10^7\ \mathrm{m}^{-1}.$$

Our measured R is higher than R_H by approximately 0.14% and higher than R_∞ by approximately 0.082%, suggesting a net systematic bias.

4.4 Result comparisons

The experimental value of the Rydberg constant is in excellent overall agreement with theoretical predictions. However, our result is slightly higher ($\approx 0.14\%$) than the theoretical R_{H}. This small discrepancy is predominantly driven by systematic errors in the experiment, rather than statistical variance.

To illustrate this, consider a constant zero-offset error of $\Delta z = +0.02$ mm in the micrometer reading due to source replacement. Re-evaluating the wavelengths and resulting R values under this uniform shift yields the adjusted values shown in Table 2.

Table 2: Recalculated Rydberg constant assuming a systematic shift of $\Delta z = +0.02$ mm.

Line	n	$z + \Delta z$ (mm)	λ' (Å)	R ($10^7\,\mathrm{m}^{-1}$)	σ_R ($10^3\,\mathrm{m}^{-1}$)
H_α	3	33.51(1)	6564(2)	1.09689	3.34
H_β	4	24.58(1)	4856(2)	1.09829	4.52
H_γ	5	21.92(1)	4342(2)	1.09670	5.04
H_δ	6	20.66(1)	4097(2)	1.09836	5.35

This conservative systematic shift yields a new weighted mean of $R \approx \boxed{1.09780 \times 10^7\ \mathrm{m}^{-1}}$, demonstrating how sensitive the calculated constant is to minor mechanical offsets in the scale.

5 Conclusion

In this experiment, the discrete Balmer emission lines of hydrogen were measured using an empirically calibrated Czerny–Turner monochromator. A quadratic calibration was required to account for small mechanical non-linearities in the sine-bar mechanism. The resulting weighted mean value of the Rydberg constant is

$$R = 1.09828(22) \times 10^7\ \mathrm{m}^{-1},$$

which agrees with quantum mechanical predictions at the 10^{-3} level. The measured value lies approximately 0.14% above the finite-mass constant R_H, indicating that the dominant uncertainty is systematic rather than statistical.

Although the result is consistent with R_H once systematic effects are considered, the present precision does not allow a clean experimental separation between R_H and R_∞. The reduced-mass shift is therefore not resolved at high significance in this measurement.

The observed offset is most plausibly explained by a small micrometer zero error, residual mechanical backlash, or minor changes in illumination geometry when switching between the mercury and hydrogen sources. A uniform offset of $\Delta z = +0.02\,\mathrm{mm}$ shifts the weighted mean to $R = 1.09780 \times 10^7\ \mathrm{m}^{-1}$, partially accounting for the discrepancy but not fully eliminating it.

Future improvements would include explicit zero-order alignment to determine the micrometer reference position, stricter control of the slit geometry, and increased calibration density near the Balmer wavelengths to reduce local interpolation error.

References

[1] *OP13: The grating monochromator*, Year 1 Physics Lab Script, University of Oxford.

[2] NIST, *Fundamental Physical Constants (CODATA 2022 recommended values)*, `https://physics.nist.gov/cuu/Constants/` (accessed 12 Mar 2026).

Appendix

Table 3: Spectrometer calibration measurements (mercury lamp)

Wavelength (Å)	Reading 1 (mm)	Reading 2 (mm)	Reading 3 (mm)	Average (mm)
4047.7	20.39(1)	20.39(1)	20.42(1)	20.40(1)
4079.0	20.55(1)	20.57(1)	20.58(1)	20.57(1)
4109.2	20.69(1)	20.72(1)	20.71(1)	20.71(1)
4340.4	21.90(1)	21.92(1)	21.89(1)	21.90(1)
4348.7	21.95(1)	21.95(1)	21.95(1)	21.95(1)
4359.5	22.00(1)	22.02(1)	22.01(1)	22.01(1)
4917.3	24.88(1)	24.89(1)	24.89(1)	24.89(1)
4961.7	25.12(1)	25.12(1)	25.11(1)	25.12(1)
5027.0	25.46(1)	25.46(1)	25.46(1)	25.46(1)
5047.0	25.56(1)	25.54(1)	25.55(1)	25.55(1)
5462.2	27.73(1)	27.72(1)	27.72(1)	27.73(1)
5677.4	28.84(1)	28.84(1)	28.84(1)	28.84(1)
5771.2	29.32(1)	29.33(1)	29.34(1)	29.33(1)
5792.2	29.43(1)	29.44(1)	29.44(1)	29.44(1)
5860.9	29.82(1)	29.81(1)	29.82(1)	29.82(1)
5891.6	29.96(1)	29.95(1)	29.96(1)	29.96(1)
6074.4	30.91(1)	30.89(1)	30.92(1)	30.91(1)
6125.2	31.17(1)	31.18(1)	31.17(1)	31.17(1)
6236.1	31.75(1)	31.75(1)	31.77(1)	31.76(1)
6718.3	34.29(1)	34.32(1)	34.30(1)	34.30(1)
6909.4	35.33(1)	35.33(1)	35.33(1)	35.33(1)

Table 4: Spectrometer hydrogen line measurements

Spectral Line	Reading 1 (mm)	Reading 2 (mm)	Reading 3 (mm)	Average (mm)
H_α	33.50(1)	33.50(1)	33.48(1)	33.49(1)
H_β	24.57(1)	24.55(1)	24.56(1)	24.56(1)
H_γ	21.90(1)	21.90(1)	21.91(1)	21.90(1)
H_δ	20.66(1)	20.62(1)	20.64(1)	20.64(1)