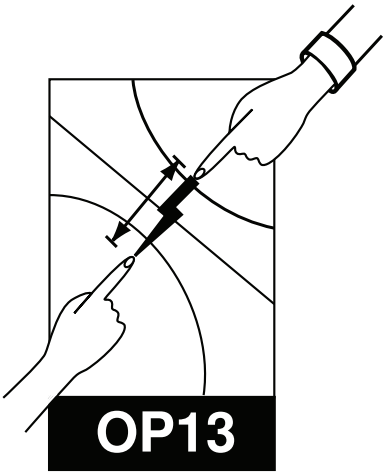
The grating monochromator



Edited MT, JJL January 2019

OP13



This experiment uses optical pointers or pins. Always take care when using sharp objects, especially when working in the low ambient light levels required in the Optics laboratory. Always leave the optical pins with their pointed end towards the wall and moved clearly out of the way when not in use.



Caution — be aware that the source lamp can get **very hot**, so please handle it with care.

1 Introduction

This experiment has two aims. Firstly, to give experience with experimental techniques which are widely applicable in spectroscopy. Secondly, you will investigate the spectrum and energy level structure of atomic hydrogen, including calculating the Rydberg constant for hydrogen. Your report should include a description of the aim of the experiment, the observations made and answers to the calculations and questions. If you do not understand any section of this manuscript please ask a demonstrator.

Before you start the experiment, a demonstrator will go through the important points and explain the optical system of the monochromator. You should, of course, have already read through the script before arriving in the lab.

1.1 Experimental steps

This lab will proceed in the following manner:

- Before lunch: We align the optics and calibrate the monochromator using the Mercury spectrum. This provides us with an instrument we understand.
- After lunch: We will then predict and measure the Hydrogen spectrum (using our calibration).

2 Spectroscope and monochromators

When a collimated beam of light of wavelength λ is incident on a reflection grating at an angle α, constructive interference occurs in beams diffracted at the angles β which satisfy the equation

$$d\,(\sin\alpha \pm \sin\beta) = p\lambda \tag{1}$$

as shown in figure 1. The order number p is usually a small integer and d is the spacing of the grating rulings.

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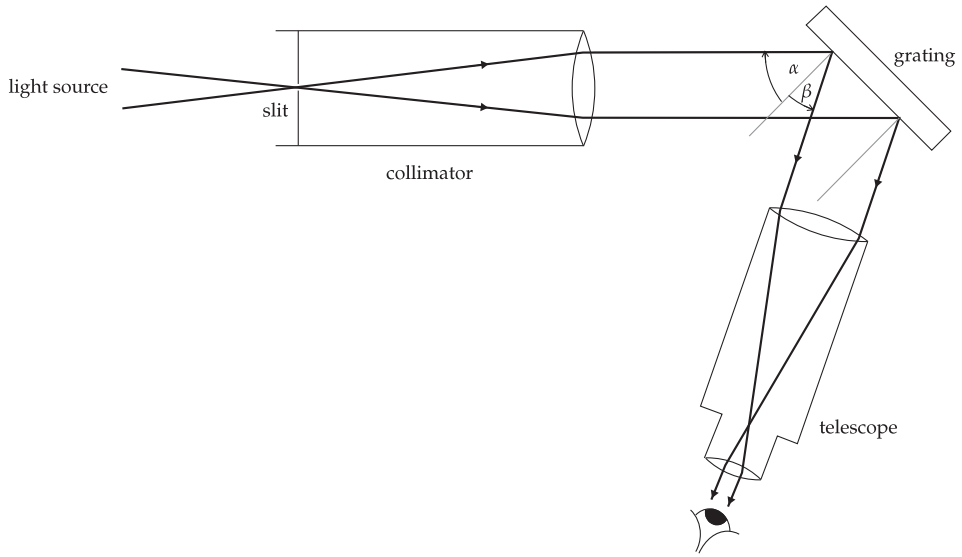


Figure 1: A simple spectroscope, with α and β as the angles of incidence and diffraction. Note that here, as in all instruments of this type, the grating consists not of slits but reflecting strips, that is, it works in reflection not transmission.

The +ve sign is used when the incident and diffracted beams are on the same side of the grating normal, and the -ve sign when they are on opposite sides of the normal.

In a grating spectroscope a system of lenses, forming a collimator and telescope, is used to produce an image of a fine entrance slit. When this slit is illuminated by light containing several different wavelengths then, due to constructive interference of light diffracted by the grating, the variously coloured images of the slit are formed at different angles. When the slit is narrow these images appear as bright lines on a dark background and are known as spectral lines. These lines are characteristic of the chemical composition of the source.

For the analysis of spectra and the determination of the spacing of atomic and molecular energy levels the highest possible accuracy in wavelength measurement is required. This is usually achieved by employing large spectrographs in which the spectra are recorded photographically. However in many experiments we want to study a line of which the wavelength is already known (for example, to see how its intensity varies with discharge conditions). Under these circumstances the wavelength measurement need only be made with sufficient accuracy to enable the line to be identified unambiguously. Thus for research purposes a less precise but more compact instrument is often more convenient. It is also desirable that the wavelength of the spectral line of interest can be determined directly from a scale or dial reading. All of these advantages are found in the Czerny-Turner monochromator on which this experiment is based.

3 The Czerny-Turner monochromator

In the Czerny-Turner monochromator, concave mirrors are used to collimate the beam of light falling on the grating and to focus those beams diffracted from the grating, as shown in figure 2. This means that the working range of the instrument is not restricted by the limited transmission of glass lenses in the ultra-violet region, and that there is no chromatic aberration. The grating is rotated by an arrangement known as a sine-bar which is driven by a large micrometer. The operation of the sine-bar is described more fully in section 4; it consists of a lever mechanism in which linear displacement by the micrometer is made proportional to the sine of the angle between the grating normal and the symmetry axis of

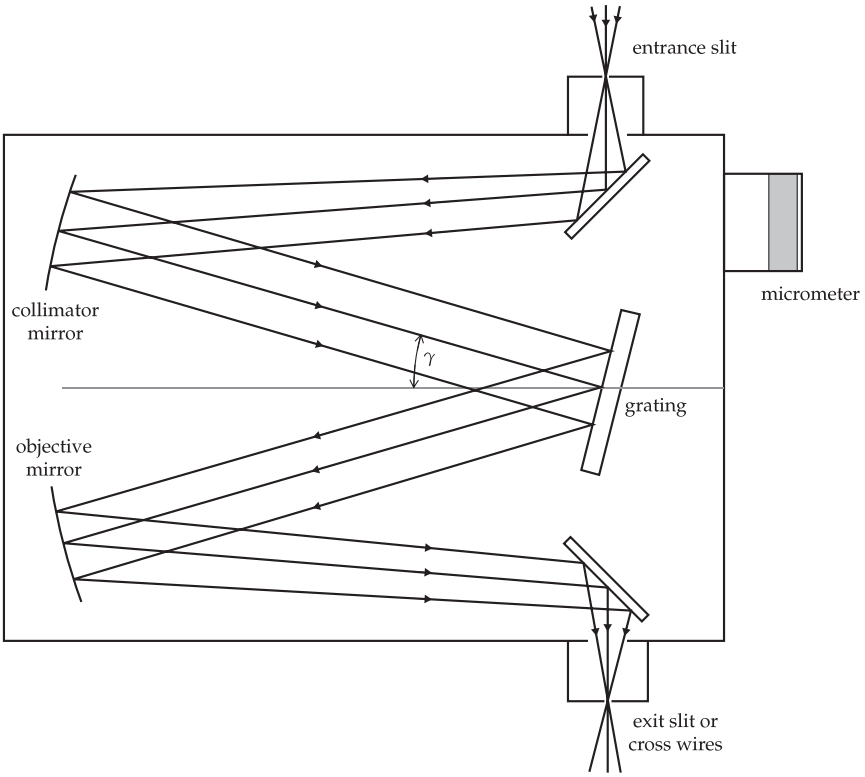


Figure 2: The Czerny-Turner monochromator, where γ is the Czerny-Turner angle.

Grating size	48 mm x 48 mm
Ruling frequency ($1/d$)	1200 lines/mm
Mirror diameter	63.5 mm
Mirror radius of curvature	439 mm
Czerny-Turner angle, γ	$25^o28'$
Length of sine-bar, l	75.24 mm

Table 1: Design parameters of the Czerny-Turner monochromator.

the instrument. An approximate value of the wavelength of any spectral line in Å (Angstrom) may, in this instrument, be obtained by doubling the micrometer reading, (1 Å = 10^{-1} nm = 10^{-10} m. Other important design parameters of these instruments are given in table 1.

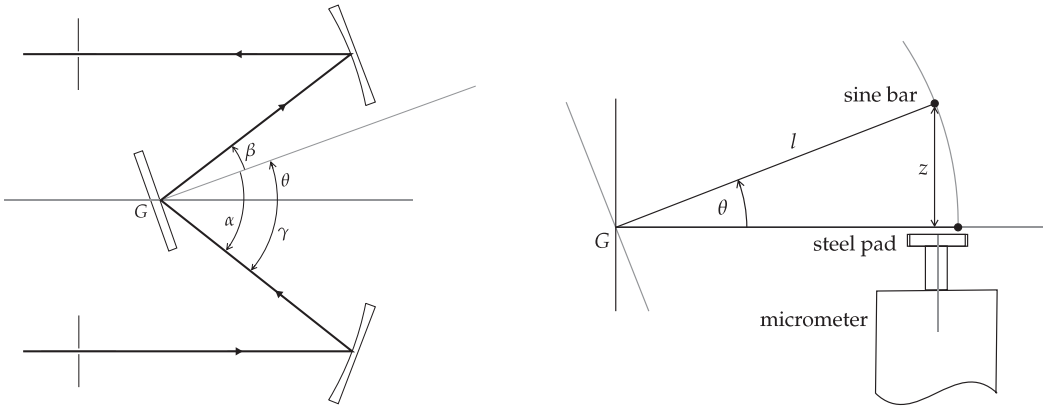
4 Theory

We return to equation 1 which gives the relation between the angles of incidence and diffraction α and β respectively, the grating spacing d, and the wavelength λ and order number p.

Equation 1 may be rewritten in terms of the Czerny-Turner angle γ (which is constant and determined by the mirror geometry) and the variable angle θ between the grating normal and the symmetry axis of the instrument. The result is

$$\sin\theta = \frac{p\lambda}{2d\cos\gamma}, \tag{2}$$

shown in in figure 3(a). The grating is rotated by a lever arm of length l which is moved by the linear



(a) Light ray path in spectrometer.

(b) Schematic of the sine-bar.

Figure 3: Further details of the Czerny-Turner monochromator, where G is the grating, γ the Czerny-Turner angle, θ the angle between the grating normal and the symmetry axis, and α and β are the angles of incidence and diffraction.

displacement z of the micrometer, as shown in figure 3(b). Hence we also have

$$\sin\theta = \frac{z}{l}. \tag{3}$$

Combining equations 2 and 3 gives

$$\lambda = z\left(\frac{2d\cos\gamma}{pl}\right). \tag{4}$$

▶ Using the data given above, calculate the combination $\frac{2d\cos\gamma}{l}$. How large is the grating line spacing d compared with a typical optical wavelength?

Thus the wavelength of light transmitted through the monochromator is a linear function of the displacement z produced by the micrometer drive. This simple proportionality is very convenient for setting on a given line to moderate accuracy. However, in practice, the sine bar drive cannot be relied upon to the precision of which the instrument is capable, so in the present experiment a calibration spectrum is used (section 6).

5 Focusing and alignment procedure

5.1 Focusing of collimator



Remember that you are responsible for your own safety when working in the laboratory. Always talk to a demonstrator or technician if you are concerned that anything may be unsafe.

To illuminate the grating with a parallel beam of light it is necessary to adjust the entrance slit to be at the focus of the collimating mirror, as follows:

Rotate the grating so that the zero-order is transmitted through the instrument. Reset the micrometer reading to zero when the zero order coincides with the cross-wire of the eyepiece.

Then rotate the grating to approximately 3200 on the micrometer. In this position the angle that the grating normal makes with the axis of the instrument is approximately equal to γ. Thus the zero order of the grating is reflected back towards the entrance aperture.

Slacken the brass lock-screw and take out the slit.

Find the focal point of the collimating mirror by the no-parallax method (see figure 4) using a steel needle held in a stand as the object. The side of the needle facing the monochromator should be illuminated by a small lamp.

Having found the focus you need to get the slit into this position. Replace it so that it coincides approximately with this focal point and open it to its widest setting.

Carry out the no-parallax procedure on the slit using its back-reflected image. When you have done this, tighten the brass lock screw **ensuring that the slit is vertical**.

Find the position of the grating at which the back-reflected image coincides exactly with the slit. Record the micrometer reading.

Using equation 3, determine the Czerny-Turner angle. Check that your result is close to the value given in section 3.

5.2 Alignment of the source

The next task is to illuminate the instrument properly using a mercury lamp and condensing lens. Two conditions need to be fulfilled. Firstly, the lamp and lens must be on the optical axis of the collimating system. Secondly, there must be a sharp image of the lamp on the slit.

In similar experiments there is often a third condition, which is to fill the grating with light by making sure the angle the lens subtends at the slit is large enough. This makes full use of the resolving power of the grating and the light-gathering ability of the instrument. However, if this is done here aberrations become a problem, and it is best instead to form an image of the source on the slit with approximately unit magnification. The grating is then not completely filled with light, but this is not serious provided that we satisfy the two conditions above.

Adjust the lens to the same height as the slit. Switch the mercury lamp on and form an image of it on the slit with approximately unit magnification (that is, the distance of the lamp from the lens is roughly equal to that from the lens to the slit). To do this, you will need to adjust the height of the lamp.

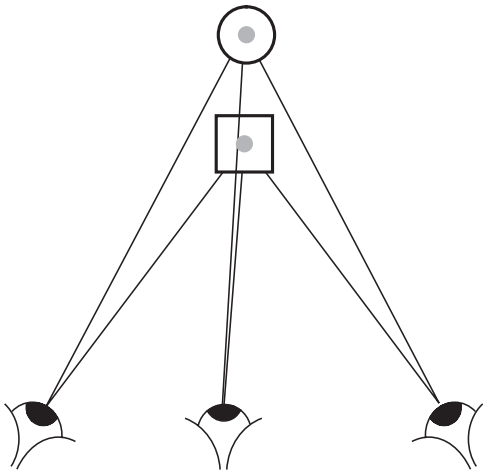
▶ Before you adjust the lamp, consult a demonstrator on how to perform this safely.

Rotate the grating so that the monochromator is set to transmit the strong green line 546 nm.

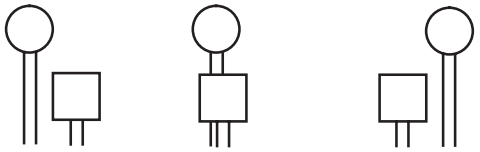
Remove the observing optic (brass cylinder containing the cross hair) from the view-port of the system.

Put your eye close to the instrument and look into the spectrometer (not at the image). Then adjust the lateral position of the source until light entering the spectrometer falls centrally on the grating.

Re-align the condensing lens between the source and entrance slit and bring an image of the source to a sharp focus on the slit. Put your eye close to the instrument again and look into it as before. There should be a uniform elliptical patch of light falling centrally on the grating. Replace the view-port.



(a) Physical situation shown from above, with three viewing angles.



(b) Appearance to the eye when viewed from the three directions.

Figure 4: When two visible entities are at different distances from an observer, their apparent alignment will depend on the direction from which they are viewed. This is called parallax. If there is parallax between the entrance slit and its image in the collimating mirror (back- reflected from the grating), then the focusing is incorrect and therefore the light is not properly collimated at the grating. If there is parallax between the output spectral line and the cross-hair, then the measured readings will be unreliable, because the position of the line will vary as you move your head.

5.3 Focusing of objective mirror

The position of the cross-hair in the exit aperture should now be adjusted. Orient it "X" rather than "+"; this makes it easier to set it on a spectral line. We require the cross-hair to be placed precisely where the spectral line comes to a sharp focus.

First move the brass cylinder (cross-hair) in and out by large amounts (centimetres) while observing the rectangular patch of light falling on it from the monochromator, in order to get a rough sense of where the focal position is (i.e. where the line is sharpest). Place it at your best estimate of the correct position, then view the cross-hair through the high power eyepiece and perform a fine adjustment using the no parallax method (i.e. no parallax between the cross and the spectral line).

N.B. you are still adjusting the cross-hair, not the eyepiece! The no parallax method is precise but can be difficult to apply owing to distortion of the image as you move your eye around to view from different directions. It is sufficient to use modest changes in viewing angle, so as to avoid this distortion.

The width of the entrance slit should now be adjusted to the optimum value. If the slit is too wide, the accuracy of the wavelength measurements will be poor. If it is too narrow, you will have difficulty in observing the weaker hydrogen lines. Bring the pair of yellow lines at 577 nm and 579 nm into view and check that you can resolve them easily, then observe a few of the weaker lines of mercury to make sure they are visible. To observe the yellow lines (and for the rest of the experiment) the green filter needs to be removed.

The apparatus should now be ready to do some measurements.

▶ **Have your experimental set up checked by a demonstrator before you proceed.**

6 The visible spectrum of hydrogen

The instrument is to be calibrated using the known wavelengths of the mercury lines shown in figure 5. If this calibration is performed carefully, a precision in the value of R_H (Finite nuclear mass Rydberg

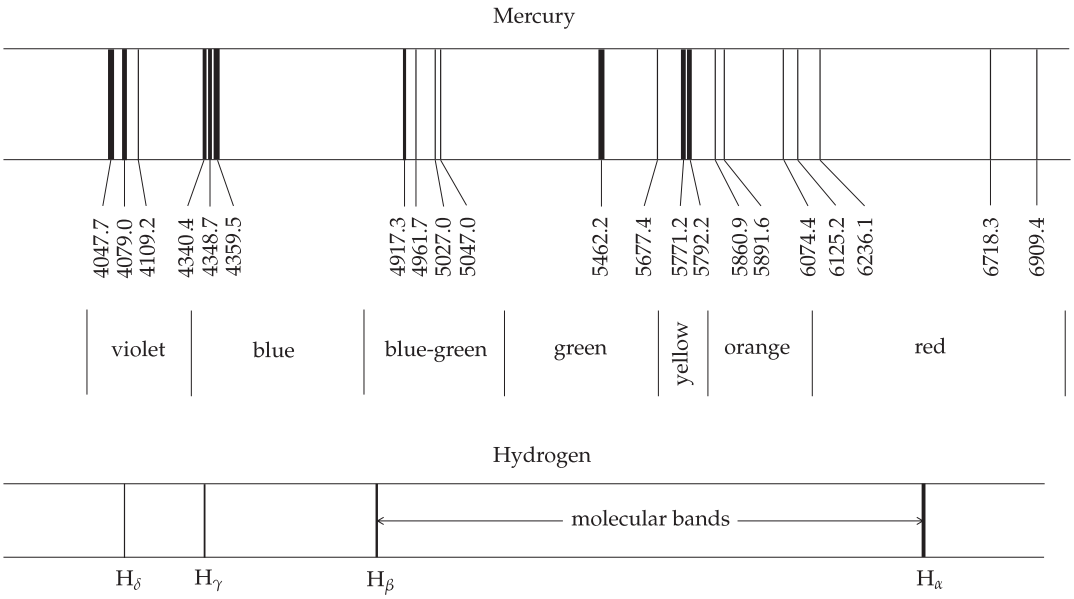


Figure 5: Schematic of mercury and hydrogen spectra. The wavelengths given are vacuum wavelengths (in Å) because they are required in this particular experiment. Note, though, that you will generally find air wavelengths quoted for visible lines as these are more usually needed in practice. For example, the well-known strong mercury green line normally referred to by its air wavelength as "the 546 nm line" appears in the diagram above as 5462.2 Å. The refractive index of air is about 1.0003.

constant) of better than 0.01% can be achieved. **Note, however, that a significant systematic error may arise if the width, and hence possibly the centre position, of the slit is changed between the calibration procedure and the measurements on the hydrogen spectrum.**

Before proceeding, make sure you understand the logic of what you are about to do. Ultimately we would like to use the monochromator to measure some unknown wavelength λ (e.g. a line in the hydrogen spectrum). What one actually records using the instrument is the micrometer reading z. Therefore we need to know the relationship between λ and z. The basic theory gave us such a relationship (equation 4), but that theory treated an ideal instrument, with no mechanical imperfection, etc., so equation 4

is obeyed only approximately in practice. We can get a much more precise equation relating z and λ by using a calibration spectrum, in this case the spectrum of mercury. By finding the value of z (micrometer reading) for a set of known λ, we can plot on a graph the accurate relation between z and λ, and then use it for interpreting further measurements of unknown wavelengths.

To do this, we rely on someone else's measurements of the mercury spectrum, and they must have used a different method to calibrate their instrument. This sort of reliance is involved in most precise scientific work.

With the mercury lamp in position, record the micrometer readings for all the visible lines. Several readings should be made for each line, but the lines must always be approached from the same side to avoid any back-lash errors. Figure 5 sets out the approximate positions and intensities of the lines, so that you should be able to identify them unambiguously. For example, the gap between the red and orange lines is similar to the gap between orange and green: this gives you an idea of how far you should expect to go past orange before seeing the final red ones.

You now need to enter your data into a computer and plot the micrometer dial reading (mm) against known wavelength (A). Fitting a linear or, if needed, a quadratic function will then give you a calibration curve for your monochromator. To help you with this task `op13.ipynb` is available which is located on the Prelims Practicals pages on Canvas (www.canvas.ox.ac.uk); look under the 'Booking an experiment and Optics lab scripts'.

The Jupyter Notebook file expects the data to be in 'comma-separated format' arranged in columns headed `wave`, `reading1`, `reading2`, and `reading3` representing your known wavelengths and three sets of dial readings that the code will average for you. You can change the headings of each column but make sure the Jupyter Notebook is also changed to reflect this.

First try a linear fit to find the linear fit values of gradient and intercept. The notebook also produces a residuals plot, i.e the difference between the experimental points and the straight line fit at each data point. You should see on this plot the random fluctuations present in the data but you may in some instances see a systematic trend in the residuals that indicates that a straight line fit is not adequate. In this case try also a quadratic fit. This should remove any systematic trend on the residuals plot for the monochromators used in the lab. Whichever fit is best you will need to obtain the standard error on the residuals. Think about how you can do this. This represents the uncertainty associated with each use of the calibration curve chosen. Appendix A gives a detailed discussion of this point.

The fitting procedure results in a formula for z in terms of λ:

$$z = a\lambda^2 + b\lambda + c \pm \sigma_{res}. \tag{5}$$

It came out this way because the wavelength was the known quantity, while the micrometer reading was the (noisy) observed quantity. Obviously, when you use this as a calibration curve for measuring unknown wavelengths, you need to invert the formula, so as to obtain an expression for the wavelength as a function of z. Fortunately a quadratic equation is easy to invert. To relate uncertainties in z to uncertainties in λ, observe that

$$\frac{\mathrm{d}z}{\mathrm{d}\lambda} = 2a\lambda + b. \tag{6}$$

Remove the mercury lamp and insert the capillary discharge tube.



Do not leave the discharge tube on for extended periods of time, as it has a short bulb life.

Without moving the cross-hair, look at the patch of light on the grating, as before (use the red line) and put the lamp in such a position that the illumination is correct

▶ **Check the new alignment with a demonstrator.**

Do not move the lens; this should remain where it was for the calibration procedure. Note that excited hydrogen atoms are only one of the many species produced in the discharge. In particular, strong emission bands of H_2 and OH^- molecules may be seen in the region between the Hα and Hβ lines. In identifying the lines, you can be guided by the approximate relationship quoted in section 3. If you are unsure of the identification of the lines, check with a demonstrator (see figure 5 for guidance).

> Measure the wavelengths of the four visible lines of atomic hydrogen taking care to distinguish the sharp atomic lines from the band-heads of the molecular spectra. If you find it hard to see Hδ, try turning off extraneous light sources and shielding your eyes so that they become dark-adapted (wide pupil diameter). It may be necessary to widen the slit somewhat for this particular measurement, but avoid resorting to that until you have completed all other measurements.

Use the calibration curve to obtain the correct vacuum wavelengths of these lines. The wavelengths of the calibration lines in figure 5 are vacuum values so your results need no correction, even though the experiment itself is carried out in air.

The wavelengths in vacuo of the Balmer lines of atomic hydrogen are found to satisfy the expression

$$\frac{1}{\lambda_n} = \tilde{\nu}_n = R_{\mathrm{H}}\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \text{ where } n = 3, 4, 5, \ldots \infty. \tag{7}$$

Calculate the separate values of R_H together with their uncertainties from your data. The errors on each value of R_H may be determined by combining the errors due to the calibration and to the uncertainty in measurement of the hydrogen lines. Finally, the separate values of R_H may be combined to give the weighted mean $\bar{R}_H$ and the standard error of the mean $\delta\bar{R}_H$, and your result should be expressed as $\bar{R}_H \pm \delta\bar{R}_H$ in either cm^{-1} or m_{-1}. Compare your result with the theoretical value (see section 7).

7 Bohr theory for atomic hydrogen

It is simplest first, as Bohr did, to take the nucleus as fixed at the origin. This corresponds to assuming that the proton is infinitely massive compared with the electron. Bohr postulated that the angular momentum of an electron orbiting the nucleus of a hydrogen atom is given by

$$m_e v r = n\hbar \text{ where } n \text{ is an integer.} \tag{8}$$

The centripetal acceleration is determined by the Coulomb attraction of the nucleus:

$$\frac{m_e v^2}{r} = \frac{Ze^2}{4\pi\epsilon_0 r^2}. \tag{9}$$

Substituting from equation 8 gives

$$\frac{1}{r} = \frac{Ze^2 m_e}{4\pi\epsilon_0 n^2 \hbar^2}. \tag{10}$$

The total energy of the system is the sum of the kinetic and potential energies:

$$E_n = \frac{1}{2} m_e v^2 = \frac{Ze^2 m_e}{4\pi\epsilon_0 r}. \tag{11}$$

Substituting from equations 9 and 10 gives

$$E_n = -\left(\frac{Ze^2}{4\pi\epsilon_0}\right)^2 \frac{m_e}{2\hbar^2}\frac{1}{n^2}. \tag{12}$$

The energy of a photon emitted in a transition from a level with quantum number n to one with quantum number n' is given by

$$h\nu_{nn'} = hc\tilde{\nu}_{nn'} = E_n - E_{n'} = -\left(\frac{Ze^2}{4\pi\epsilon_0}\right)^2 \frac{m_e}{2\hbar^2}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right). \tag{13}$$

The Rydberg constant for atomic hydrogen, $Z = 1$, is therefore given in the approximation of an infinitely massive nucleus by

$$R_\infty = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m_e e^4}{4\pi c\hbar^3}. \tag{14}$$

A calculation in which the nucleus has mass M also gives equation 14 except that the electron mass, m_e, is replaced by its reduced mass

$$\mu = \frac{m_e M}{m_e + M} \tag{15}$$

An interesting consequence of this analysis is that the Rydberg constant for deuterium differs slightly from that of hydrogen and gives rise to an isotope shift between the spectral lines of light and heavy hydrogen.

For the particular case of hydrogen, we have,

$$R_H = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{e^4}{4\pi c\hbar^3} \frac{m_e m_p}{m_e + m_p}. \tag{16}$$

where m_p is the proton mass.

Evaluate R_H to sufficient accuracy to allow critical comparison with your experimental result using the following data:

$$\begin{aligned}
\epsilon_0 &= 8.8542 \times 10^{-12}\,\mathrm{F\,m^{-1}} \\
e &= 1.6022 \times 10^{-19}\,\mathrm{C} \\
c &= 2.9979 \times 10^{8}\,\mathrm{m\,s^{-1}} \\
\hbar &= 1.0546 \times 10^{-34}\,\mathrm{J\,s^{-1}} \\
m_e &= 9.1095 \times 10^{-31}\,\mathrm{kg} \\
m_p &= 1.6726 \times 10^{-27}\,\mathrm{kg}
\end{aligned}$$

Notice that the reduced mass μ represents approximately a part per thousand correction, compared to m_e.

▶ Comment on whether your experimental result gives clear evidence that R_H and not R_∞ is observed.

▶ Now hand-write a brief summary of the experiment in your logbook. State the aims of the experiment, the method used, quantitative results, and your conclusions, in not more than about half a page. Show this to a demonstrator as part of your assessment.

Appendix A Interpreting a fitting curve

Fitting a curve to data is an extremely important scientific tool. It is mostly used to extract the values of various parameters, such as the location, height and width of a peak in the data. Various types of

question arise: is the fit a "good" fit? how well are the parameters of the fit determined by the data? what uncertainty is associated with the use of such a curve for calibration of an instrument? Here we will focus on the last question.

Suppose for the sake of argument that we are fitting a quadratic curve to a set of data, as in the monochromator experiment, and we wish to use this curve as a calibration curve. We would like to know to what degree of accuracy we can trust this calibration.

There are two issues: first, is the instrument well modelled by a quadratic function? Second, supposing that it is, how precisely have we been able to obtain the true curve? The first question is about systematic uncertainty, the second is about statistical uncertainty.

Consider first the idealised case of no systematic error. Each time we observe a known wavelength, we add one more point to our data. This builds up the number of values in our set of residuals. The standard deviation of the set will not change much as we go from, say, 10 to 20 to 30 observed wavelengths, but we should expect that this increased information should be making the calibration more reliable. It is. As long as we are sure the instrument is very well modelled by a quadratic function, then the uncertainty (1 standard deviation) to be associated with any single use of the calibration curve would be given by

$$\sigma_{cal}(\text{statistical}) = \frac{\sigma_{res}}{\sqrt{n-3}}, \tag{A.1}$$

where n is the number of data points. This is substantially smaller than σ_{res}!

However, we can't rule out the possibility that small deformities in the mechanical mechanism, such as an indentation on the pad on which the micrometer bears, would lead to a systematic departure from a quadratic function (the sine-bar length is about 75 mm, therefore it only requires an indentation of depth 10 mm to introduce a part per 10000 shift in the grating angle). This means that, no matter how many observations we make, the calibration curve will always be wrong, simply because it is the wrong function to describe the instrument. The statistical error gets smaller as more observations are made, but the systematic error does not.

How large is the systematic error, then? This is always hard to assess, and each experimental apparatus needs to be considered individually. However, it is obvious that if the true calibration curve departed strongly from the one you picked, then you would notice because the residuals would not look randomly distributed. Therefore we can argue that the systematic error is no larger than σ_{res}, but without further investigation we can't claim that it is smaller than σ_{res}.

The conclusion is that the uncertainty to be associated with any single use of the calibration curve is σ_{res} if you do no further analysis at this stage (and that is acceptable).

Optional If you want to take things further then, carry out a search for systematic effects. For example, you could group the data points. For each group, find the mean and the statistical error on the mean . Then fit a curve to these and examine whether the scatter in the residuals is consistent with the error bars.

Appendix B Equipment

- Monochromator
- Optical Bench
- Mercury Lamp
- Green Mercury Filter
- 50mm Lens
- Eyepiece

- Hydrogen Discharge Tube
- Box
- Glove
- Optical Pin
- Lamp