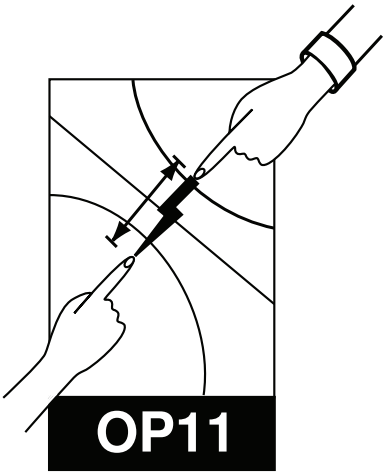
Lloyd's Mirror and Double-Slit Interference Fringes

MT January 2019





This experiment uses optical pointers or pins. Always take care when using sharp objects, especially when working in the low ambient light levels required in the Optics laboratory. Always leave the optical pins with their pointed end towards the wall and moved clearly out of the way when not in use.

1 Introduction

Two-beam interference fringes are perhaps the single most instructive topic in physical optics. You may have come across Young's arrangement for observing them. This experiment is similar to Young's, except that the two beams are produced in a different way; the principle is shown in figure 1. A mirror M forms an image S' of a single slit S, and the interference of light from the slit and from the image is observed. This interference can be observed anywhere in the shaded region: the fringes are therefore said to be non-localized. In practice, one generally observes the interference in a plane perpendicular to the mirror surface, such as OQ. Further information can be found in [1].

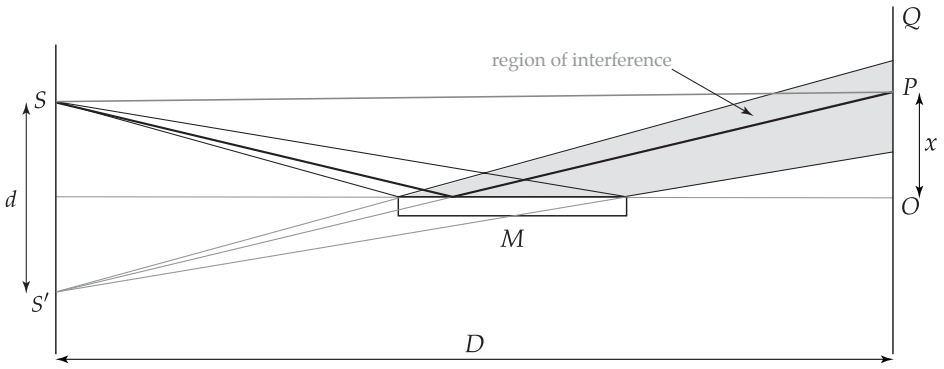


Figure 1: Optical ray diagram showing how interference is produced using one real source S, and the mirror to produce an imaginary source S'.

The experiment is in two parts. In the first, which you should complete before lunchtime, you set up the apparatus and use it to observe interference fringes under various conditions. In the second part, you make measurements with the aim of determining the wavelength of the light source. As with all experiments in the teaching laboratories, this has an importance well beyond simply learning about optics - it gives you the opportunity to develop the skills of:

(a) making relevant observations and recording them clearly

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(b) getting the best out of the apparatus,

(c) taking reliable data,

(d) assessing sources of uncertainty.

The last item listed is likely to be the least familiar to you. However, measurement of anything doesn't mean much unless one is able to estimate the level of uncertainty in the final result. If you need advice about errors or anything else, *ask a demonstrator* — that's what they're for.

You should write comprehensible notes on the experiment in your logbook as you go along, and check that the results make sense before you leave the apparatus.

2 Theory

The object of this section is to show that the separation Δx of successive fringes in the plane of observation is given by $\Delta x = \frac{\lambda D}{d}$ (see figure 1).

In Young's arrangement, constructive interference occurs when the phase difference ϕ between the electromagnetic waves from S and S' to a point P in the plane of observation is $2n\pi$, where n is an integer. However, in Lloyd's experiment an additional phase change of π occurs upon reflection at the air/glass boundary. Taking this into account and considering light of wavelength λ, the phase difference between the waves from S and S' in figure 1 will be given by

$$\phi = \frac{2\pi}{\lambda}(S'P - SP) + \pi \tag{1}$$

Thus at the point P there is a superposition of waves of the same frequency differing in phase by ϕ giving an electric field amplitude of

$$\begin{aligned} E(P) &= E_0[\cos(\omega t) + \cos(\omega t + \phi)] \\ &= 2E_0 \cos(\omega t + \frac{\phi}{2}) \cos(\frac{\phi}{2}) \end{aligned} \tag{2}$$

The time-averaged intensity at P is proportional to the average of the square of the electric field, i.e.

$$I(P) = 2E_0^2 \cos^2(\frac{\phi}{2}) \tag{3}$$

which is a maximum when $\phi = 2n\pi$. An approximation for the path difference can be obtained thus:

$$SP = [D^2 + (x - \frac{d}{2})^2]^{\frac{1}{2}} \tag{4}$$

$$S'P = [D^2 + (x + \frac{d}{2})^2]^{\frac{1}{2}} \tag{5}$$

$$S'P - SP = D\left(\left[1 + \left(\frac{x + \frac{d}{2}}{D}\right)^2\right]^{\frac{1}{2}} - \left[1 + \left(\frac{x - \frac{d}{2}}{D}\right)^2\right]^{\frac{1}{2}}\right) \tag{6}$$

Providing $x \pm \frac{d}{2} \ll D$ we have

$$S'P - SP \approx D\left(\left[1 + \frac{1}{2}\left(\frac{x + \frac{d}{2}}{D}\right)^2\right] - \left[1 + \frac{1}{2}\left(\frac{x - \frac{d}{2}}{D}\right)^2\right]\right) = x\frac{d}{D} \tag{7}$$

Substituting into equation 1 we find that

$$\phi = \frac{2\pi}{\lambda}\frac{xd}{D} + \pi \tag{8}$$

Hence the n^{th} bright fringe occurs at

$$x_n = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d} \tag{9}$$

The fringe separation is therefore

$$\Delta x = \frac{\lambda D}{d} \tag{10}$$

and the *spatial frequency* f (the number of fringes per unit length) is given by

$$f = \frac{1}{\Delta x} = \frac{d}{\lambda D} \tag{11}$$

The mirror micrometer drive gives $\frac{d}{2}$ to within an additive constant. If D is known, one can thus find f for different values of the mirror position and from the slope of a straight line graph determine λ.

3 Setting up the apparatus

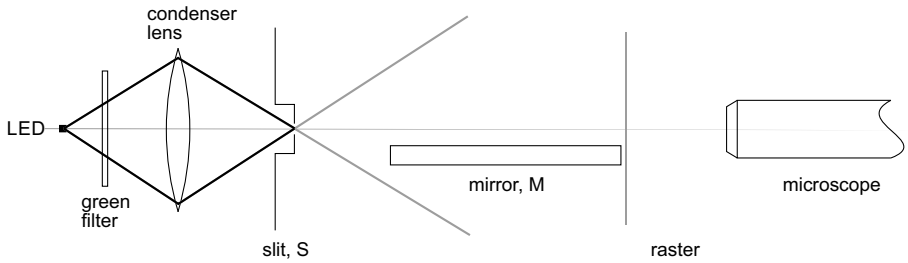


Figure 2: Setup of the apparatus to view the interference fringes, complete with ray diagram from light source.

Set up the apparatus as shown in figure 2. The slit is illuminated by an LED with a condenser lens. M is a glass plate, the front surface of which is flat to a quarter of a wavelength and acts as a plane mirror.



Please do not scratch or otherwise damage the mirror.

The rasters are transparencies in 35 mm slide holders and aren't needed yet.

3.1 Use of optical pins

The idea of an optical pin is that its point defines a line - the optical axis. The first time any optical component is placed on the bench it is lined up using the pin so that its axis coincides with the optical axis. A long optical component like the mirror, needs to have both ends lined up with the axis.

The first thing one has to decide is: where should the axis be? There is no need to try to put it exactly over the centre of the bench, any line of holes in the breadboard will do; the important thing is that its height should be within the range of adjustment of all the components you are going to use. In the case of Lloyd's Mirror, the mirror does not move vertically, so this defines the height of the axis.

3.2 Lining up the Lloyd's mirror apparatus

There are different ways of going about this: the following is one possibility. Before you start, though, please note:



It is very easy to scratch optical components with the pins. When aligning a component, place the pin-point close to, but not touching, the component.

Set up a pin so that its height is the same as the centre of the edge of the mirror. Put the slit on the bench, then bring the pin close to it and adjust the height until the centre of the slit is aligned with the pin. Then use the pin to align the condenser lens. The raster mount could also be aligned at this stage, but it is so uncritical it is not worth bothering.

Make sure the mirror is parallel to the line of holes that define the axis, using the tilt adjusters on the mirror mount.

It is not a good idea to poke the optical pin too close to the LED. Fortunately, if the condenser lens and the slit are on axis, and the image of the light source falls on the slit, then the light source is correctly positioned. When the condenser lens is positioned correctly you should be able to see a focused image of the rectangular LED on the slit. The LED has a grid of dots on it, which you should be able to see when it's focused.

Open the slit wide and with a piece of white paper or card check that a uniformly illuminated cone of light passes through the slit. Move the slit along the bench as necessary to achieve this, and make any fine adjustments necessary to the light source position to get the best illumination. Close the slit down again.

Put the mirror on the bench, around $10-20$ mm from the slit. Using the micrometer adjustment, withdraw the mirror surface a millimetre or so from the optical axis. Put an eye close to the mirror surface and look at the slit and its image, which should be a couple of millimetres apart. Tilt the mirror if necessary to make the two parallel - but think first which screw to use (one of the two will destroy the parallelism with the optical axis).

Place the eyepiece so that the edge of the front surface of the mirror is in focus. Two-beam interference fringes should now be visible.

▶ Have your setup checked by a demonstrator.

4 Initial observations

This section can be understood in comparatively simple physical terms, but if you are finding it difficult to understand, talk to a demonstrator to make sure you follow what is going on.

4.1 Effect of changing slit width and mirror displacement

Describe the pattern, and investigate the effect of changing the slit width and the sideways mirror displacement. If you never get more than a few fringes, you probably haven't made the slit and its image parallel - try the effect of tilting the mirror.

4.2 Destructive interference

▌Restore the set-up so that you have a good pattern of fairly widely spaced fringes.

The fringe observed at the mirror's edge is in a position of zero path difference. Nevertheless, there is destructive interference because of the phase change on reflection. Verify this.

4.3 White light source

▌Replace the green light source with a white light source and observe the interference pattern.

▶ Describe and explain what you observe, remembering that the eye is sensitive to wavelengths between about $400\,\mathrm{nm}$ (blue) and $700\,\mathrm{nm}$ (red).

4.4 Superposition

This part of the experiment will be shown and described to you (and the other students) by the demonstrator.

▌A mercury lamp will be used for illumination. The mirror position will be adjusted to give a large number of closely spaced fringes. The mercury lamp also has a green filter which will now be removed so that the full spectrum of the lamp can be observed.

▶ Describe the pattern, which is a superposition of the individual patterns set up by the mercury spectral lines in the visible region (see wall chart). Explain how the periodic fuzzy, coloured regions of the pattern originate. Hint: what would you expect if mercury had only two spectral lines?

4.5 Diffraction fringes

So far, all your observations have been in the plane of the edge of the mirror.

▌Using a green light source, move the microscope back along the bench; describe and explain what happens to the interference pattern as the distance changes.

Note that new fringes of non-uniform spacing appear in the vicinity of O, outside the region in which the two-beam pattern can be seen. Describe these fringes. They are due to diffraction (what at?) and must not be confused with the two-beam pattern.

▶ Check your results with a demonstrator if you haven't already done so.

5 Finding the wavelength of the green light source

To find the wavelength of the green light we need to measure the fringe spacing as a function of mirror position $\frac{d}{2}$ for some distance D of the observation plane from the plane containing the slit and its image (equation 11). It would be tedious and inaccurate to measure the distance occupied by a given number of fringes, so we use the moiré fringe technique instead (a copy of [2] is in each bound version of this manuscript). The fringes are observed in a plane occupied by a photographically produced raster of equally spaced straight lines, arranged to be approximately parallel with the fringes. When the spatial frequencies of the two systems are nearly equal, a 'beat' pattern appears which can be used to make them match very accurately.

Place the raster holder close to the end of the mirror, as shown in figure 2. Insert one of the rasters; the one with about 10 lines/mm is probably best for setting up. Block the light from the light source, (e.g. by putting a piece of paper in front of the condenser lens), and focus the microscope on the raster (illuminate if necessary) so that the lines are clearly visible. The raster now defines the plane of observation OQ in figure 1. You will therefore need to measure its distance D from the slit at some point. An accurate method of doing this is suggested below; for the moment, however, make a measurement with a metre rule so that you know at least roughly what it is.

Move the raster vertically until the grid covers half to three quarters of the area visible through the microscope and translate the mirror sideways until the fringe spacing is approximately equal to that of the raster. Vertical moiré fringes should now be observable. As the spatial frequency of the interference fringes approaches that of the raster, the moiré fringes should become more and more widely spaced. They will generally not disappear completely because the grid lines and the interference fringes will not be exactly parallel; the moiré fringes become horizontal when the spatial frequencies are equal. The micrometer reading of the mirror translator should then be noted; take at least five independent readings with the setting always approached from the same direction to avoid backlash errors. The procedure should be repeated for all the grids supplied. *Plot a rough graph of your results as you go along to check all's well.*

Fainter moiré patterns are observable when the spatial frequency of one of the two systems is an integer multiple of the other. These can be used to give additional data points.

Finally, measure D, the distance from the slit to the raster, when the eyepiece is focused on the raster.

6 Results and analysis of experimental errors

The results can be conveniently displayed as a straight line graph, the slope of which depends only on the distance from the slit to the observation plane (i.e. D) and the wavelength of the light. Plot your data and apply an unweighted straight line fit to it, finding out the errors in the fit.

6.1 Random errors

The reason for using an unweighted fit is not obvious, since the standard deviations of the measurements of mirror position y probably vary considerably. However, this is not because the measurements varied significantly in accuracy.

▶ Suggest other possible reasons.

The standard error $\sigma(D)$ in the distance D should also be estimated. Remember that if the accuracy is limited by the measuring instrument, you can't use statistics! This error should then be combined with the standard error on the slope. This gives the standard error in the wavelength result which you should express in the form $\lambda \pm \Delta\lambda$.

6.2 Systematic errors

In many experiments these are important, and they are much harder to assess than random errors. There is no simple way to quantify them, or even to be sure one has thought of all effects which might give rise to them. Try to think of possible sources of systematic error in this experiment.

One is a possible misalignment of the plane of the raster by an angle q from the perpendicular to the optical axis. Show that this leads to a fractional error of about $\frac{q^2}{2}$, and estimate the likely maximum percentage systematic error from this effect in your experiment.

▶ Check your results and error analysis with a demonstrator

▶ Now hand-write a brief summary of the experiment in your logbook. State the aims of the experiment, the method used, quantitative results, and your conclusions, in not more than about half a page. Show this to a demonstrator as part of your assessment.

Optional

7 Effect of finite slit width

There are many subtleties associated with interference phenomena which we have so far avoided in this experiment. This is an optional section on one of them, which you might like to try: it extends the theory in section 2, and gives an insight into the concept of coherence.

> Using an appropriate raster, set the system up to give around 15 bright fringes mm^{-1}. Then widen the slit. You should already have seen that the first few fringes remain visible, but the higher order ones blur out. Adjust the slit width until the 10^{th} fringe disappears completely. You should then notice that the fringes improve slightly again before disappearing once more at around the 20^{th}.

Two points: first, this isn't going to work unless you had more than 20 clear fringes with the slit narrow. Second, do not agonise over trying to do this very precisely - convince yourself that it happens, and make a single setting of the slit width.

> Remove the slit, and determine its width. This is most easily done by measuring the diffraction pattern it produces when placed in a beam of 633 nm light from a helium-neon laser.

For safety reasons you must consult a demonstrator or technician before doing this.



If the slit width is W, the angular separation of the minima either side of the principal diffraction maximum is $\frac{2\lambda}{W}$.

Back to the interference fringe pattern. To interpret your results, consider the finite slit to be divided up into thin elements. Because of the mirror, each of these produces a two-beam interference pattern; the centres of the patterns due to different elements coincide, but the spacing is different from one to the next. Since this is only a semi-quantitative part of the experiment, let us ignore the complication of the π phase change. Then if the n^{th} bright fringe produced by the element nearest the mirror coincides with the $(n+1)^{th}$ produced by the element furthest away, there will be uniform intensity in the observation plane in the vicinity of these fringes.

Draw simple diagrams to illustrate this. Your first reaction may be that the fringe disappearance should occur when the fringe numbers are n and $n+\frac{1}{2}$, not n and $n+1$; but remember that all the elements in between the extremes are present as well. By simple application of the theory in section 2, show that the slit width required for the n^{th} fringe to disappear is given by

$$w = \frac{\lambda D}{2n\Delta x} \tag{12}$$

Compare this prediction with your measurement.

▶ **Can you explain why the fringes reappear between the positions of the 10^{th} and 20^{th}?**

Appendix A Equipment

- Optical Bench
- Post Holders x 6
- Tuning Fork Bases x6
- 60cm Ruler
- Microscope
- Raster Holder
- Mirror
- Adjustable Slit
- 100mm Lens
- Green LED
- Optical Pin
- Lamp
- Rasters
  - 10 l/mm
  - 15 l/mm
  - 20 l/mm
  - 25.59 l/mm
  - 30 l/mm
  - 35 l/mm
  - 40 l/mm
  - 45 l/mm
  - 50 l/mm

Bibliography

[1] E. Hecht, *Optics*, 4th edition, Pearson (2003)

[2] G. Oster and Y. Nishijima, Moiré Patterns, *Scientific American*, 208, (1963), p. 54. doi: 10.1038/scientificamerican0563-54