Instruments and Techniques for Atmospheric Remote Measurement

Yes- this is on the syllabus…
The class next week provides a worked example of the main concepts

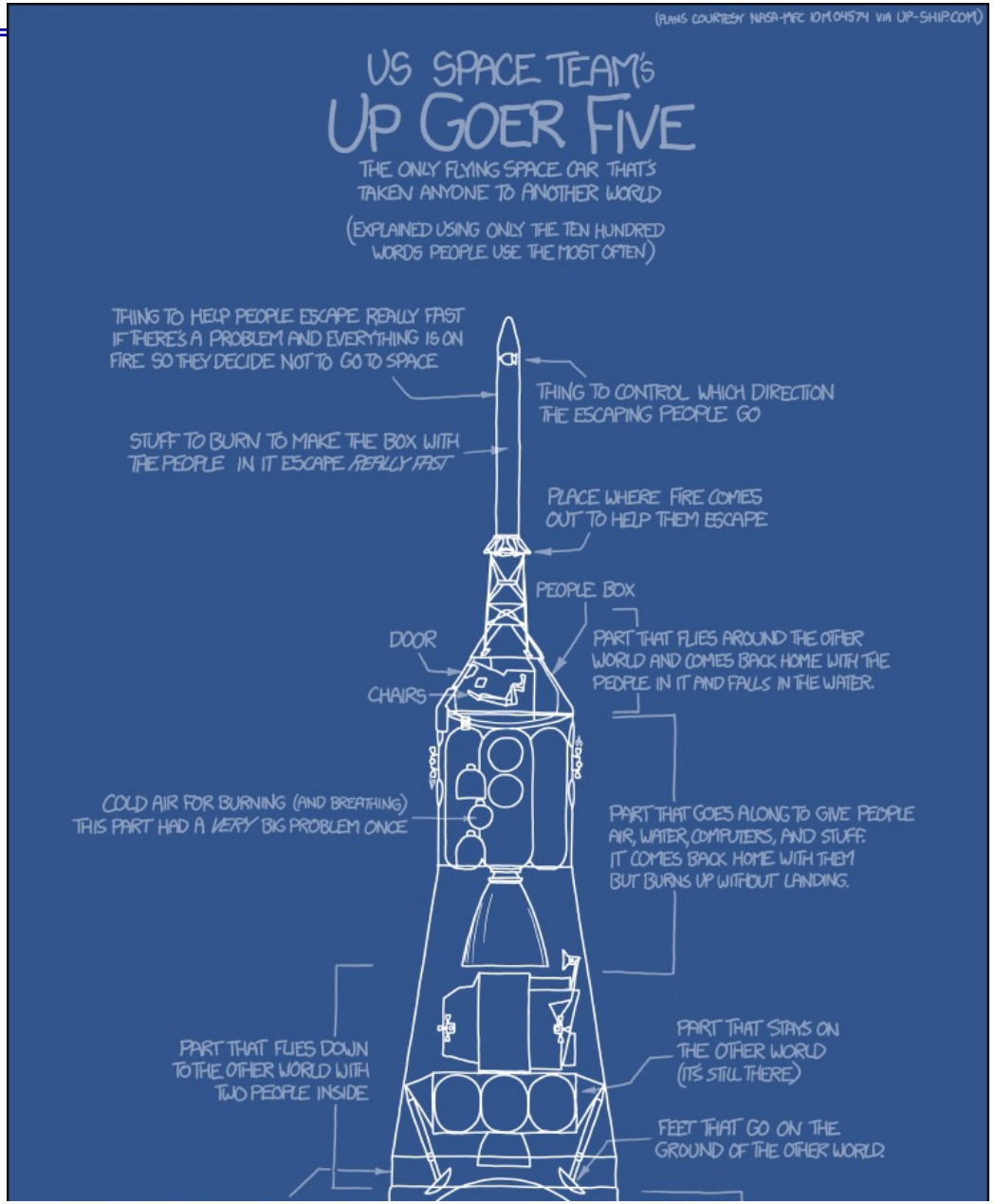
Carly Howett

Neil Bowles

AOPP - 2022

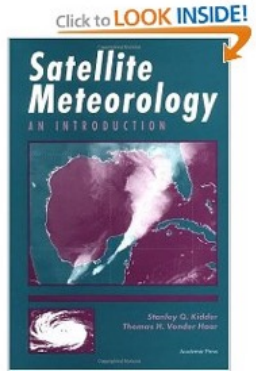
Jargon…

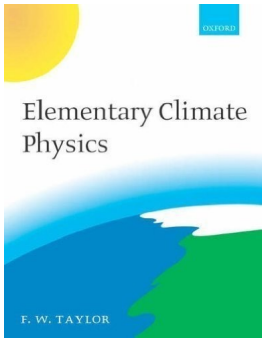
- Lots of acronyms…
- Important ones include:
- Channel = Chunk of EM spectrum measured by an instrument
  - E.g. 15 micron channel means a filter that selects a region of the infrared spectrum around 15 microns
  - Similar to ‘pass band’
- I will not expect you to remember specific instrument names!
- Most instrument types will be familiar from 2nd year optics..

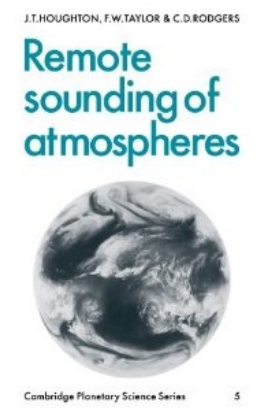


Books

- **"Satellite Meteorology", Kidder and Vonder Haar Academic Press 1995**
  - **Very comprehensive on tropospheric instruments; good on orbits and small amount on stratosphere.**
  - **Expensive to buy**
- **"Remote Sounding of Atmospheres" Houghton, Taylor Rodgers**
  - **Showing its age but covers some bits well (plus other parts of the course)**
- **"Physics of Atmospheres" J T Houghton**
  - **Has one relevant chapter and covers the basic concepts.**
- **"An introduction to Atmospheric Physics" D G Andrews**
  - **Good for the basics of remote sounding.**
- **"Elementary Climate Physics" F W Taylor**
  - **Chapter 9 covers some basic instrumentation and noise concepts.**
- **The APO1 and AP03 Remote Sounding lab script from the 3rd year atmos lab**
  - **In the third year lab, AP01 is a bit of a slog, for C5 it is surprisingly very informative. Seriously, check it out.**
- **These notes**







Lecture outline

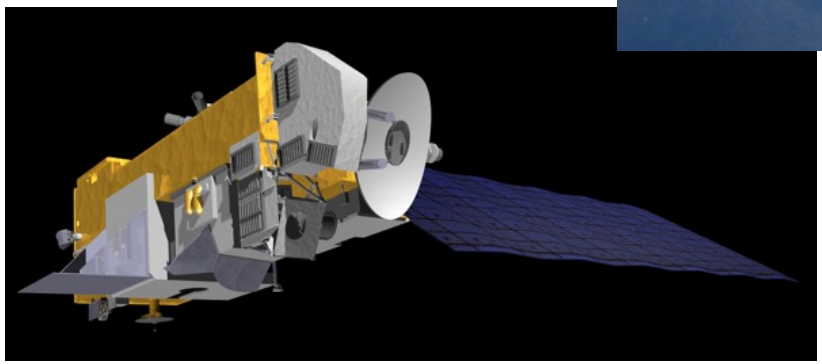
- We will be covering:
- Lecture 1
- What are we measuring? radiative transfer revision
- Satellite orbits and Geometries
  - Orbits
    - Geostationary
    - Low Earth Orbit
  - Low Earth Orbit Viewing geometries
    - Downward
    - Limb



Images: ESA/NASA

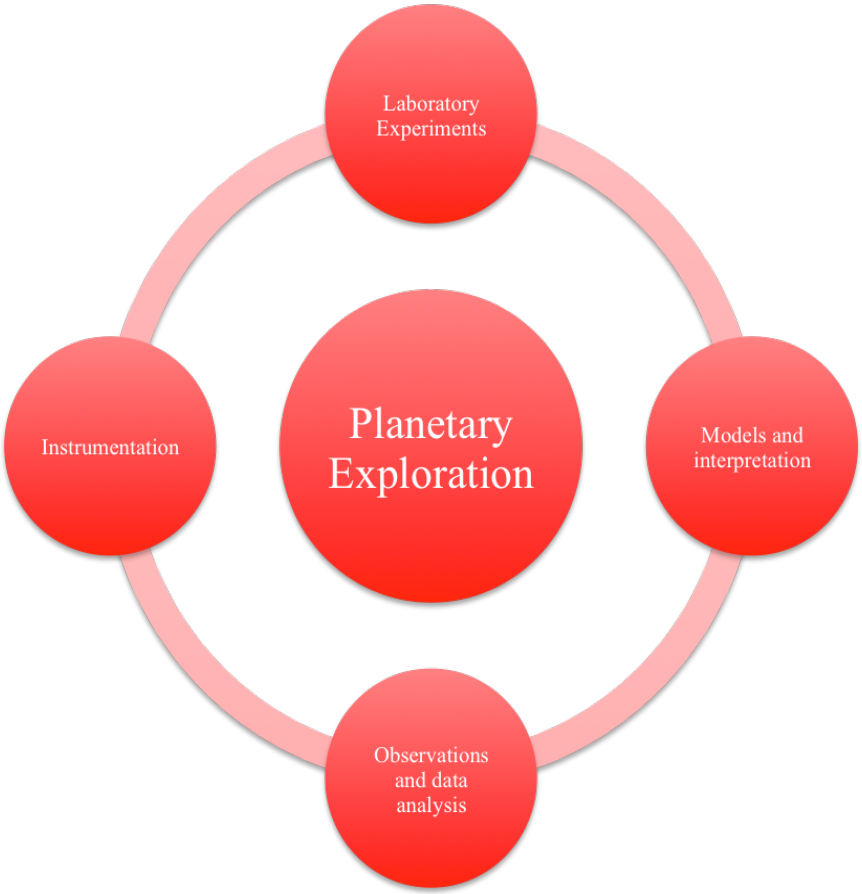


- Instruments
  - Filter radiometers
  - Michelson Interferometers
  - Microwave Sounding
  - Grating Spectrometers
  - Detectors
  - Instrument Design
  - Calibration
- A simplified design example class –
  - Mars Climate Sounder



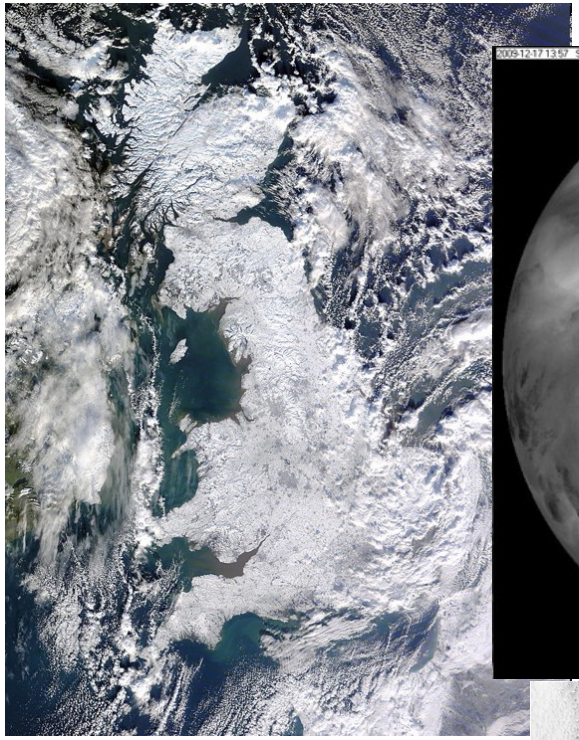
Aims

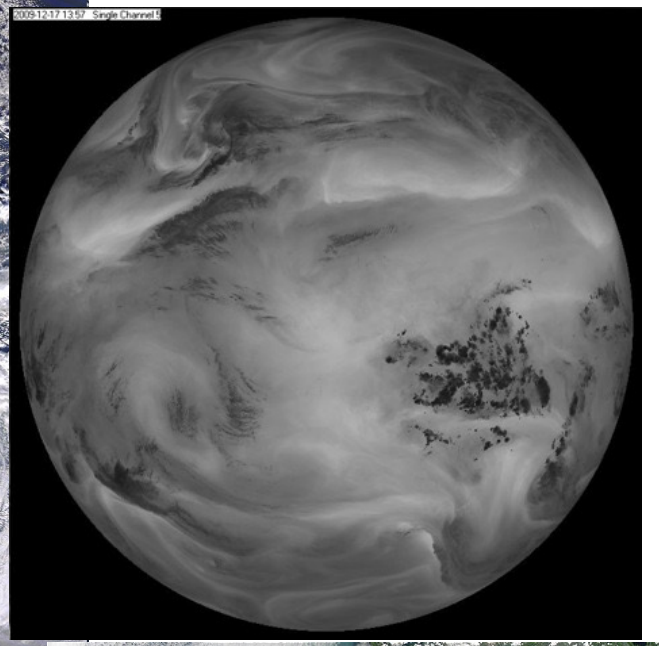
- **The aim of these lectures and class is to show how the design and operation of satellite remote sensing instruments is informed by science questions/observation requirements that then feed back into the instrument design.**

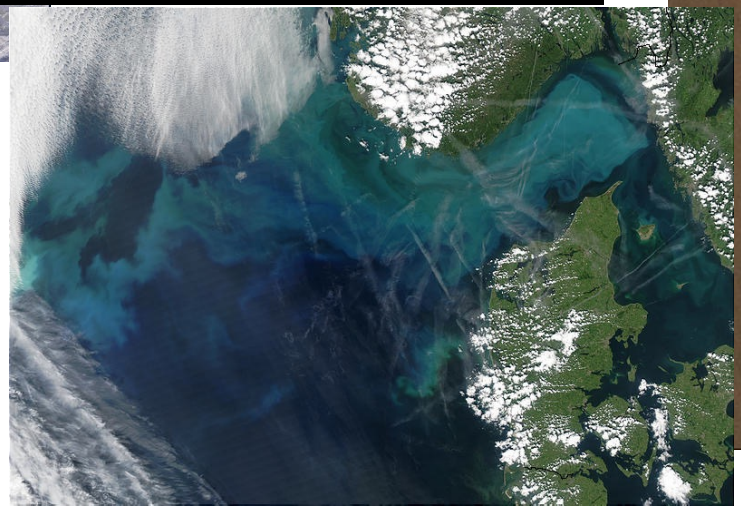


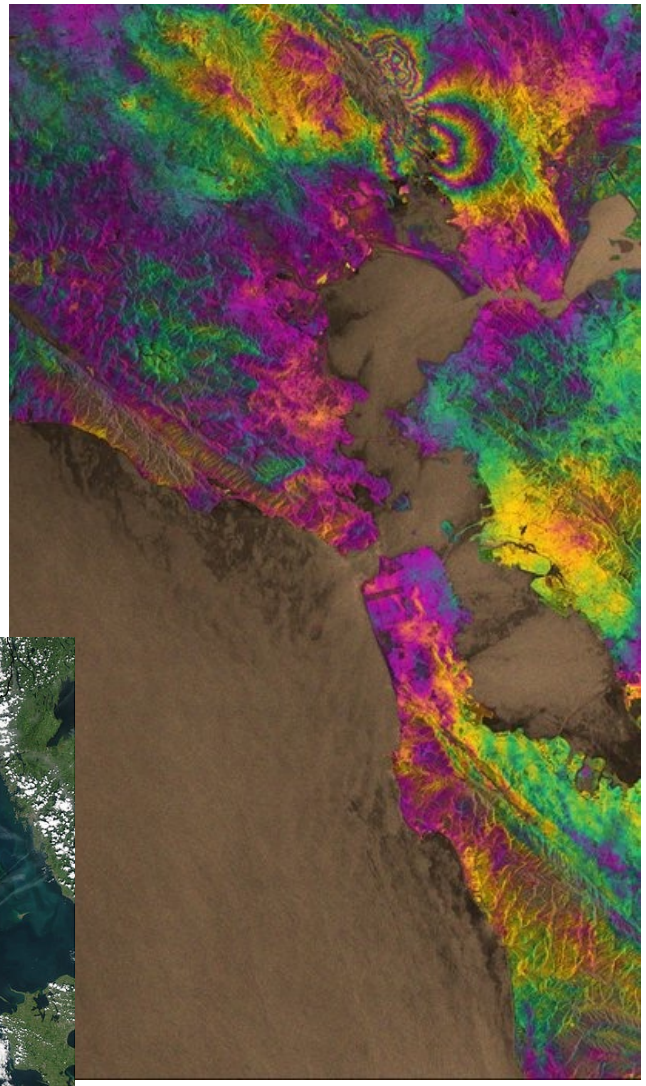
- **Keep in mind the radiative transfer sections of the course!**

Types of Satellite observations









Types of Satellite
Big ones…
Sentinel 1 (ESA), Aura (NASA), MetOP (ESA)





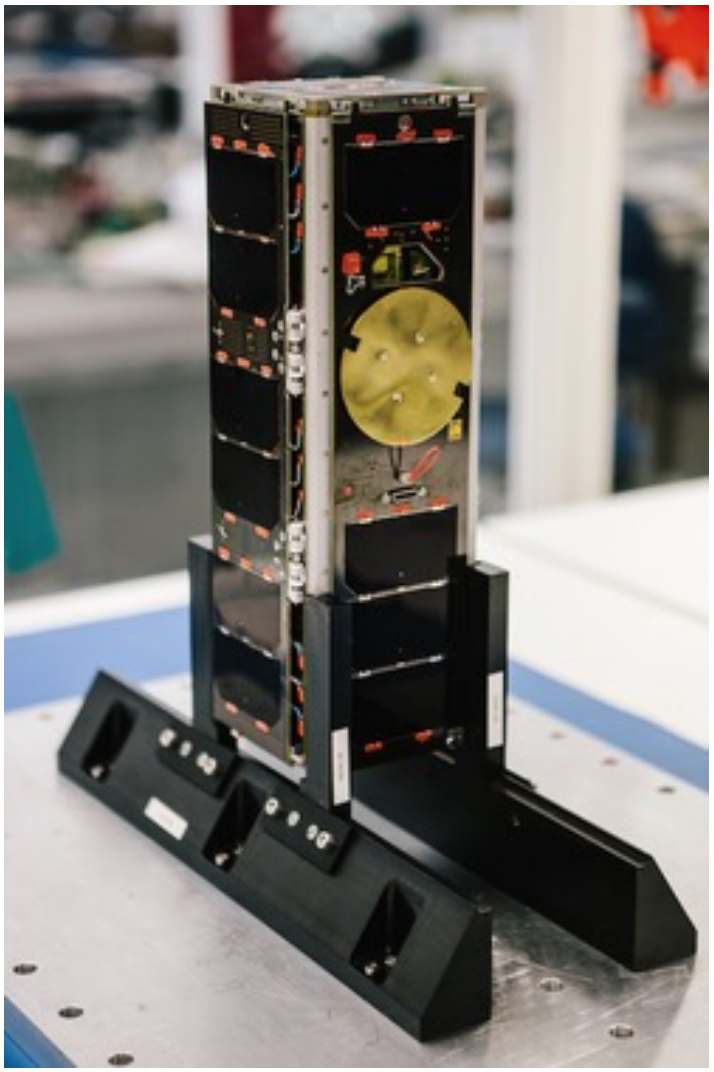


Size of a minibus approximately.

Small/tiny ones



TechDemoSat-1 150 kg class small satellite (image: SSTL, CMS instrument team from Oxford and RALSpace shown).



Ukube-1 3U (30 cm long) cubesat (image:Clyde Space).

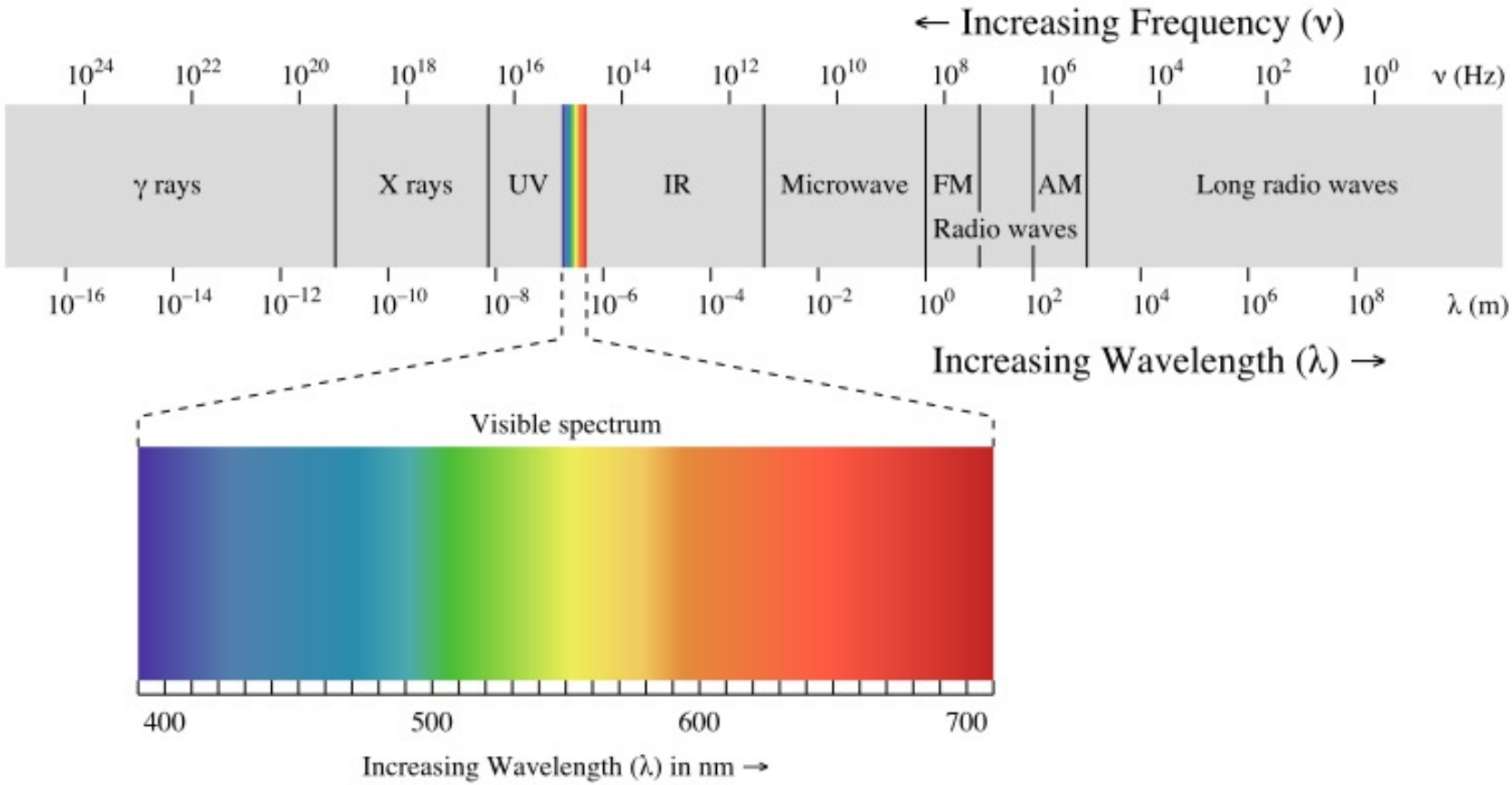
What are the most important things to consider when building a remote sensing instrument?

- **Science Questions!**
  - If it is a scientific mission, lots of other reasons though…
  - Why do we want to measure something?
  - Climate monitoring?
  - Weather forecasting?
  - Geo prospecting?
  - Disaster Monitoring?
- **These lead to *requirements.***
  - Orbit
  - Resolution (spectral and spatial)
  - Wavelength range (UV, Visible, IR)
  - Integration time
- **To make measurements**
  - Spectra
  - Radiances
  - Images

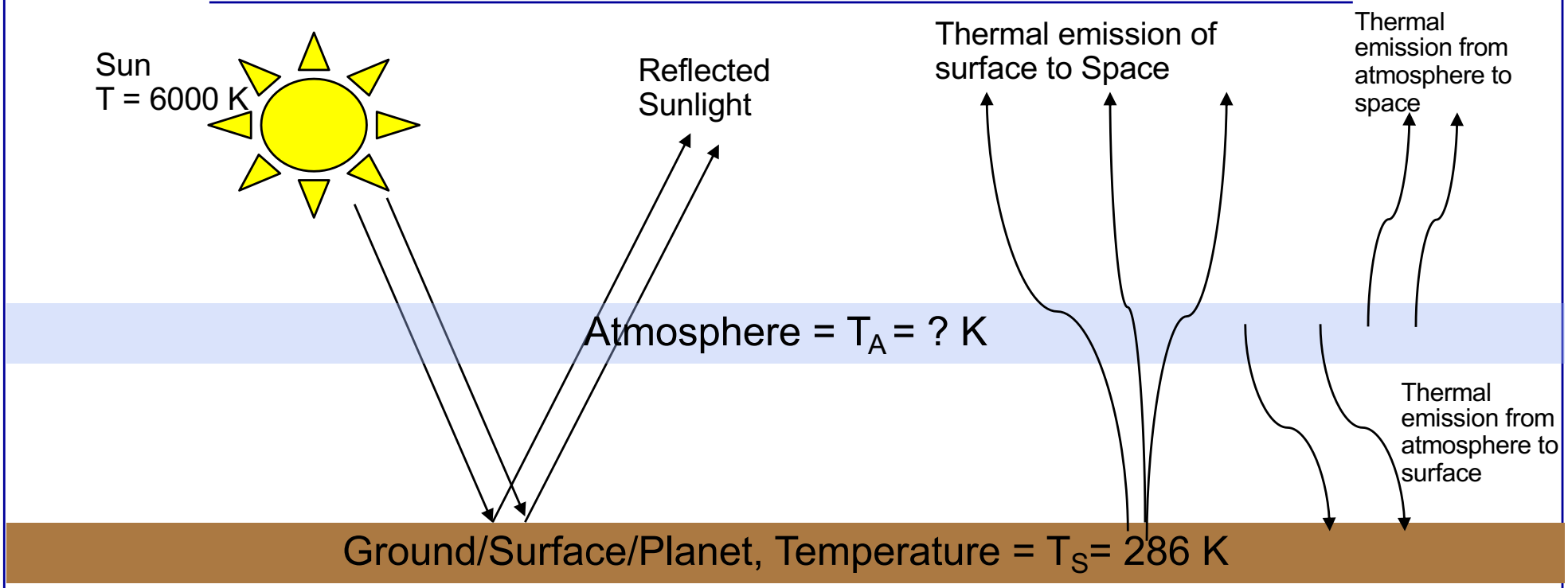
- **To get Data Products**
  - Ozone abundance
  - Climate data
  - Pollution monitoring
  - Etc.
- **To answer the Science Questions!**
- **Every step along this path is more complicated than you might think…**

What are we trying to measure?

- **In these lectures and class we will mainly study observations in the IR and visible.**

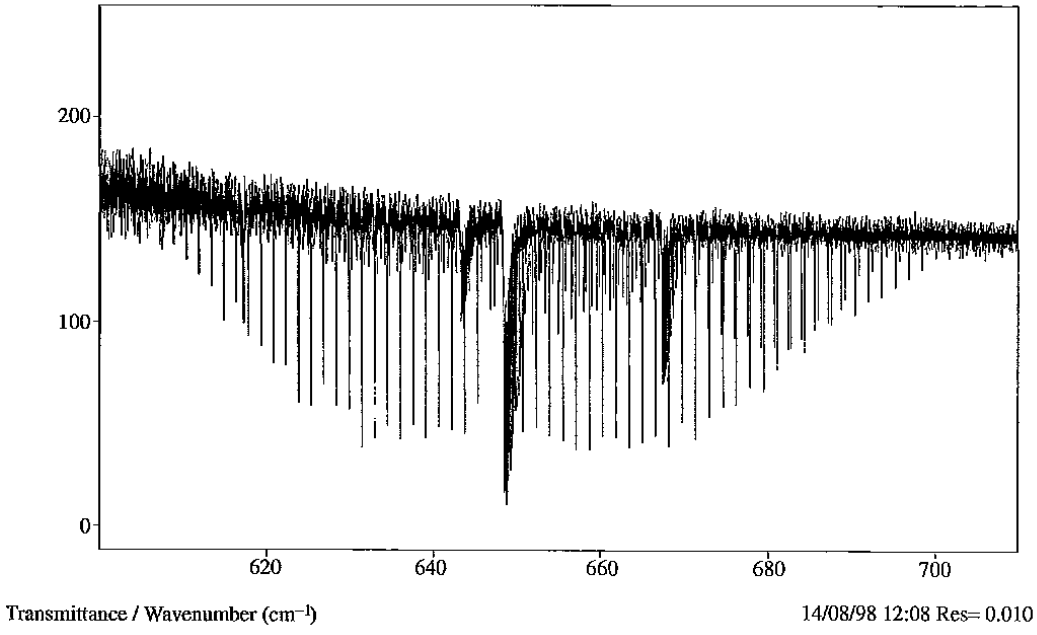


What are we measuring in visible and infrared remote sensing?

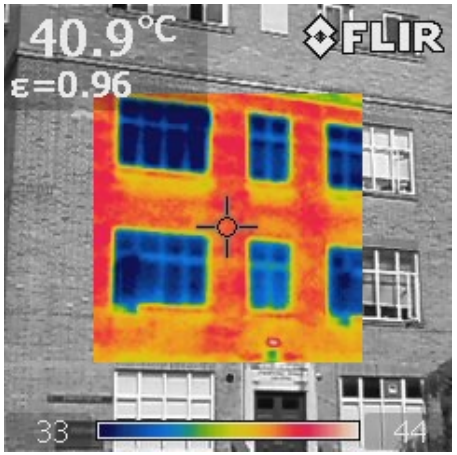


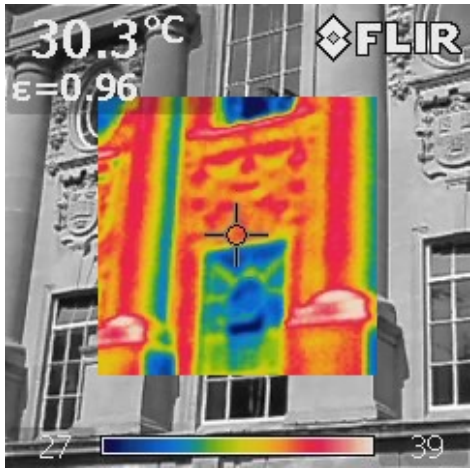
- **Recap: We are measuring electromagnetic radiation (energy).**
  - **Either reflected Sun light from the surface or clouds**
  - **Or thermal emission from the surface or atmosphere**
  - **Or a combination of the two..**

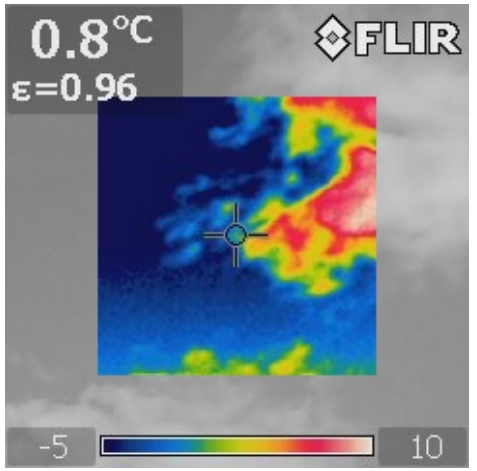
Why measure in the Infrared?



Strong CO_2 band at 667 cm^{-1} (15 μm)





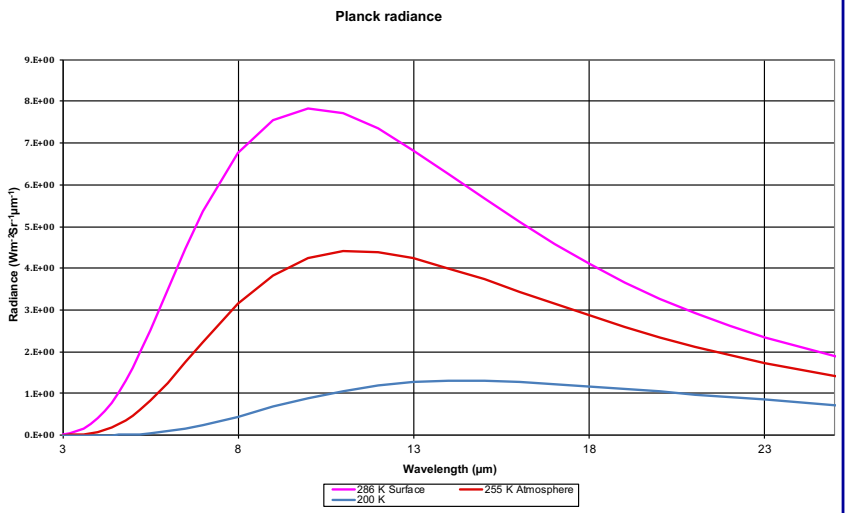


What are we measuring? (Planck functions and blackbodies)

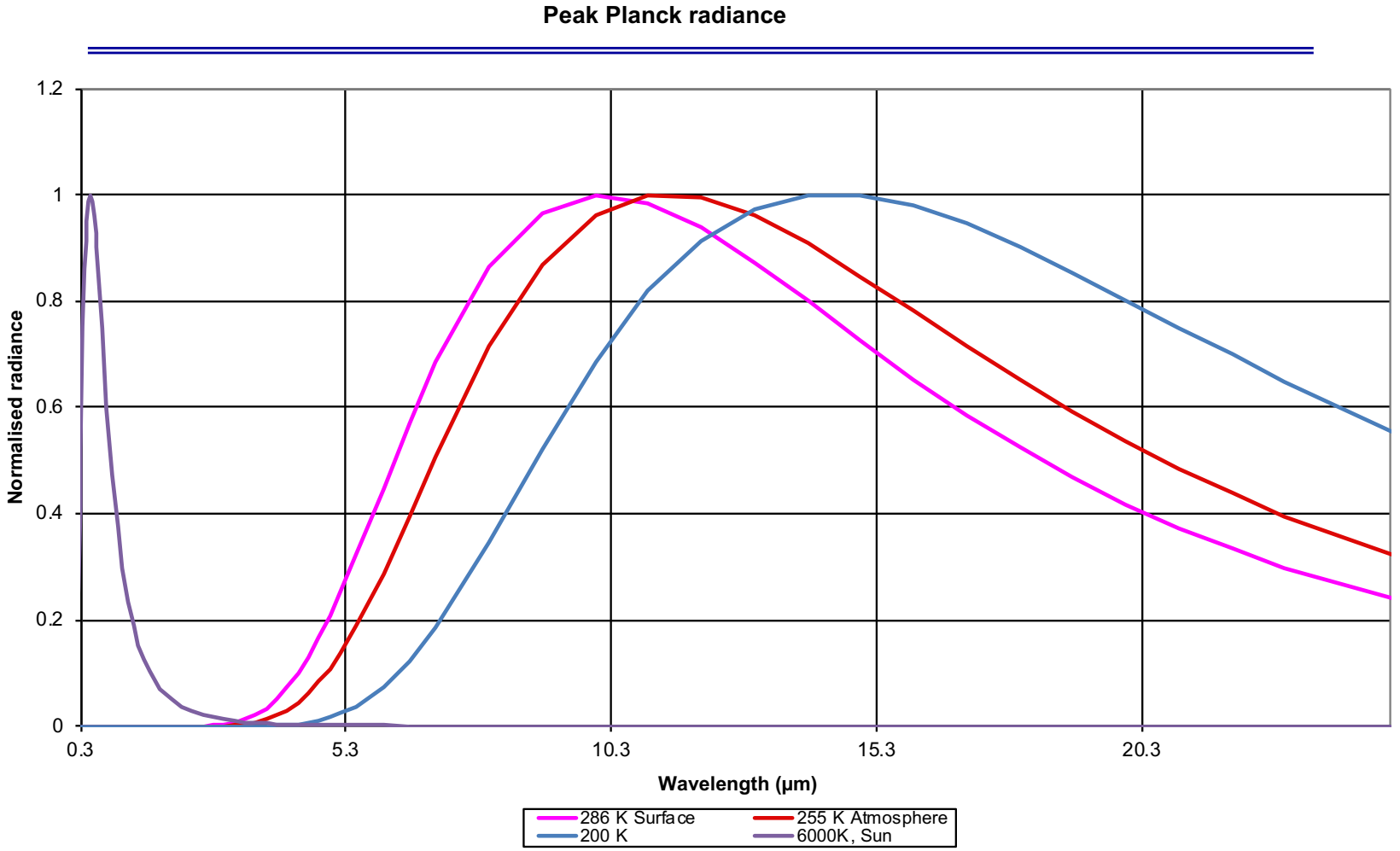
- **Any thing with a temperature above 0 K (absolute zero) emits radiation.**
- **The wavelength dependence of this (spectrum) is determined by the Planck Function.**
  - **h = Planck's constant, λ=wavelength, k_B = Boltzmann's constant, T = Temperature.**

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda k_B T)} - 1}$$

- **This can look complicated, but there are a couple of key results to keep in mind (next slides).**
- **The Planck function has units of spectral radiance:**
  - **$Wm^{-2}sr^{-1}m^{-1}$**
  - **Or $Wm^{-2}sr^{-1}\mu m^{-1}$ depending on your spectral unit.**
- **It describes:**
  - **A flux (Wm^{-2}) per solid angle (sr^{-1}) in a spectral interval (e.g. per μm)**
  - **At a particular temperature.**



Why do we care?



- **The peak of the Planck/Blackbody curve is given by Wien's displacement law: $\lambda_{Max} = \frac{2898}{T}$ μm**
- **E.g. 286 K = 2898/286 = 10.13 μm, 6000 K (Sun) = 0.483 μm.**
- **Useful for picking where in the spectrum to look!**

So, what does this mean for e.g. looking at the Earth from Space?

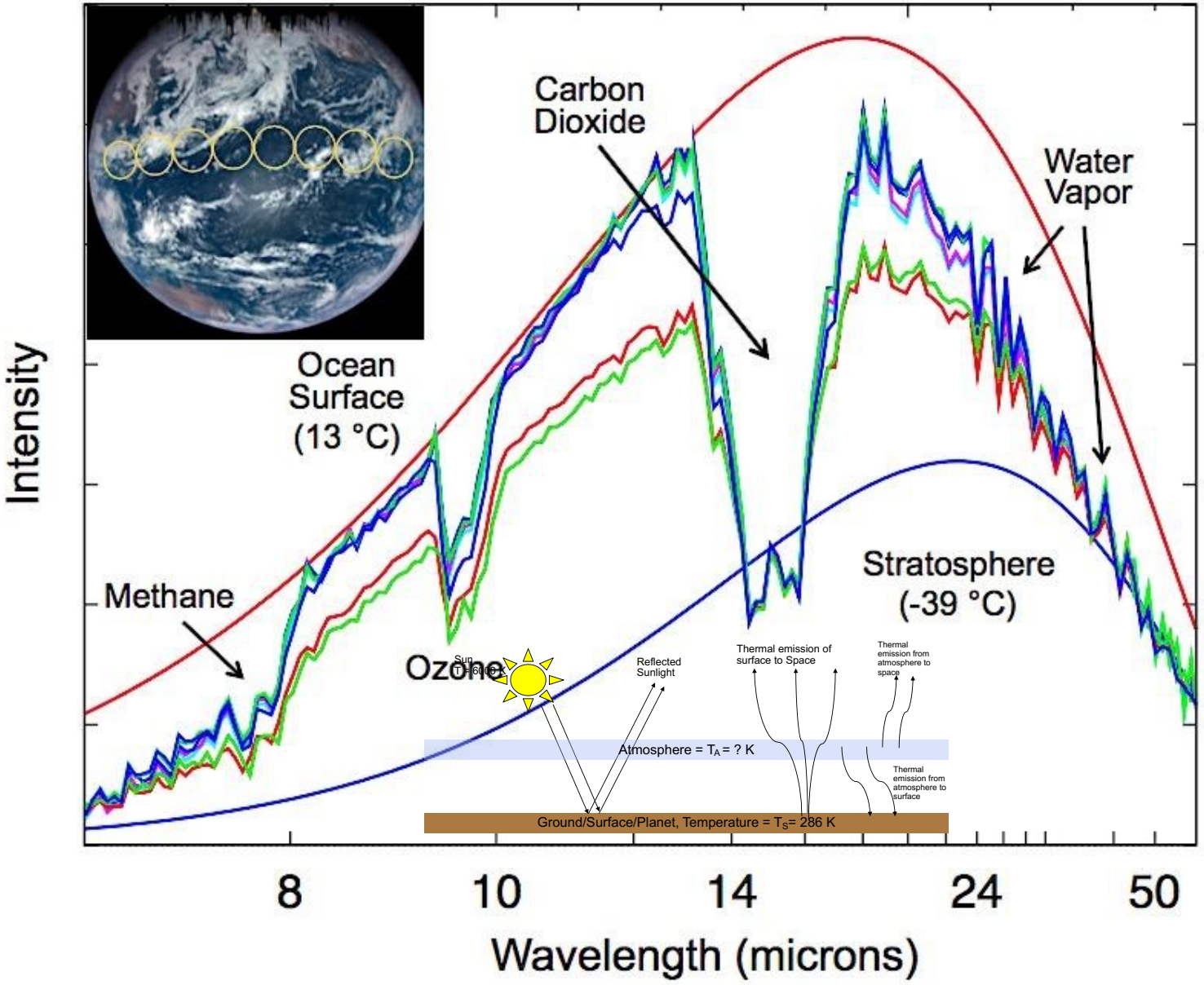


Image by NASA/Goddard Space Flight Center/University of Arizona (Earth image) and Arizona State University (OTES spectrum)

Putting the Maths back in…

$$L^{\uparrow} = \int_0^{\infty} B(z) \frac{dT(z,\infty)}{dz} dz + B(0)T(0,\infty)$$

emission of dz

transmittance from z to space

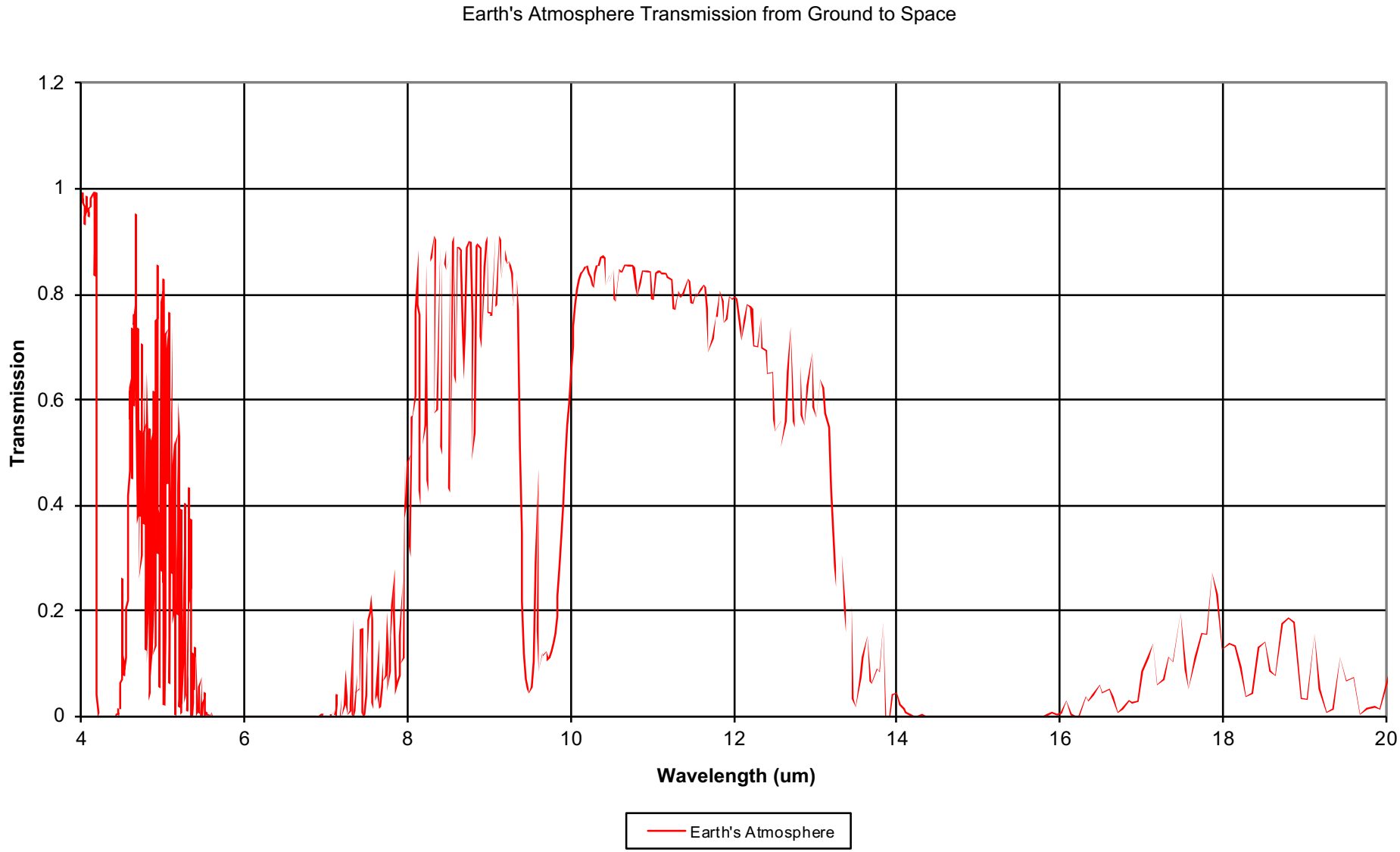
surface emission

transmittance of whole atmosphere

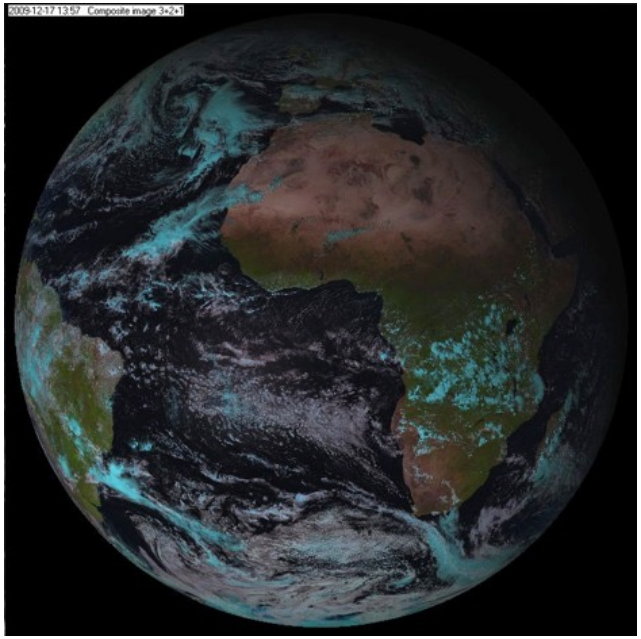
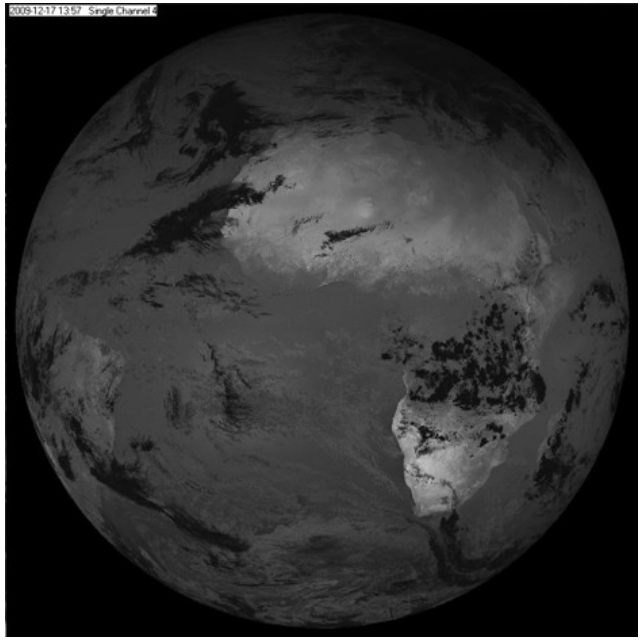
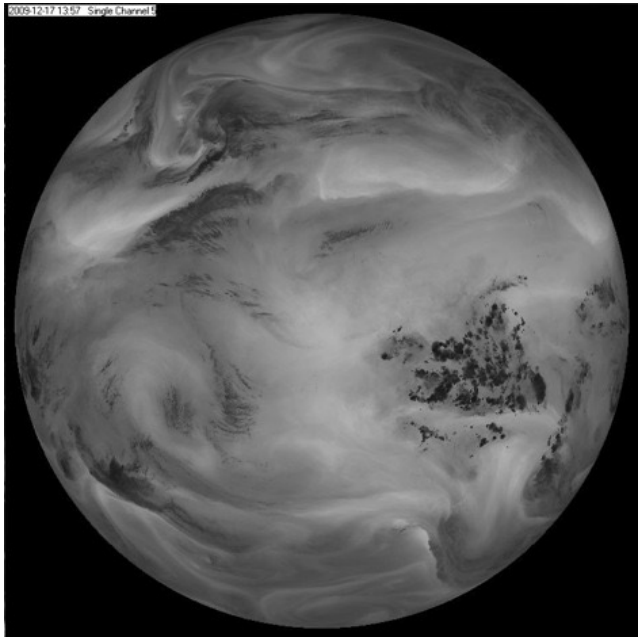
- This is what our instrument measures.
- The radiance observed from the atmosphere in different spectral bands.
- Spectral variation in the absorption properties of the gases in the atmosphere give emission from different altitudes.

- Allows us to ”slice” through different pressure levels in the atmosphere

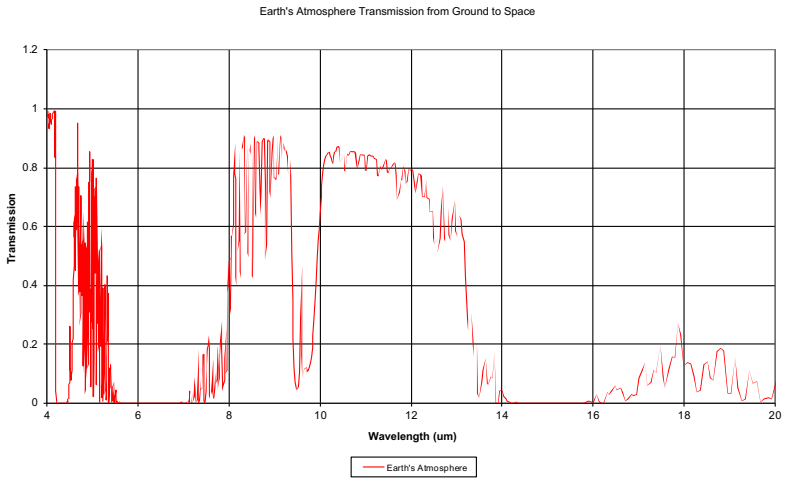
E.g. Earth Lots of spectral features, as we are viewing the ground through the atmosphere



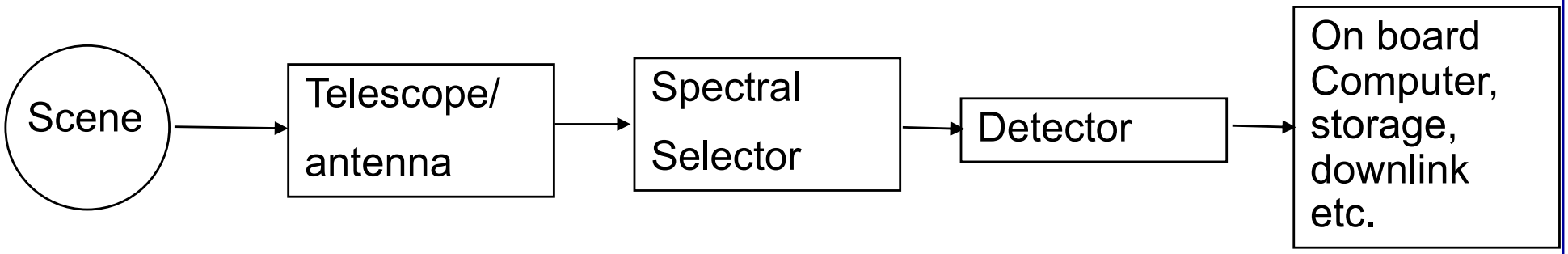
What this looks like in practice…
(Demo)

Images from EUMETSAT, visible composite (left), 3.9μm (centre) and 6.2μm water vapour (right)



A simple example – the thermal-IR camera

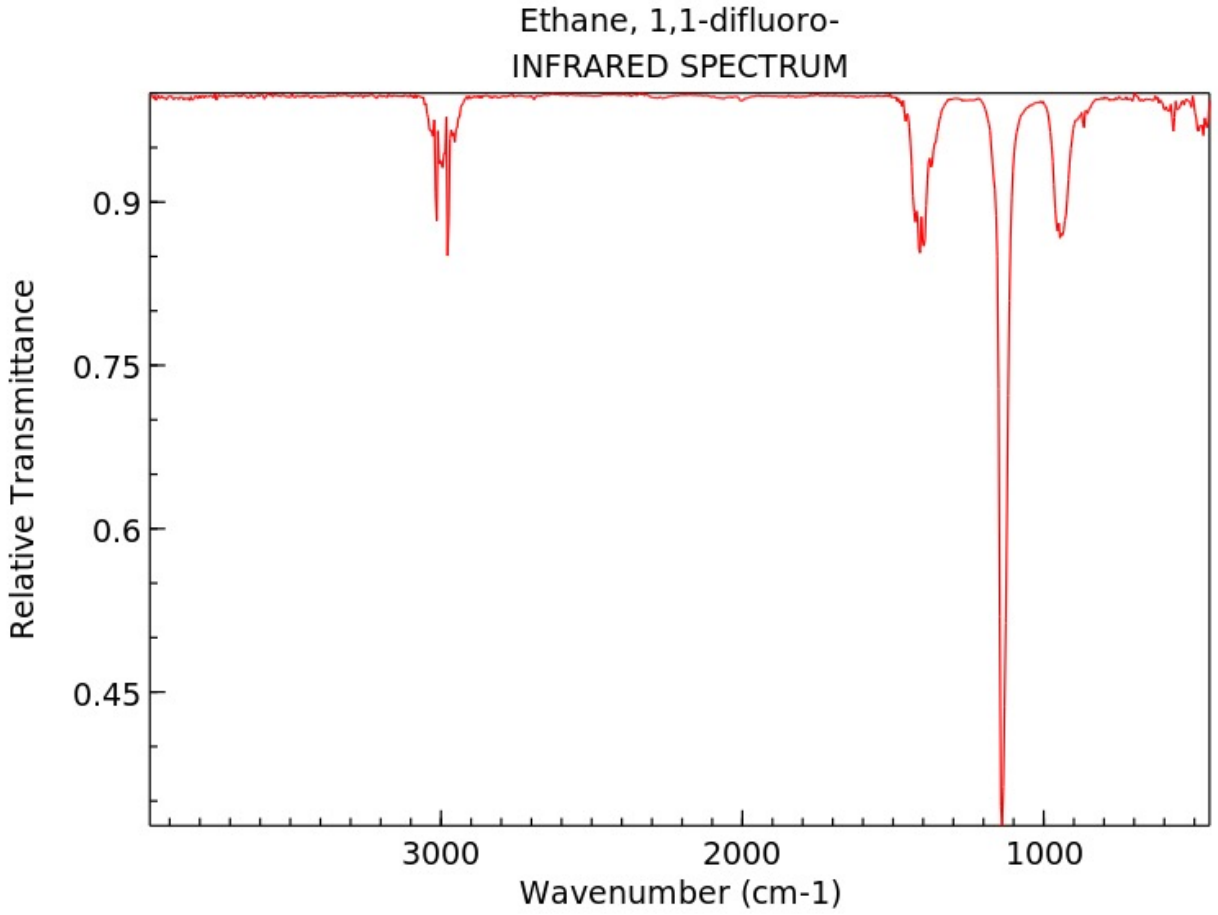


- **All instruments have multiple *sub-systems* in common**
- **For IR camera:**
  - **Telescope = lens**
  - **Spectral selector = broadband IR filter 7 – 14 μm**
  - **Detector = infrared detector array**
  - **Onboard computer = SD card storage, USB etc.**
- **Same principles apply to satellite instruments.**
- **Lets do a demo!**



Imaging warm things through balloons

- One balloon contains regular air
- The other contains “Dustoff” – 1,1-difluoroethane
- Dust off has a very strong absorption feature at ~1130 cm^{-1}/~8.8 μm
- Nice PQR structure almost :-)



NIST Chemistry WebBook (https://webbook.nist.gov/chemistry)

Can now put this into the radiative transfer equation

$$L^{\uparrow} = \int_0^{\infty} B(z) \frac{d\mathcal{T}(z,\infty)}{dz}\, dz + B(0)\mathcal{T}(0,\infty)$$

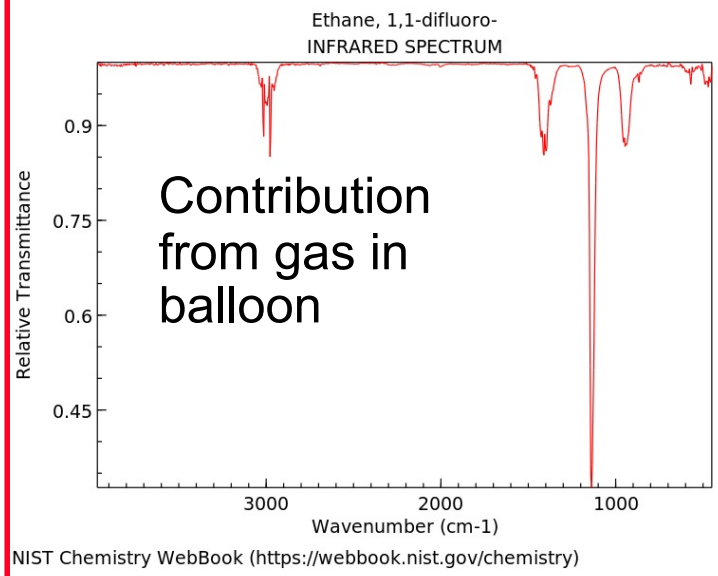
Radiance measured by IR camera

emission of dz

transmittance from z to space

surface emission

transmittance of whole atmosphere

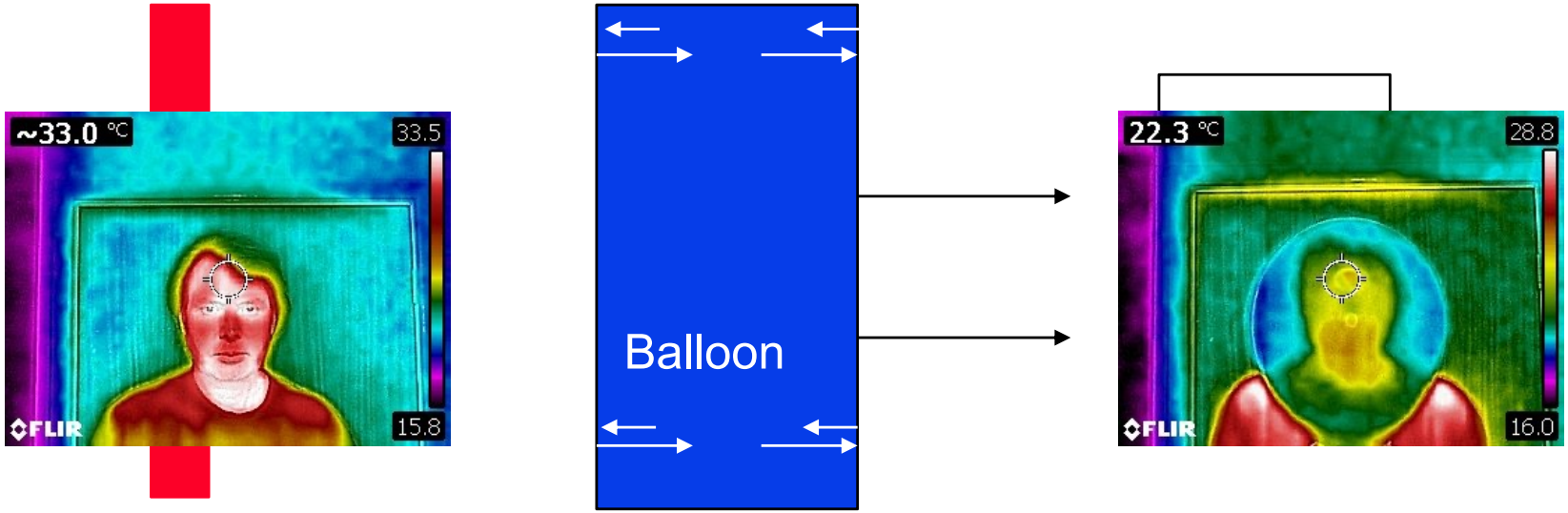


Contribution from gas in balloon

Contribution from IR source (person's face) attenuated by gas in the balloon.

Demo!

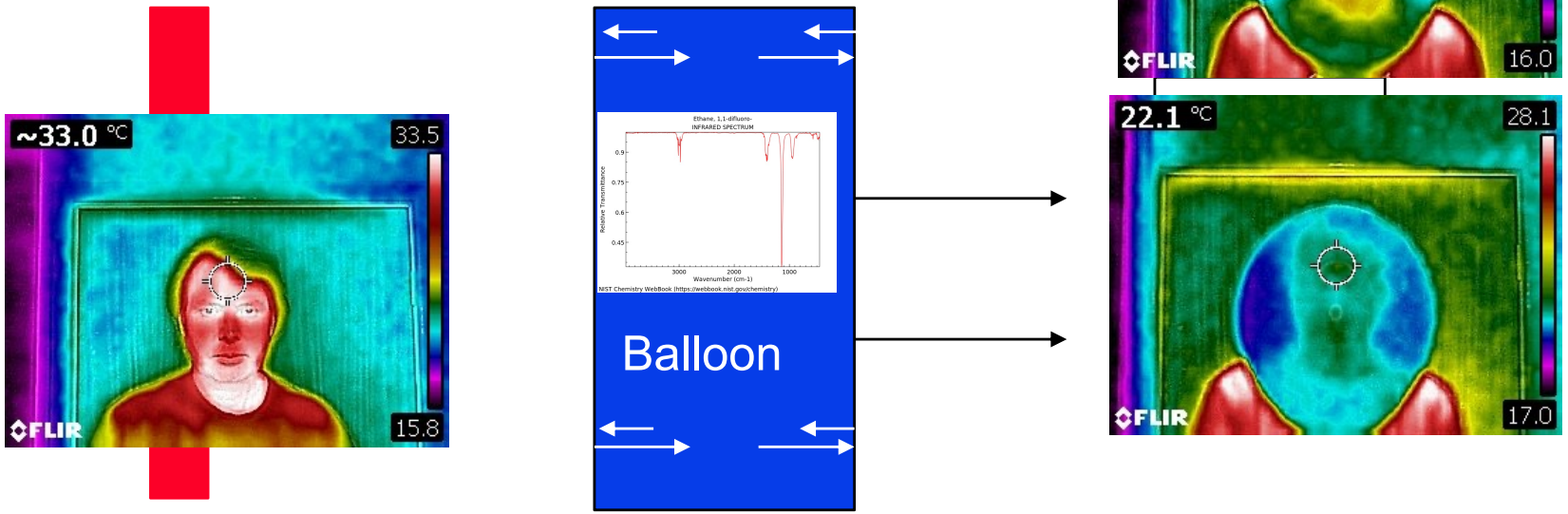
- **What should happen…. Radiances add c.f. the two stream approximation for an atmosphere.**



There will also be reflections off each boundary of the balloon but we will ignore these for simplicity. Also, the camera is hot so will also reflect/absorb at the balloon.

Demo!

- **What should happen…. Radiances add c.f. the two st n atmosphere.**



There will also be reflections off each boundary of the balloon but we will ignore these for simplicity. Also, the camera is hot so will also reflect/absorb at the balloon.

Summary: Note the temperature change

- **When the balloon with air is inserted between the person's face and the camera, the recorded temperature drops.**
  - **Attenuation by the balloon plastic plus the slight increase in air pressure (more water vapour and CO_2 absorption.**
  - **Drops 33 to 28 ish C?**
- **When the "Dustoff" filled balloon is used, temperature drops a lot.**
  - **More like ~25 C**
  - **This is because the radiance from the object (face) is now being attenuated strongly by the gas.**
  - **Bulk of the radiance is now coming from the balloon.**
  - **Measured temperature is now dominated by the balloon, most likely the surface nearest the camera.**
- **For the Earth, its like looking in the CO_2 band compared with an atmospheric window.**

$$L^{\uparrow} = \int_0^{\infty} B(z)\frac{d\mathcal{T}(z,\infty)}{dz}\,dz + B(0)\mathcal{T}(0,\infty)$$

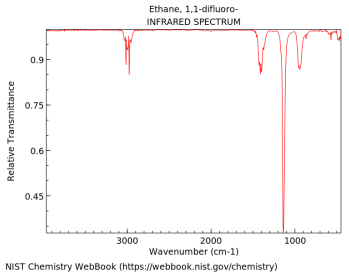
Radiance measured by IR camera

emission of dz

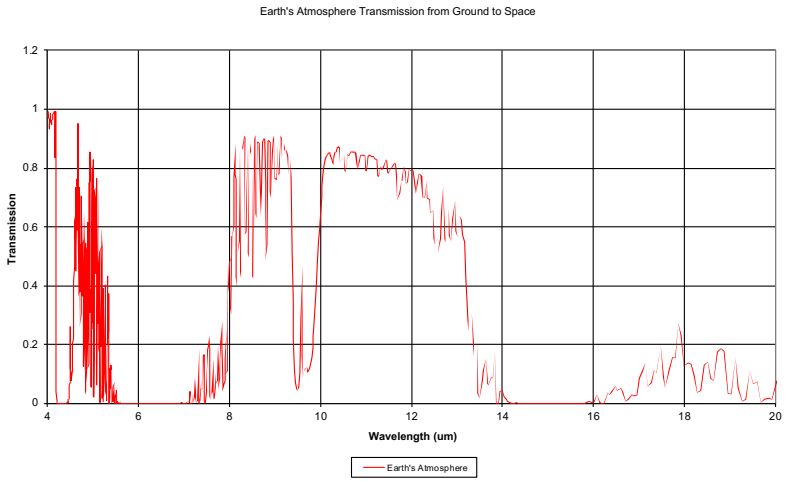
transmittance from z to space

surface emission

transmittance of whole atmosphere



Contribution from IR source (person's face) attenuated by gas in the balloon.



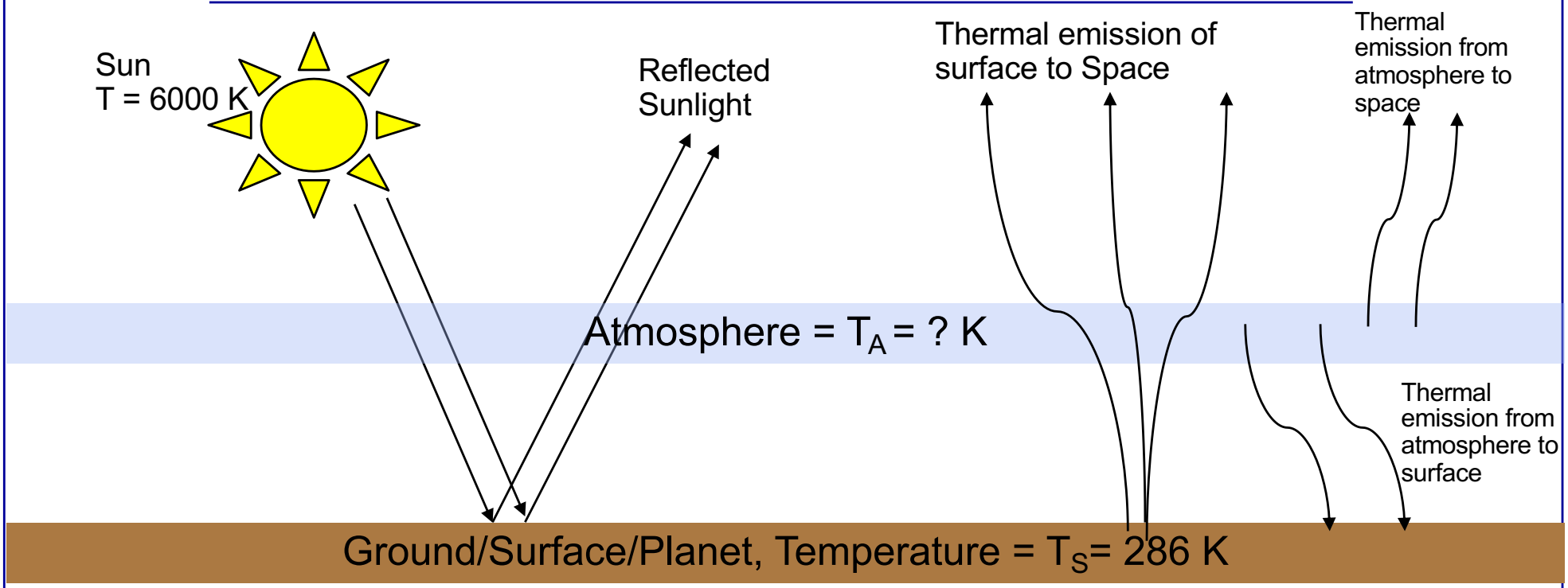
OXFORD

Final comment – optical depth

- **Inserting the "Dust-off" gas filled balloon increases the attenuation in the single from the source to the detector (thermal camera)**
- **This is connected to the concept of *optical depth* that you will have seen in the Radiative Transfer part of the course.**
- **Image below shows the effect of increasing optical depth in the visible during a dust storm on Mars (image:NASA/JPL/Caltech)**
- **I.e. transmission is proportional to exp(-τ)**



What are we measuring in visible and infrared remote sensing?



$$L^{\uparrow} = \int_0^{\infty} B(z) \frac{d\mathcal{T}(z,\infty)}{dz} dz + B(0)\mathcal{T}(0,\infty)$$

emission of dz

transmittance from z to space

surface emission

transmittance of whole atmosphere

OXFORD

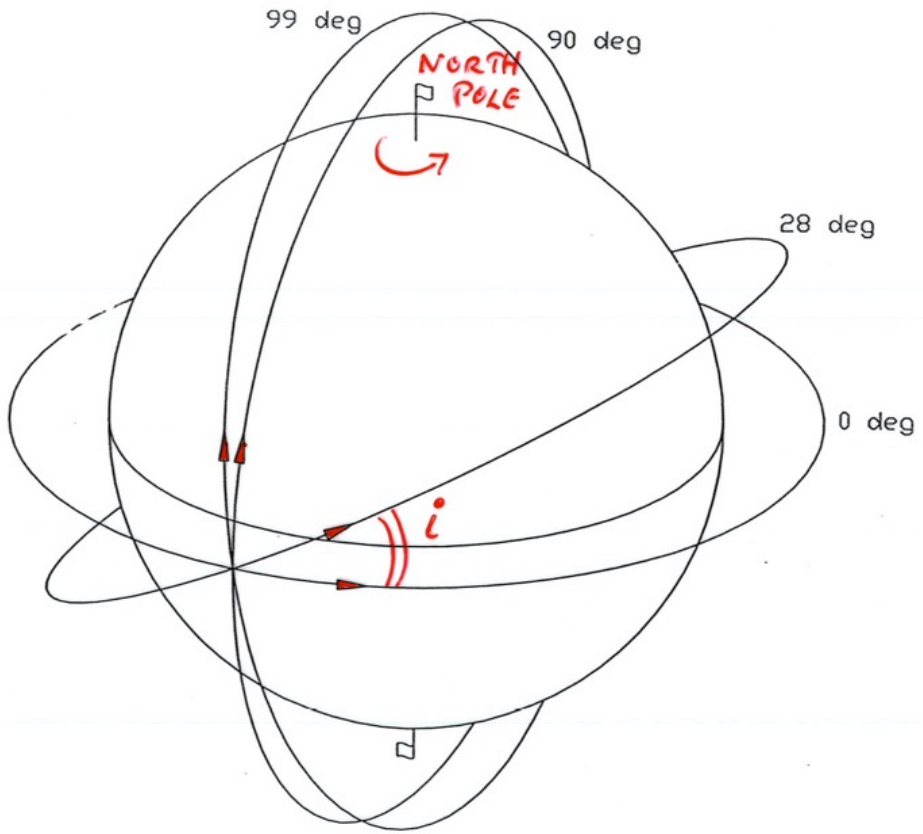
Back to the spacecraft: Where do we place our satellite? Orbits, instruments and design

- **Science Questions!**
  - **If it is a scientific mission, lots of other reasons though…**
  - **Why do we want to measure something?**
  - **Climate monitoring?**
  - **Weather forecasting?**
  - **Geo prospecting?**
  - **Disaster Monitoring?**
- **These lead to *requirements.***
  - **Orbit**
  - **Resolution (spectral and spatial)**
  - **Wavelength range (UV, Visible, IR)**
  - **Integration time**
- **To make measurements**
  - **Spectra**
  - **Radiances**
  - **Images**

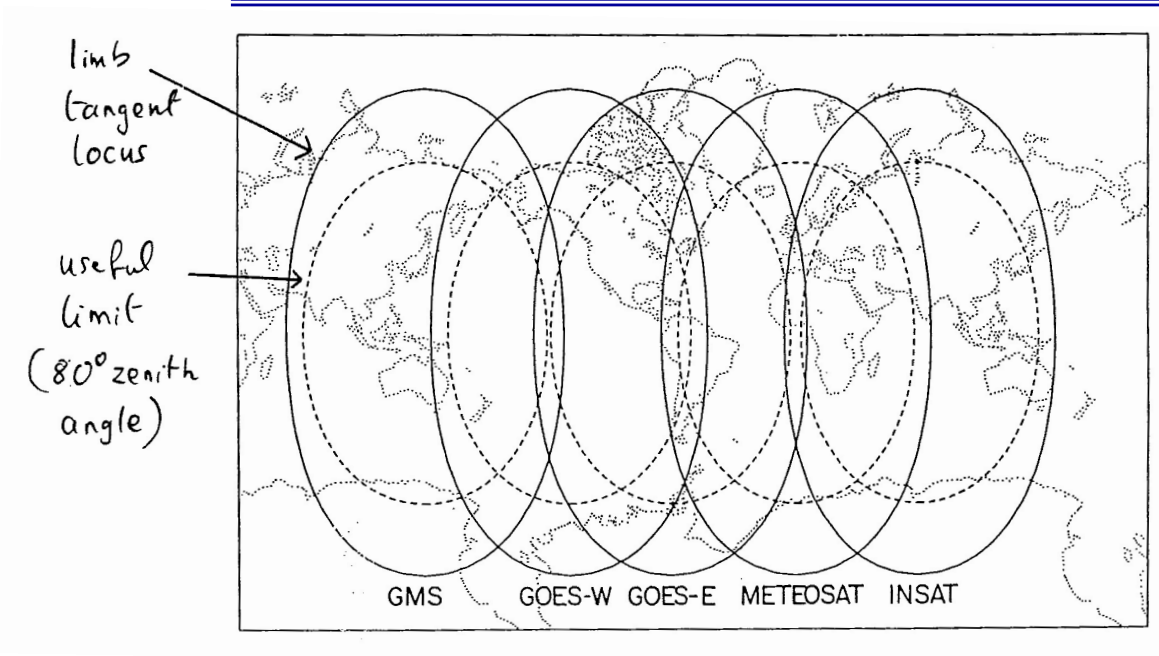
- **To get Data Products**
  - **Ozone abundance**
  - **Climate data**
  - **Pollution monitoring**
  - **Etc.**
- **To answer the Science Questions!**
- **Every step along this path is more complicated than you might think…**

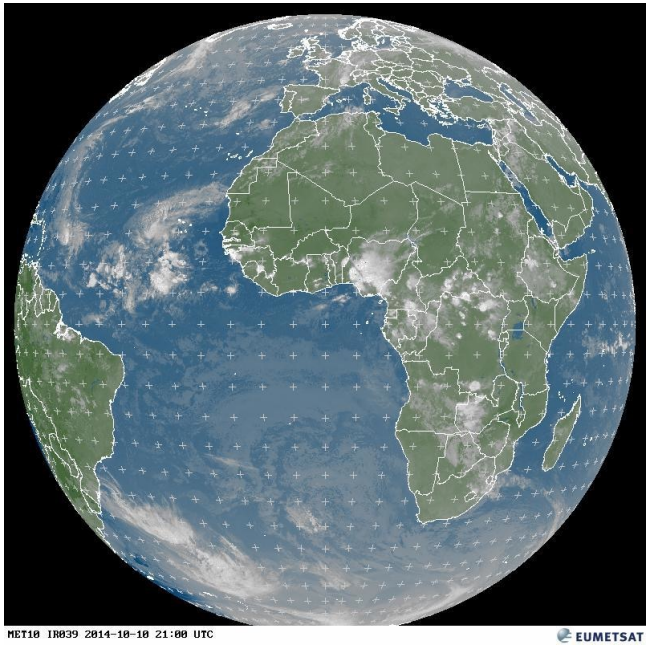
Satellite Orbits

- **Orbits are elliptical with the planet at one focus**
- **For Earth Observation orbits are normally chosen to be circular.**
- **Station Keeping – to maintain the orbit altitude or position satellites use gas jets every few days to every few weeks. This is one limit to the lifetime of the satellite.**
- **Orbital inclination**
  - **This the angle between the orbit plane and the equatorial plane**
  - **>90^{o} means the *opposite* direction to the Earths rotation.**
- **Typical Orbital Altitudes:**
  - **Geostationary orbit ≈35000 km**
  - **Low Earth Orbit (LEO) 300-1000km**
- **We shall consider each in turn**



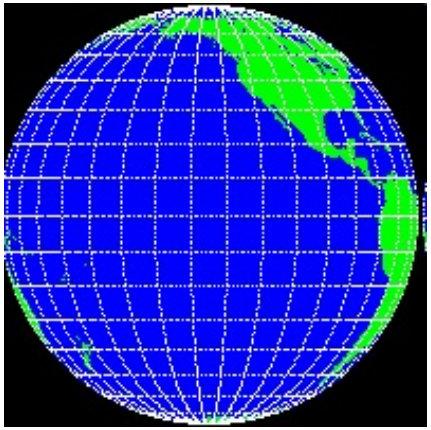
Geostationary Orbit





- **Choose the altitude so that the period is one sidereal day (23hr 56 min or 8616 sec)**
- **Orbits at the Earth rotation rate**
  - **Stays at the same longitude**
  - **Requires height of 35000 km above the surface**
  - **Normally select 0^{O} inclination so the satellite stays on the equator.**
- **Advantages**
  - **Can observe as often as needed, e.g. every 30 minutes or more often over hurricanes etc.**
  - **Satellite can also act as a communication hub.**
- **Disadvantages**
  - **Sees little poleward of 60^{O}**
  - **Needs ≥ 3 satellites to view all around the Earth**
  - **Much further away than LEO so needs bigger, more expensive instruments an launch vehicles.**

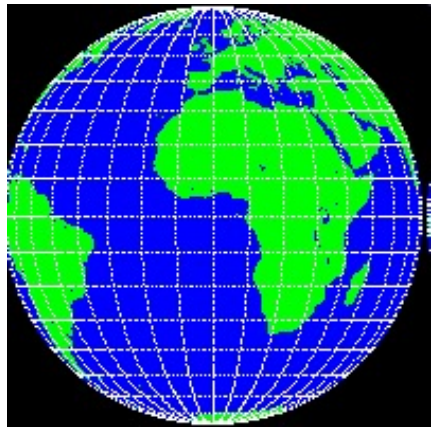
Examples of Current Geostationary Meteorological Satellites



GOES W 135^{O} W



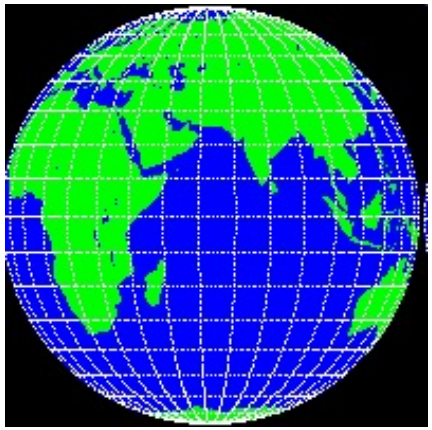
GOES E 75^{O} W



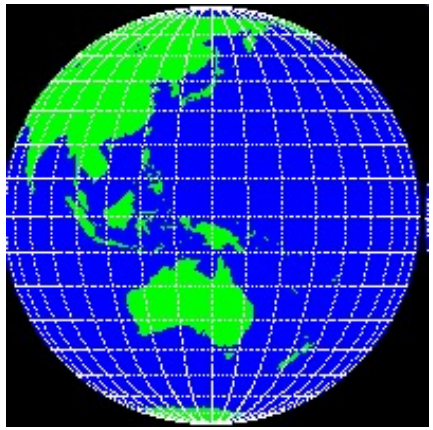
Meteosat 0^{O}

Images from:

http://www.sat.dundee.ac.uk/pdusfaq.html



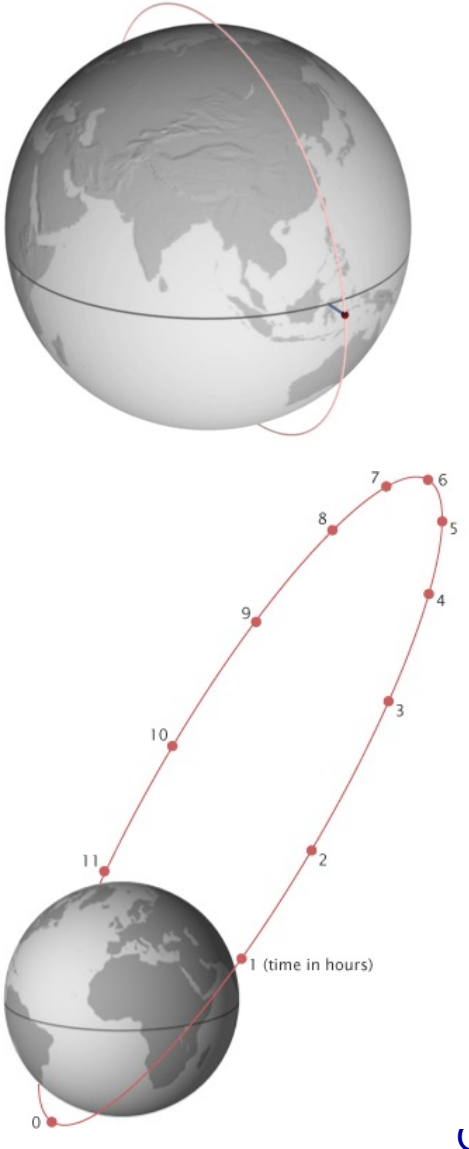
IODC 63^{O} E
(Meteosat)



GMS 140^{O} E

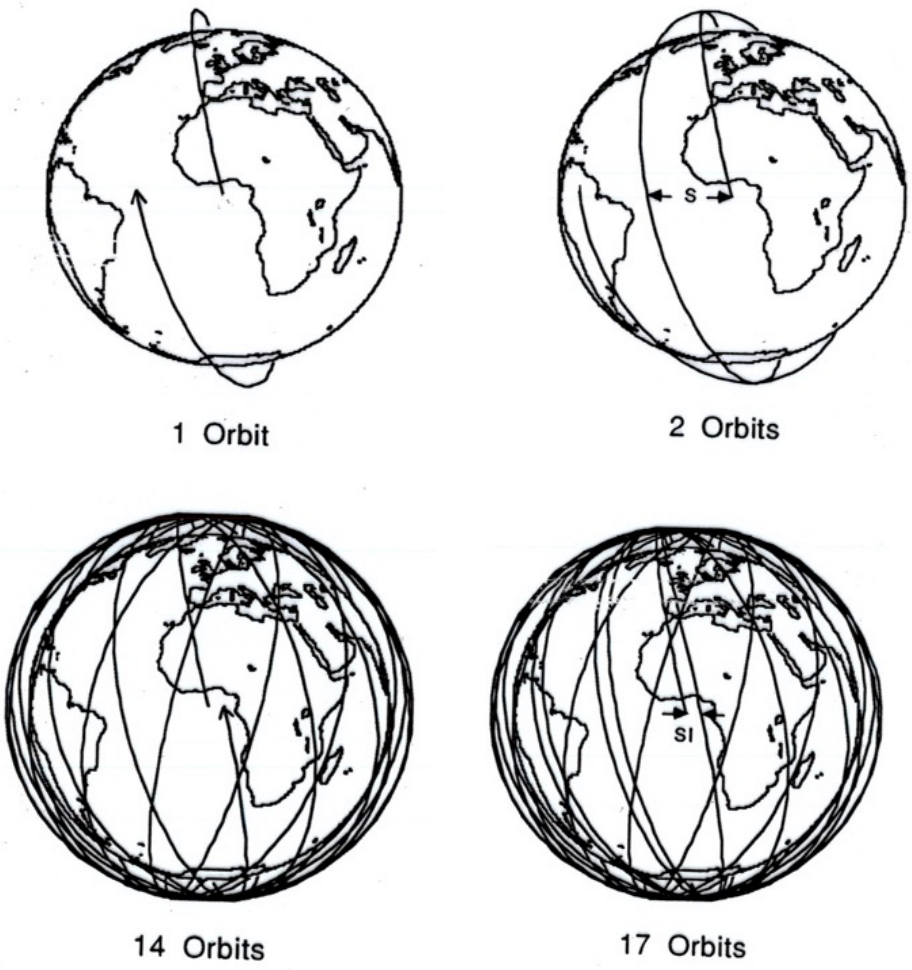
What about Polar coverage?

- **Two typical options.**
- **Low Earth Polar orbit, popular for Earth Observation spacecraft.**
  - **Much more on this later**
- **High elliptical orbit**
  - **Useful for types of applications normally associated with e.g. geosynchronous orbits.**
  - **Kepler's laws of planetary motion. Satellite sweeps out equal area in equal time, so spends most of its time 'hovering' over the high latitudes.**
  - **Popular with e.g. Russia where high latitude coverage is required for communications etc. (e.g. Molniya orbit)**
- **(Images and animation courtesy of NASA (earthobservatory.nasa.gov)**



Low Earth Orbit

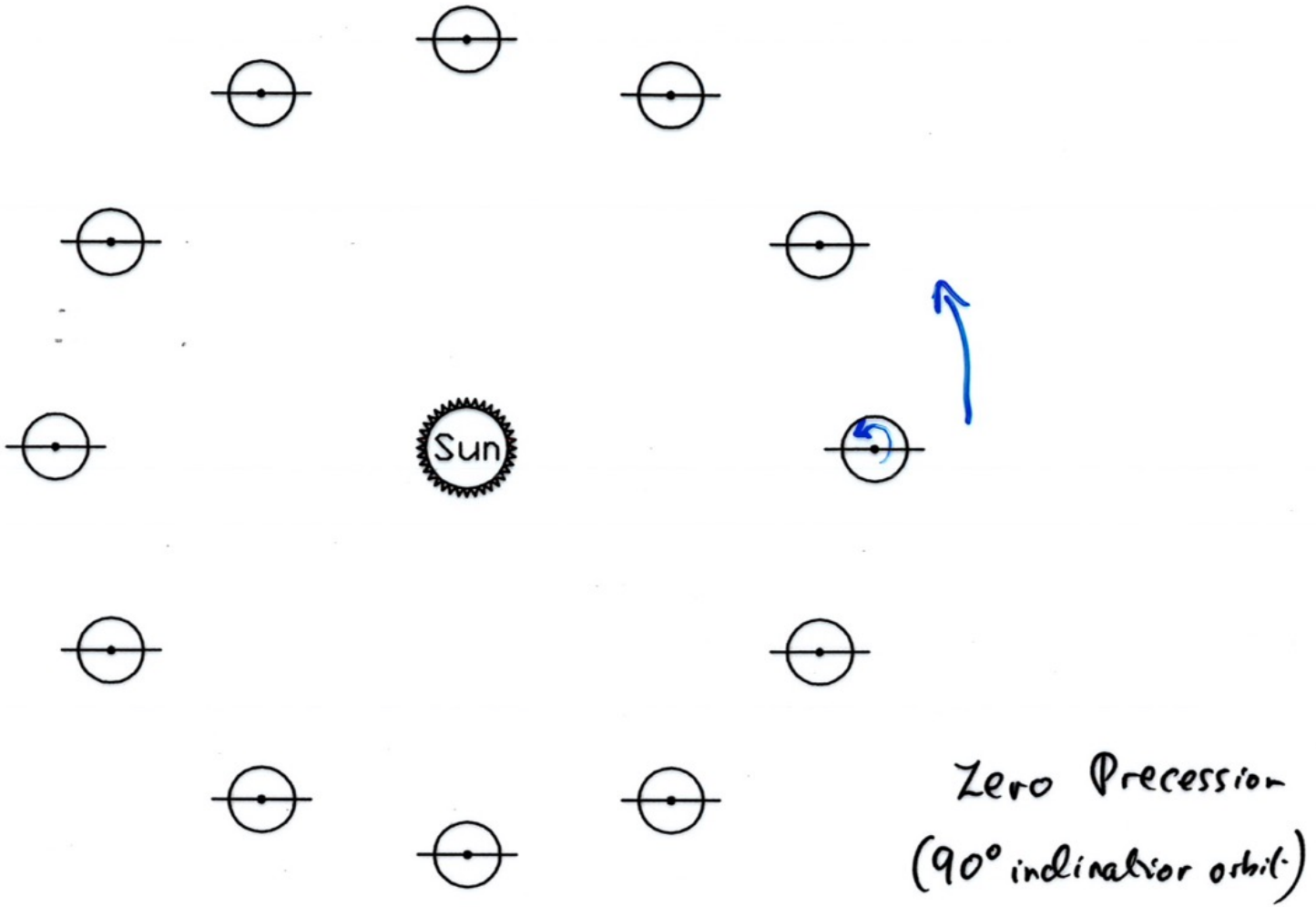
- **Typically 700-1000km altitude**
  - **Orbital periods ≈100-105 minutes**
- **Provides global coverage as the Earth 'turns' beneath the orbit, e.g.**
  - **824km altitude orbit has a 6079 sec period**
  - Earth turns $\frac{6079}{86164} \times 360°$ in 1 orbit = $25.40°$
  - **Subtract the orbit precession in 6079 sec since the orbit is Sun Synchronous (see later)**
  - $25.4° - 0.07° = 25.33$
  - **Hence the orbits are 25.33° longitude apart**
  - **Thus there are** $\frac{360}{25.33} = 14.21$ **orbits/day**
  - **Normally there are 14-15 orbits/day for LEO**
  - **Distance S= 25.33°**
  - **Distance S1=0.21*25.33°**



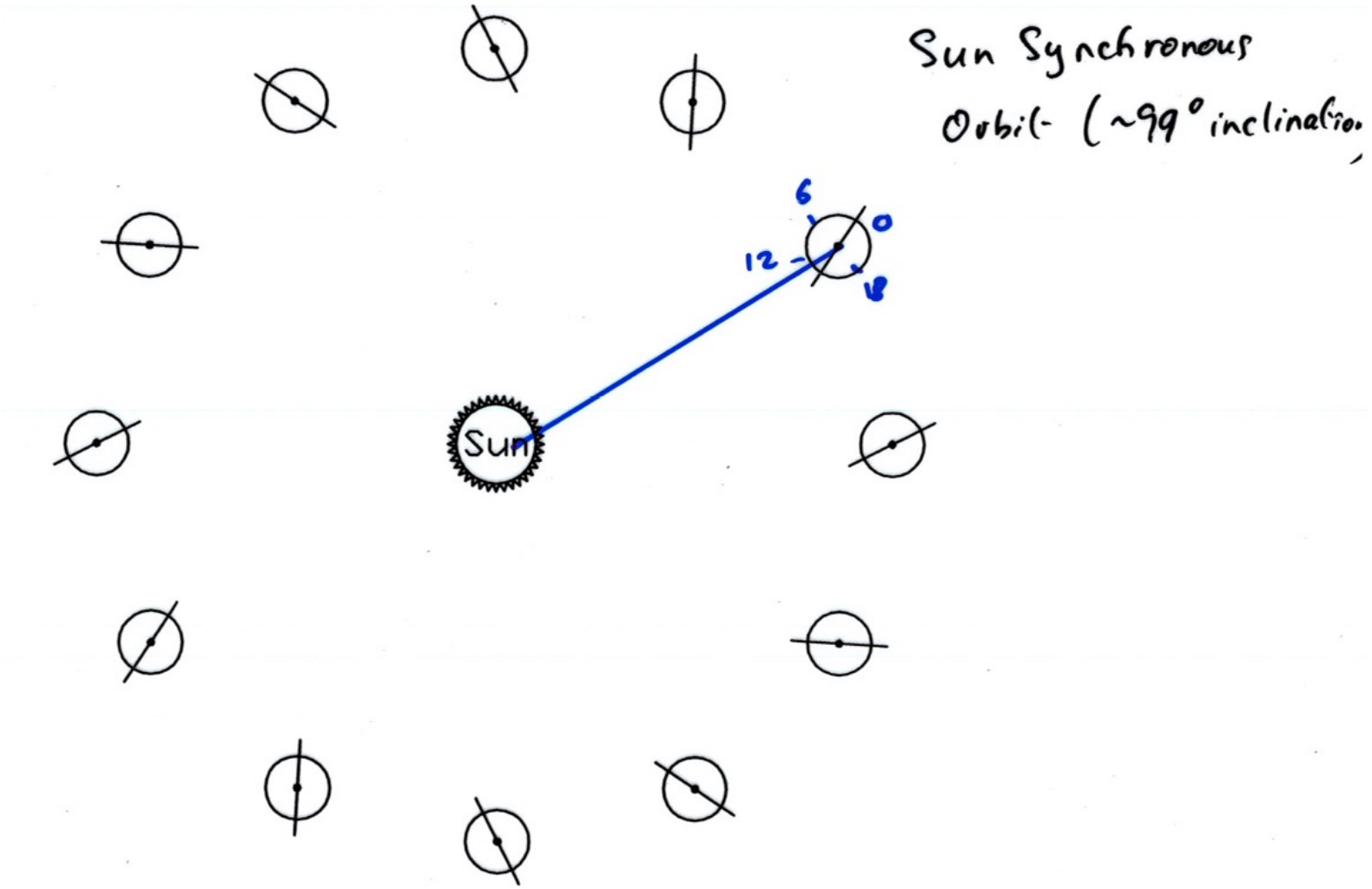
Low Earth Orbit (2)

- **Some applications need an exact repeat of ground track after some number of days, e.g. Radioaltimetry for measuring ocean height.**
  - **E.g. 14 3/16 orbits/day gives a repeat after 16 days**
  - **The ground track is adjusted to achieve this.**
  - **Accurate to hundreds of metres or better cross-track.**
- **Orbit Precession**
- **Orbits in general precess because the Earth is not spherical.**
- **90° inclination**
  - **no precession relative to the inertial frame**
- **> 90° inclination**
  - **orbit plane rotates anti-clockwise relative to the inertial frame looking down on the North Pole**
- **< 90° inclination**
  - **orbit plane rotates clockwise relative to the inertial frame looking down on the North Pole**
- **≈99° plane rotates 360° in 1 year in the same direction as the Earth goes around the Sun.**
  - **Same orientation to the Earth-Sun line always.**
  - **Depends weakly on altitude - 99.4° for 1000km, 98° for 600km**
  - **–Called "Sun Synchronous" orbit**

Orbital Precession
Zero Precession



Sun Synchronous
≈99° inclination

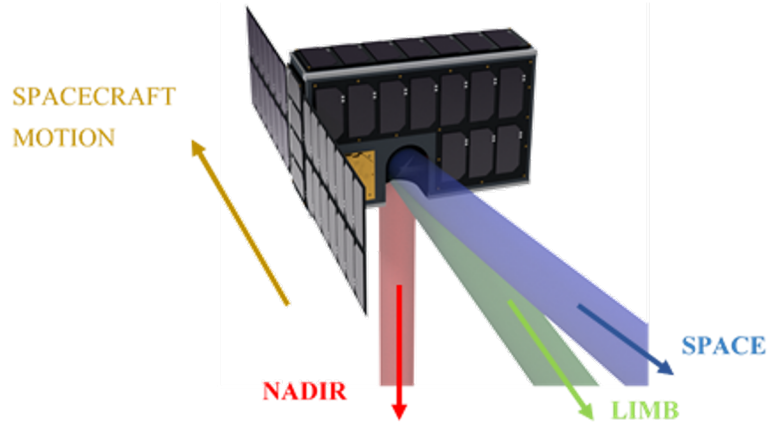


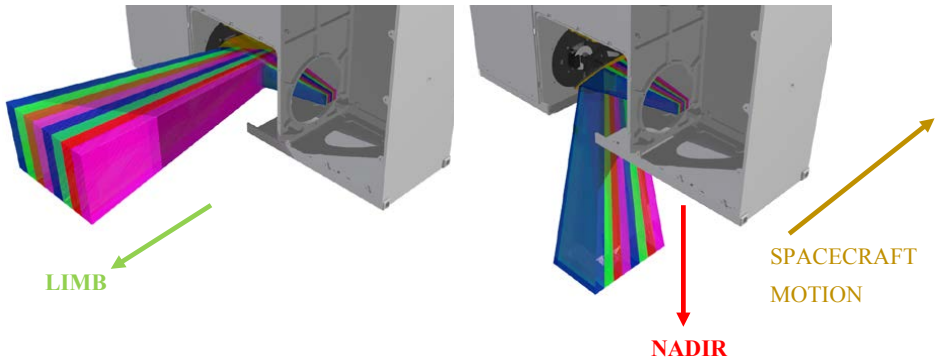
More Sun Synchronous Orbit Action..

- **For Sun Synchronous Orbits the satellite crosses the equator northbound at the same *local* time each orbit throughout the year, and southbound 12 hours *local* time later (e.g. a 3PM/3AM orbit)**
- **The time is chosen to give the best observations based on**
  - **Cloudiness (more clouds in the afternoon overland)**
  - **Diurnally varying stratospheric chemistry etc**
- **Advantages**
  - **Same local time observing times each day**
  - **Same Sun geometry on the instruments and spacecraft at all times – helps thermal control of the spacecraft by allowing the dumping of heat on the dark side, provides solar power and light on the other.**
- **Disadvantages**
  - **Cannot sample diurnal cycles with one satellite**
  - **Inclination must be ≈99°**
  - **Solar occultation instruments see a very limited latitude range**
- **Other Orbits**
  - **Most Earth Observation orbits are Sun Synchronous, but not all.**
  - **E.g. Upper Atmosphere Research Satellite (UARS) was in a 57° 585km circular orbit**
  - **Local times moved forward 20min/day**

Field of View
Angles, distance, ground track

- **For a remote sensing instrument vital numbers to know are:**
- **Instantaneous Field of View (IFoV)**
  - **The footprint of a detector due to the *angular resolution of the instrument***
  - **This is set by the fundamental optical properties of the instrument, things like focal ratio, telescope diameter, detector size etendue (AΩ) etc.**
  - **It is doesn't take into account integration time ("shutter speed"), it's the best angular resolution the instrument can achieve without post-processing.**
  - **Usually quoted in mradians (10^{-3} mrad) or μradians (10^{-6} μrad). E.g. an IFoV of 260 μrad.**
- **Total Field of View (FoV)**
  - **The complete field of view of the instrument.**
  - **Usually the sum of all the pixels in a particular dimension**
  - **Annoyingly often quoted in degrees. E.g. 640 x 480 pixels with IFoV = 260 μrad = 0.1664 x 0.1248 rad = 9.5 x 7.2 degrees**





Field of View to ground track spatial resolution

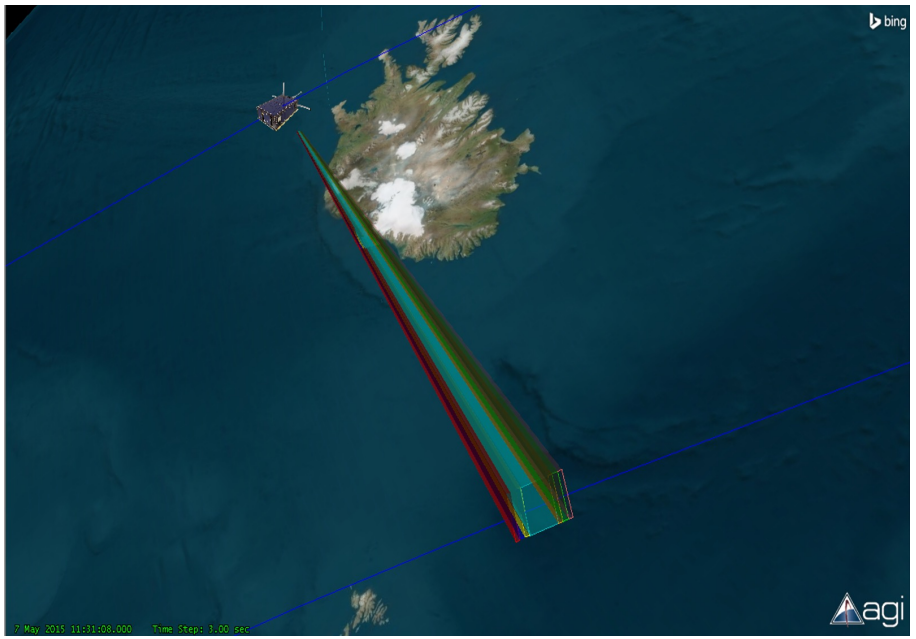
- If we know the orbit altitude and IFoV we can convert the ground track resolution into a distance.
- If θ is the half angle of the IFoV then:

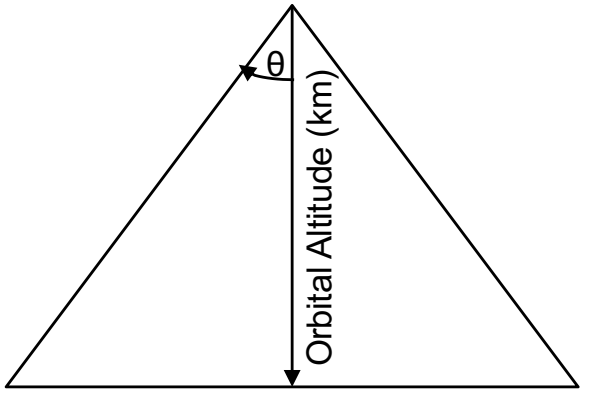
$$\tan\theta = \frac{ground\ track}{2 \times orbital\ altitude}$$

- So using the examples before and a 700 km orbit:
- IFoV = 260 µrad
  - IFoV =$\tan(130\ \mu rad) \times 2 \times 700$
  - IFoV = 182 m

- **This gives the GROUND SAMPLE DISTANCE, a fundamental measure in remote sensing data**
- Next, we work out the time it takes the spacecraft during this time:
- Orbital velocity for 700 km altitude = 7.5 km/s (see earlier equation).

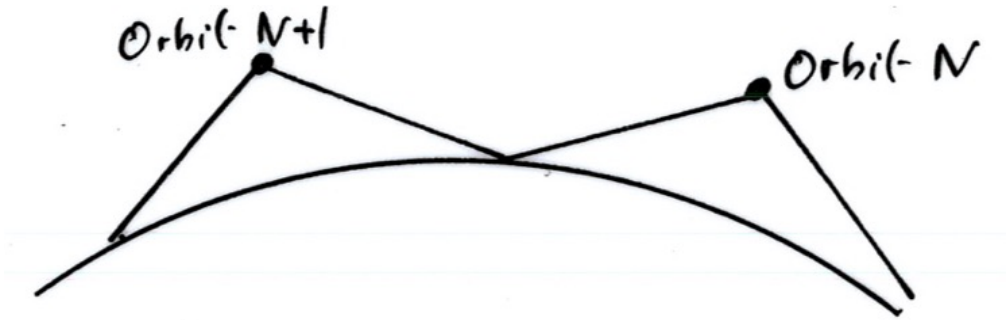
- Integration time:
  - Time to travel ground track = 182/7500 = 0.024 s
  - Read out rate = 1/0.024 = 41 Hz





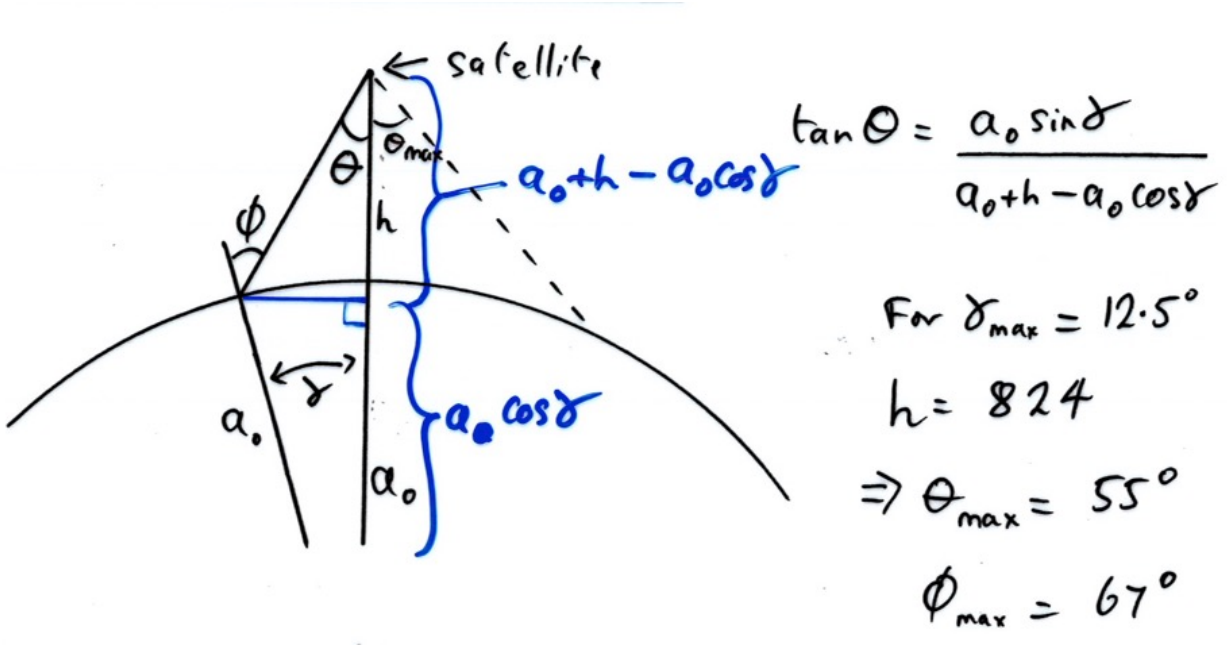
Pick your Orbit….
Factors affecting orbit height for LEO

- **Telescope size – lowest altitude is preferred (more mass=more money)**
- **Orbital decay due to atmospheric drag- >700km is preferred to (UARS decayed 1km every few months)**
- **Swath width at the equator**
  - **At low altitude you need bigger scan angles to achieve the same spatial coverage on the surface, leading to a limb darkening with the weighting functions moving up in altitude.**
  - **Prefer altitudes of >800km.**



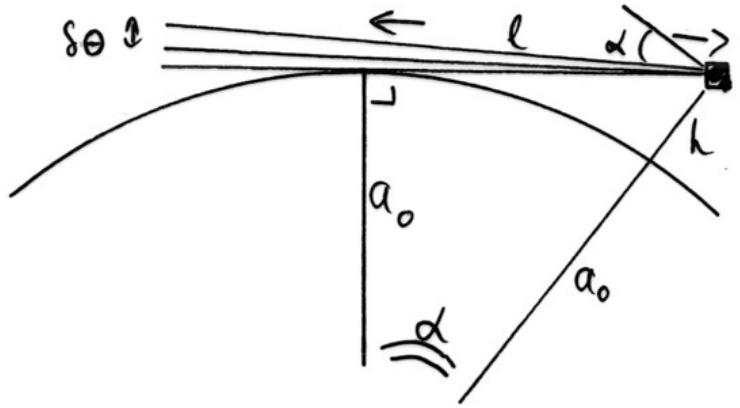
- **Orbit repeat time, as described for radioaltimetry this may affect the precise choice for some payloads**
- **Launch vehicle, cheaper for lower orbits**
- **The inclination required for Sun Synchronous is a negligible factor so the typical range is <u>700 to 1000km</u>**
- **Formation flying – pick same orbits to allow simultaneous coverage from multiple spacecraft.**

Low Earth Orbit
Vertical or Downward Viewing



- **From the diagram, the large effects on the transmittances, weighting functions etc at the edge of the scan can be seen since $1/\cos\varphi_{max}=2.56$**
- **Scanning options:**
  - **None – nadir view only θ=0, e.g. BUV instruments. Good for simple instruments where you do not need cross-track spatial resolution.**
  - **Cross track scanning – mechanically scan single view from side to side normal to the velocity vector, e.g. by rotating the mirror. Pixel size will change across the scan (AP03 lab experiment from last year!) and the *weighting functions move up and down.***

Limb Viewing Geometry



$$\cos\alpha = \frac{a_0}{a_0 + h}$$

$h = 705$

$\Rightarrow \alpha = 26^\circ$

$l = 3100$ km.

- **1km vertical distance at the tangent point corresponds to** $\delta\theta = \frac{1}{3100}\text{ radians} = 0.02^\circ$
  - **i.e. very small.**
- **Options:**
  - **Scan up and down the limb mechanically. Scan to tangent point altitude >150km to obtain the zero radiance calibration. Scan range of 3-4° is normally adequate, e.g. instruments such as HIRDLS, ISAMS, MLS.**
  - **Use a detector array to stare at the atmosphere (e.g. CLAES or Mars Climate Sounder). Need to occasionally scan up to get a zero radiance calibration.**
- **Horizontal scanning:**
  - **MLS, CLAES looked sideways (90° to velocity)**
  - **ISAMS looked sideways ±4° to use the satellite velocity vector to cancel the Earth's rotation velocity vector**
  - **HIRDLS looked backwards at +20 -40° to get better horizontal coverage.**

Example Operational Low Earth Orbit Satellites

- **US TIROS-N/ATN**
  - **Sun Synchronous Orbit, 870 km, 98.73° inclination orbit, 102.14 min period**
  - **Instruments:**
  - **Advanced Very High Resolution Radiometer (AVHRR/3)**
  - **Advanced Microwave Sounding Unit (AMSU)**
  - **High resolution Infrared Radiation Sounder (HIRS/4)**
  - **Solar Backscatter Ultraviolet Spectral Radiometer (SBUV/2)**
  - **Space Environment Monitor (SEM/2)**
  - **Microwave Humidity Sounder**
- **EUMETSAT METOP-A/B**
  - **Sun synchronous Orbit, 817 km, 98.7° inclination orbit, 101 minute period**
  - **Instruments**
  - **AVHRR/3**
  - **SEM**
  - **Microwave Humidity Sounder (MHS)**
  - **Infrared Atmospheric Sounding Interferometer IASI**
  - **Global Ozone Monitoring Experiment (GOME)**
  - **SBUV/2**
  - **Advanced Scatterometer (ASCAT)**
  - **Global Navigation Satellite System Receiver for Atmospheric Sounding (GRAS)**

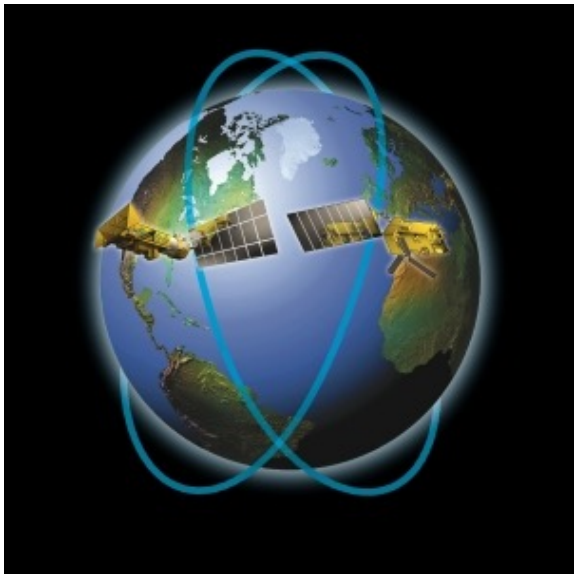
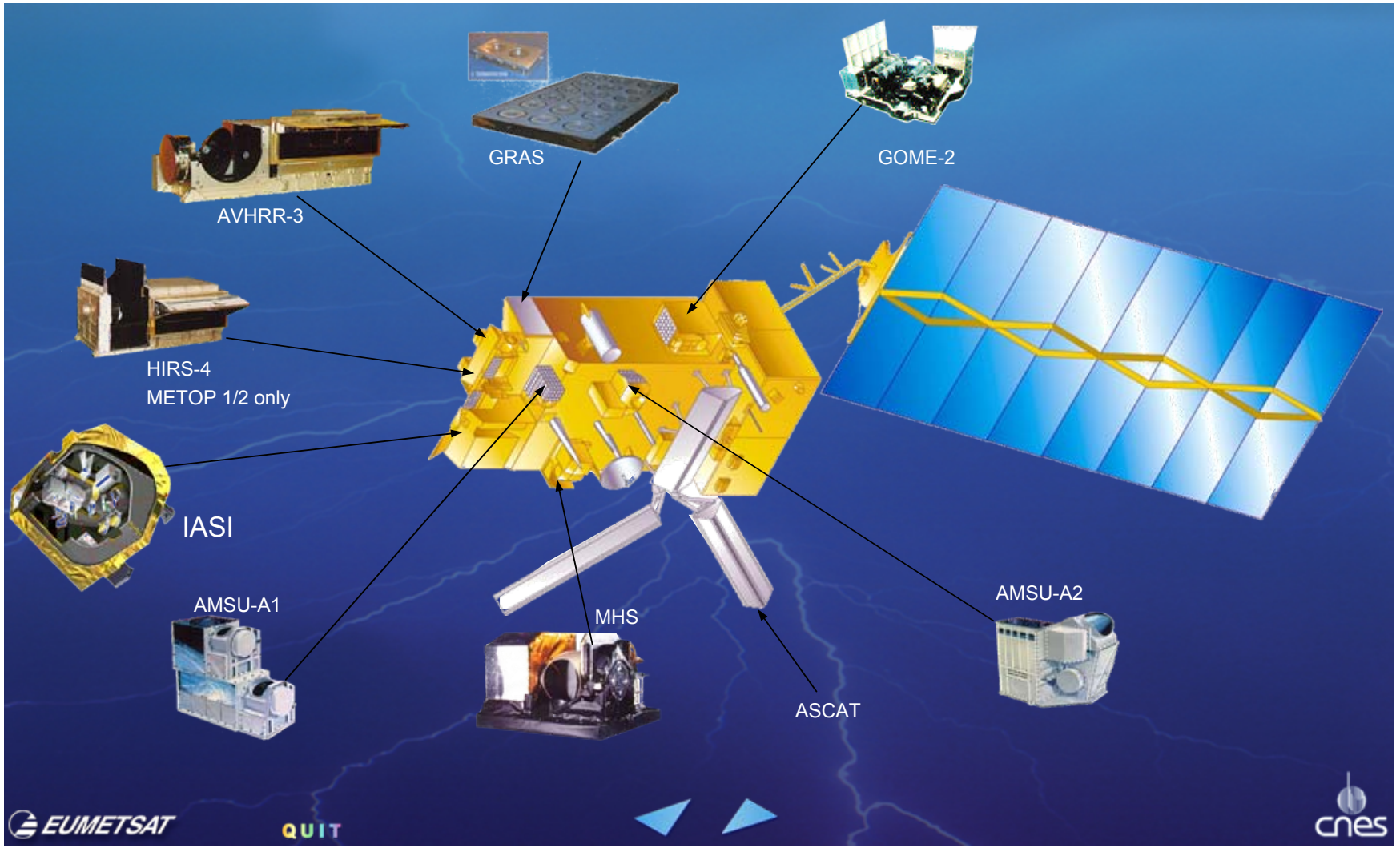


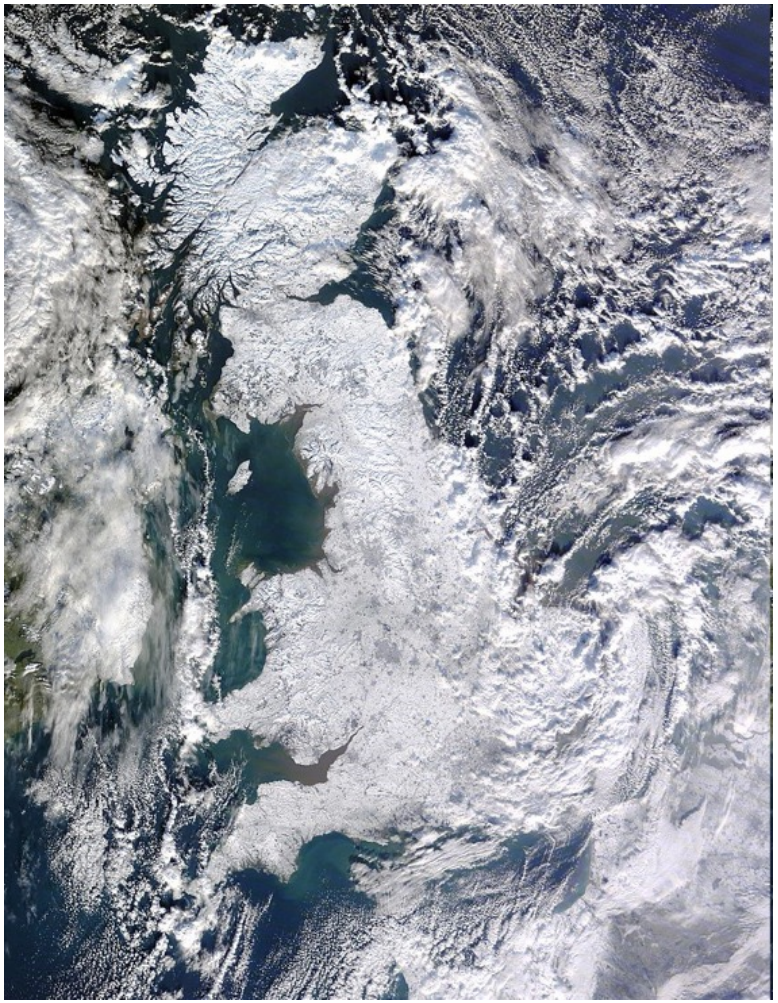
Image: EUMETSAT, METOP and NOAA satellites in complimentary orbits. METOP shares instruments with the equivalent NOAA Satellite. METOP 9:30AM orbit

EUMETSAT METOP-A



METOP-A Instrument locations, Image: EUMETSAT/CNES

MODIS Action
2010 Snow

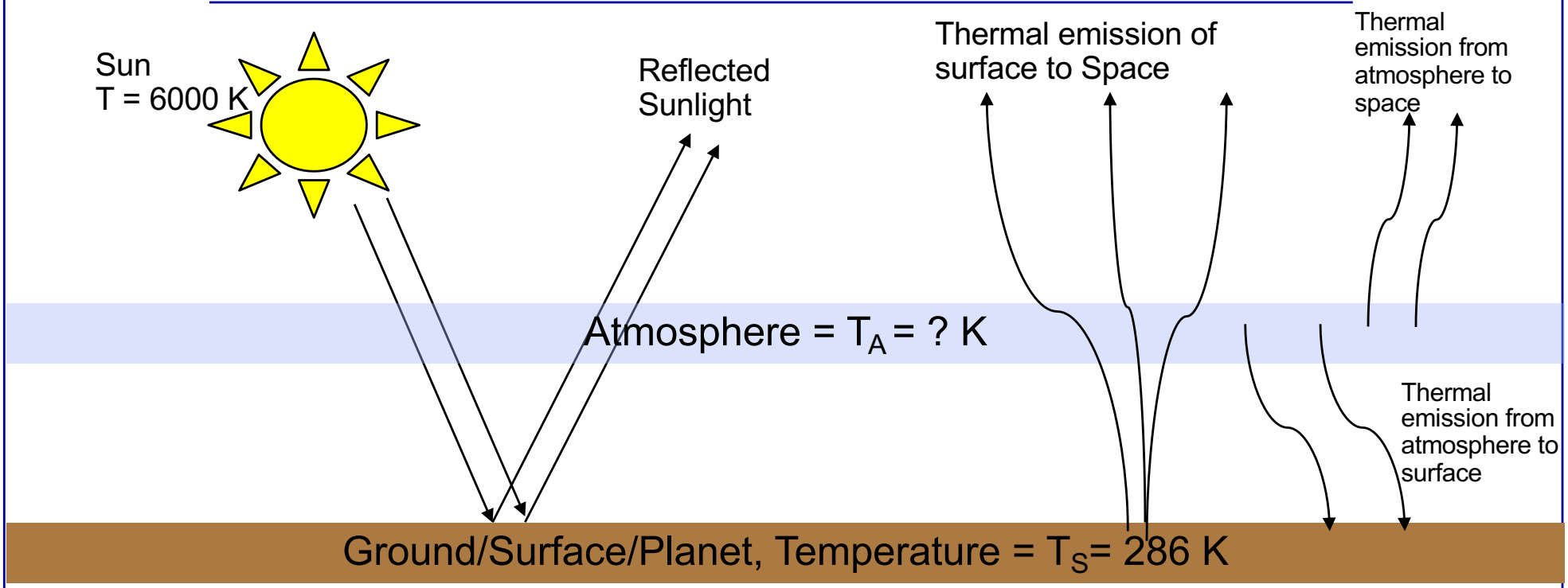


- **Measured by MODIS, the moderate resolution imaging spectrometer, also on NASA's Terra satellite (image NASA/GSFC).**

Which Geometry is best?

- **Limb**
  - **Poor horizontal resolution (several hundred km)**
  - **Very good vertical resolution, but you need a narrow, very well defined field of view.**
  - **Pointing accuracy is essential, small angles=large distances. Need very good spacecraft stability.**
  - **Cannot see below the cloud level**
- **Occultation**
  - **Same restrictions as limb viewing**
  - **Also, you only get two profiles per orbit (sunrise, sunset)**
  - **Sun is a very powerful source at the Earth (but should work well at Mars…)**
- **Vertical (e.g. nadir)**
  - **Very good horizontal resolution possible**
  - **Poor vertical resolution in the atmosphere (≥10km weighting function width)**
  - **No difficult spacecraft stability requirements.**
  - **Shorter path length reduces the number of species that can be measured**
    - **E.g. Ozone, tropospheric water vapour, tropospheric CO, etc**
- **However, normally there is no choice for the problem at hand**
  - **E.g. Limb for trace gases, vertical for sea surface temperature.**

Summary 1: What are we measuring in visible and infrared remote sensing?



$$L^{\uparrow} = \int_0^{\infty} B(z) \frac{d\mathcal{T}(z,\infty)}{dz} dz + B(0)\mathcal{T}(0,\infty)$$

emission of dz

transmittance from z to space

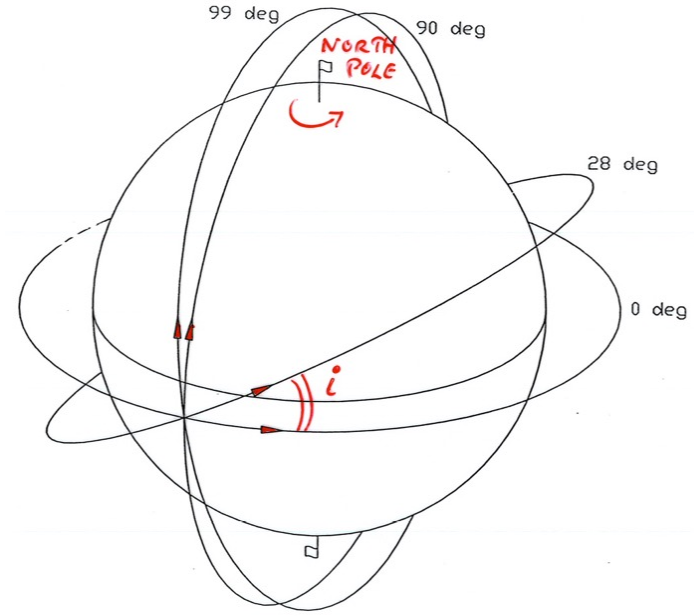
surface emission

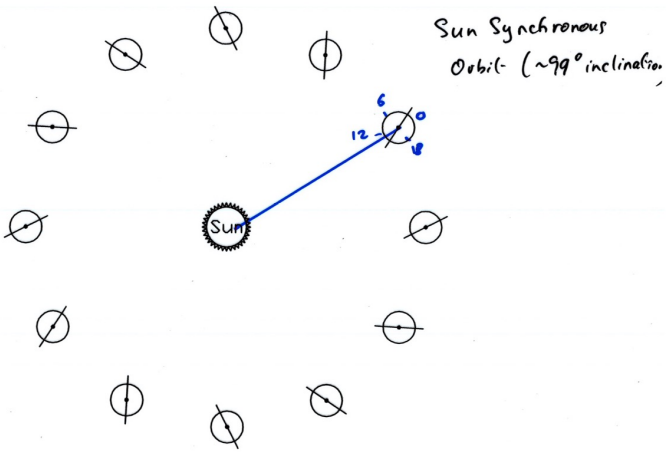
transmittance of whole atmosphere

OXFORD

Summary (2)

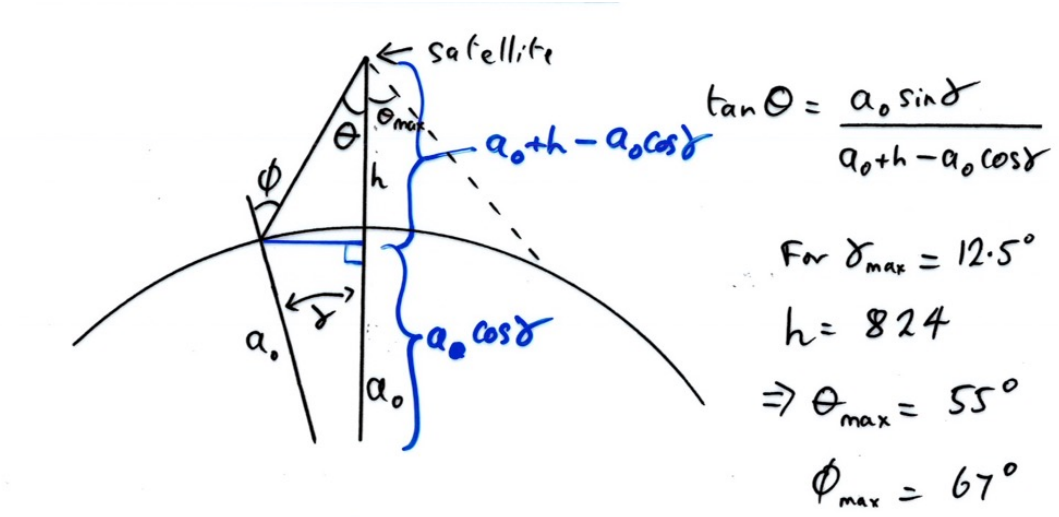
- **Orbits:**
  - **Typically two types used.**
  - **Low Earth Orbit**
  - **Geostationary (~35800 km)**
- **Low Earth Orbit**
  - **700 - 1000km**
  - **Sun Synchronous**
  - **≈99° plane rotates 360° in 1 year in the same direction as the Earth goes around the Sun.**
  - **Same orientation to the Earth-Sun line always.**
  - **Same local time (e.g. 3PM, 10AM)**
  - **More straightforward spacecraft design**

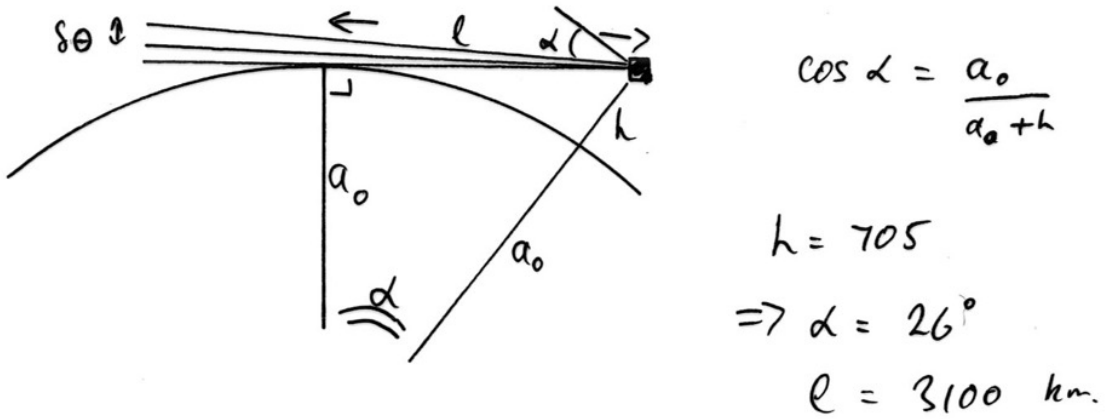




Summary (3)

- **Viewing geometries**
- **Nadir/side scanning**
  - **Good ground track resolution - imagers**
  - **Limited vertical resolution**
  - **Strong/abundant species only**
  - **Numerous examples**
- **Limb scanning**
  - **Long effective path length, good for trace gases**
  - **Poor ground track resolution.**
  - **Good vertical resolution**
  - **High pointing stability requirement.**
- **Solar Occultation**
  - **Special case of limb scanning**
  - **Transmission spectrum of atmosphere**
  - **Sun/star as a source**

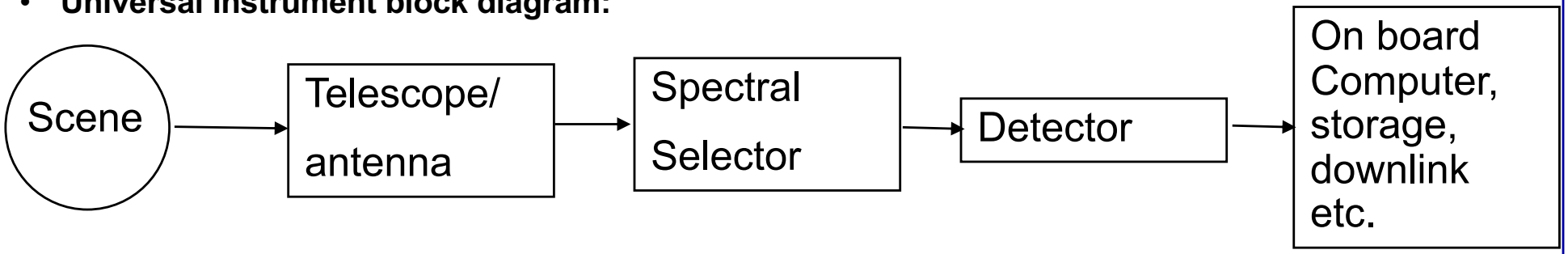




End of Lecture 1

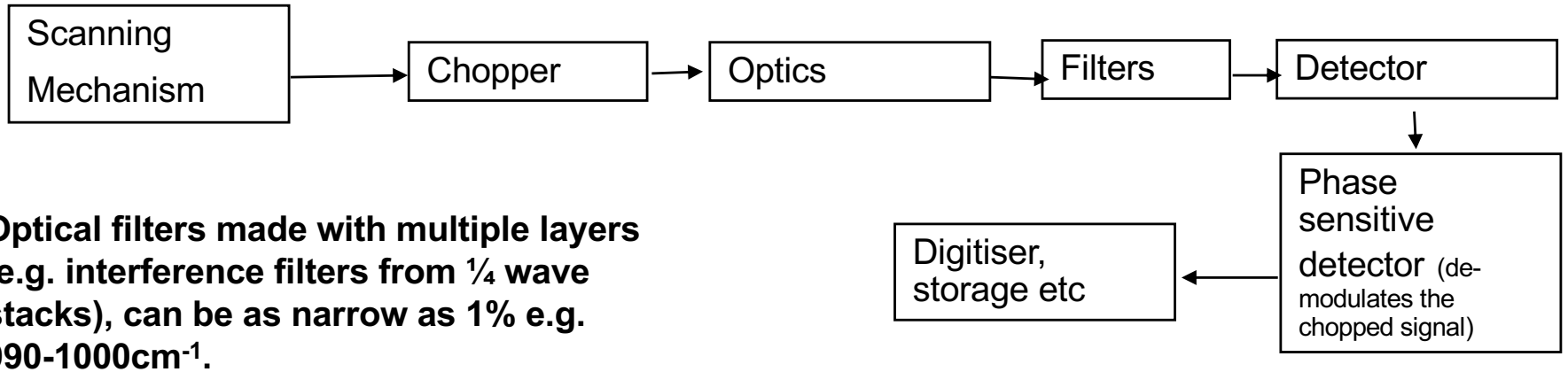
Instruments

- **Universal instrument block diagram:**



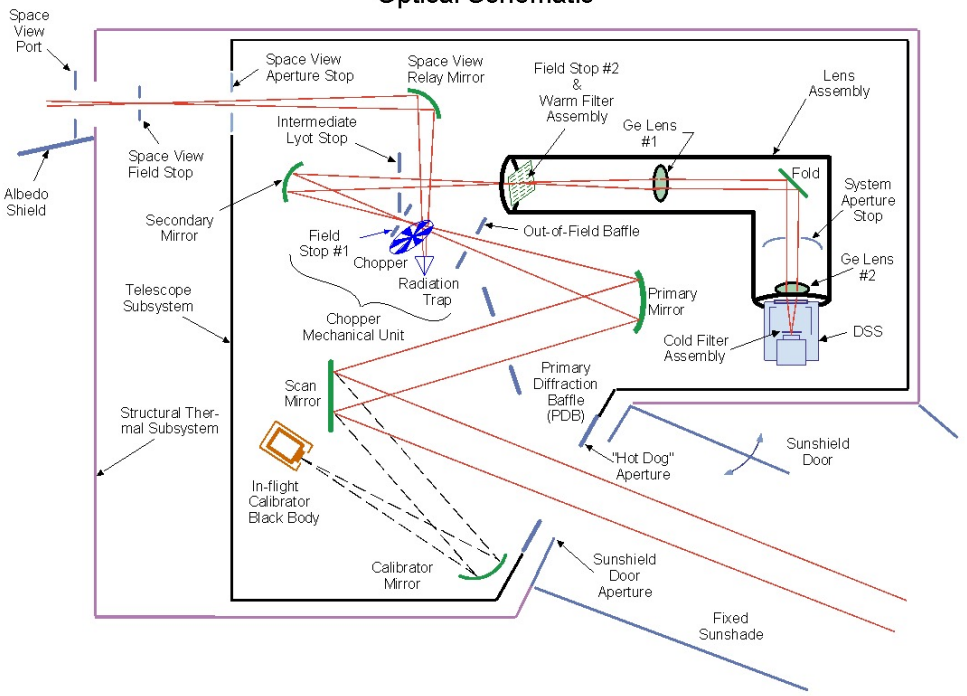
- **Spectral selection and detection:**
- **Thermal Infrared**
  - **Filter Radiometer**
  - **Gas Correlation Radiometer**
  - **Grating Spectrometer**
  - **Interferometer**
  - **Fabry-Perot etalon**
- **Thermal Microwave**
  - **Microwave filters and receivers**
- **Ultra Violet**
  - **Grating spectrometer**

Filter Radiometers

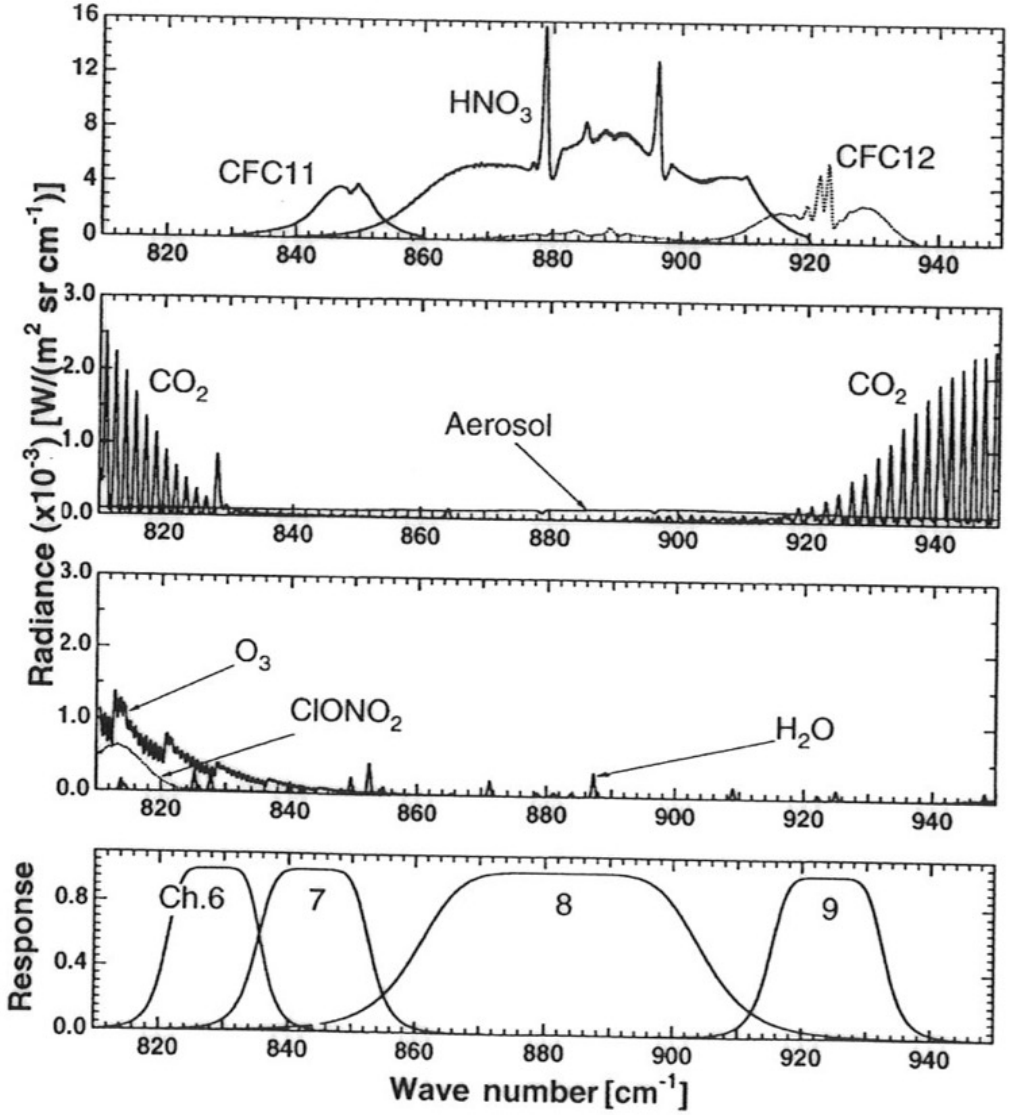


- **Optical filters made with multiple layers (e.g. interference filters from ¼ wave stacks), can be as narrow as 1% e.g. 990-1000cm^{-1}.**
- **Can timeshare a channel by switching different filters, e.g. using a filter wheel before the detector.**
- **Advantages**
  - **'cheap' robust and relatively uncomplicated design.**
- **Disadvantages**
  - **Low spectral resolution so cannot resolve single lines**
- **Examples include most surface imagers, LIMS, HIRDLS limb sounders**
- **AP01 in the Atmos UG lab!**

Optical Schematic



Example of Spectral Selection: HIRDLS on Aura



15 km tangent spectra

ch 8 targetted at HNO_3

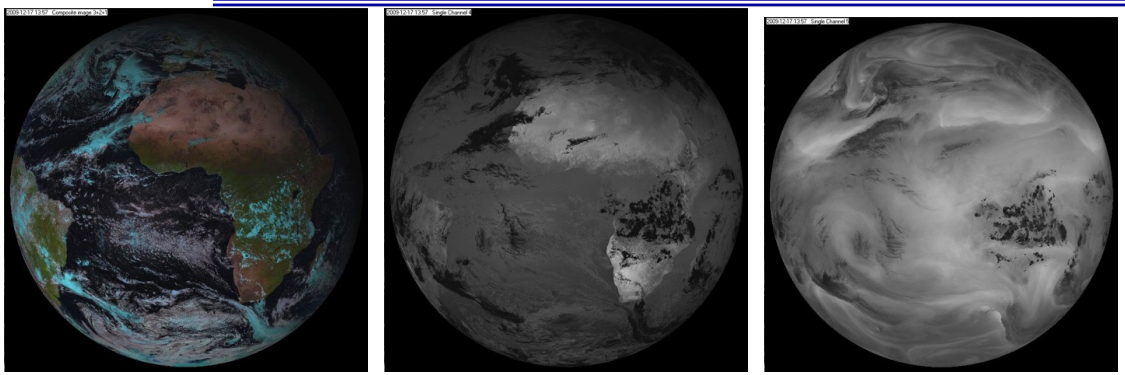
ch 9 " " CFC12

etc

HIRDLS filters (limb sounder)

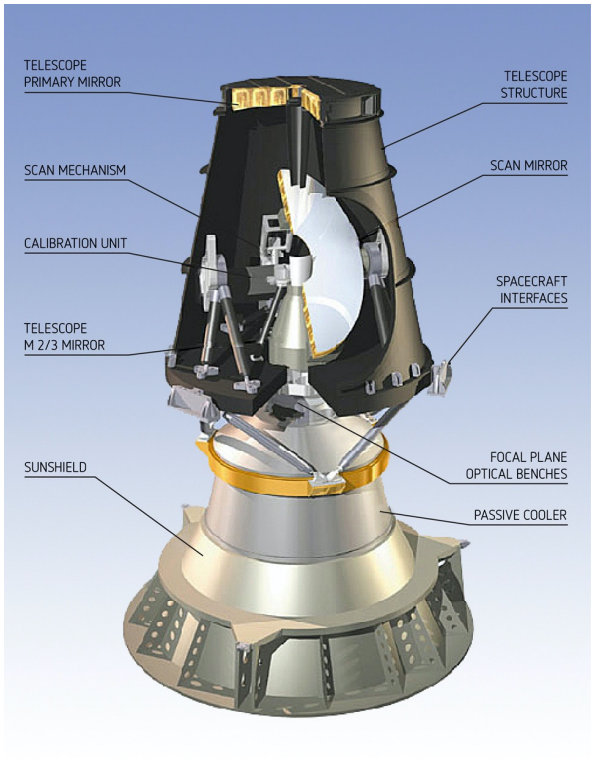
HIRDLS has 21 different filters

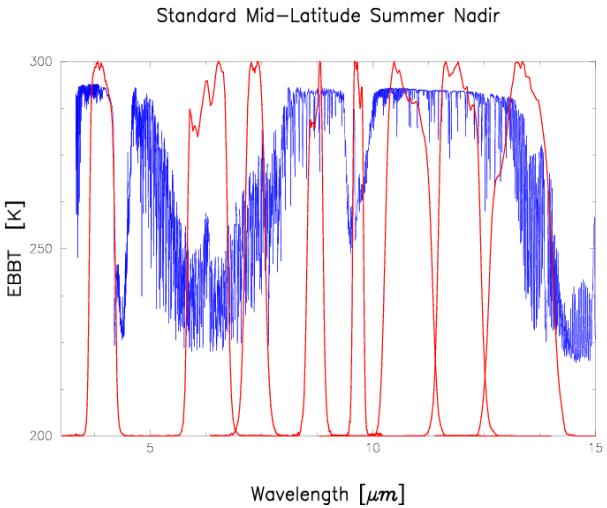
Final Filter Radiometer Images
SEVIRI on Meteosat Second Generation, Geostationary Orbit

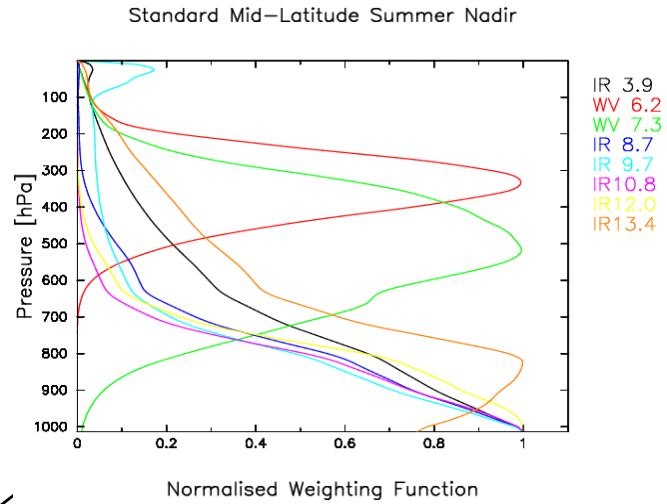


Images from EUMETSAT, visible composite (left), 3.9μm (centre) and 6.2μm water vapour (right)

Plots from Holmlund et al. 2004







$$L^{\uparrow} = \int_0^{\infty} B(z) \frac{d\mathcal{T}(z,\infty)}{dz} dz + B(0)\mathcal{T}(0,\infty)$$

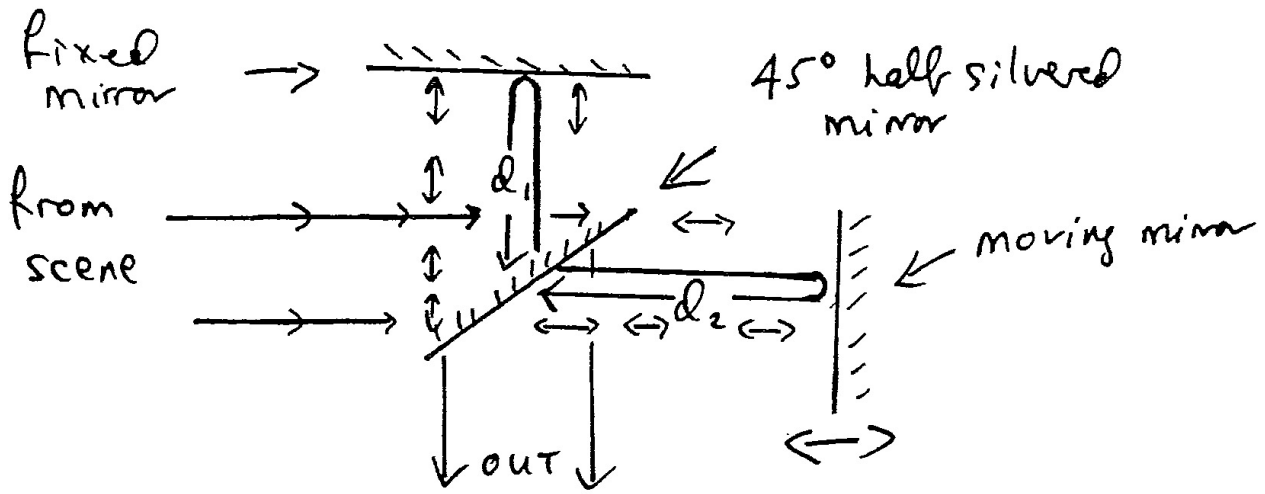
emission of dz

transmittance from z to space

surface emission

transmittance of whole atmosphere

Spectrometers
Michelson Interferometer



d_1 and d_2 are round-trip distances from 45° mirrors.

- **Fourier transform spectrometer – see 2nd year optics.**
- **For monochromatic input frequency ω wavenumber ν**

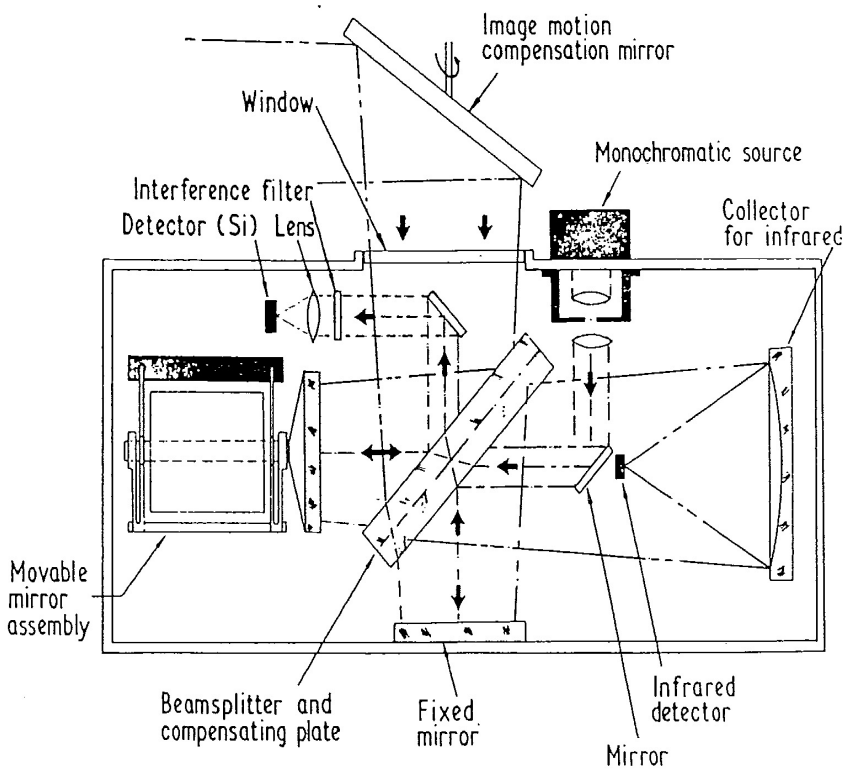
$$Output = a\sin\left(\omega t - 2\pi\upsilon(d_1 + x)\right) + a\sin\left(\omega t - 2\pi\upsilon(d_2 + x)\right)$$

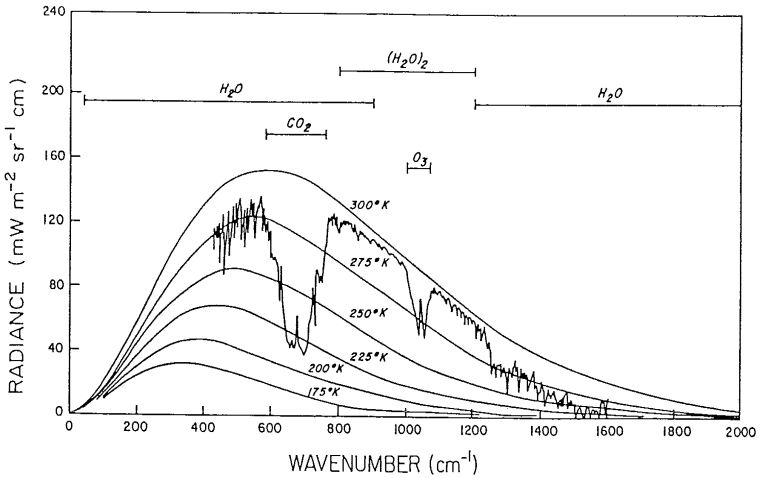
$$Output = 2a\sin\left(\omega t - 2\pi\upsilon\left(x + \frac{d_1 + d_2}{2}\right)\right)\cos\left(\pi(d_1 - d_2)\right)$$

$$Power = a^2\left(1 + \cos\left(2\pi\nu(d_1 - d_2)\right)\right)$$

- **Hence monochromatic input leads to cosine wave pattern at the detector as a function of the path difference d_1-d_2**
- **Linear – hence the output power is cosine transform of the input spectrum $a^2(\nu)$**

Example of an Interferometer
IRIS –this one is a bit of a classic





IRIS spectrum
15°N 145°W
27/4/1970

central spikes in CO_2 and O_3 is stratospheric emission (since hotter)

- **IRIS was a nadir viewing instrument flown on Nimbus 3 and 4 in the early 1970s.**
  - **Nadir viewing, 500-200cm^{-1} (5-20μm) with 5cm^{-1} resolution, 150km pixel size on surface**
- **Versions were also flown to Mars on Mariner 9, and on Voyagers 1 and 2 to the outer planets.**
- **CIRS instrument on Cassini was descendent of the design.**
- **Modern Earth observation instruments include MIPAS (Envisat), IASI (Metop), TES (Aura), ACE occultation spectrometer**

IASI - METOP
(EUMETSAT – CNES)

- **'Medium' resolution operational nadir Fourier Transform Interferometer**

- **Maximum optical path difference ± 2cm**
  - **Resolution 0.5 cm^{-1}**
- **4 off-axis pixels, field of view is ± 48˚**
- **Spatial resolution per pixel 12 km**

- **Covers 645 to 2760 cm^{-1} in 3 bands**
  - **B1: 8.26 – 15.5 µm**
  - **B2: 5.0 – 8.26 µm**
  - **B3: 3.62 – 5.0 µm**

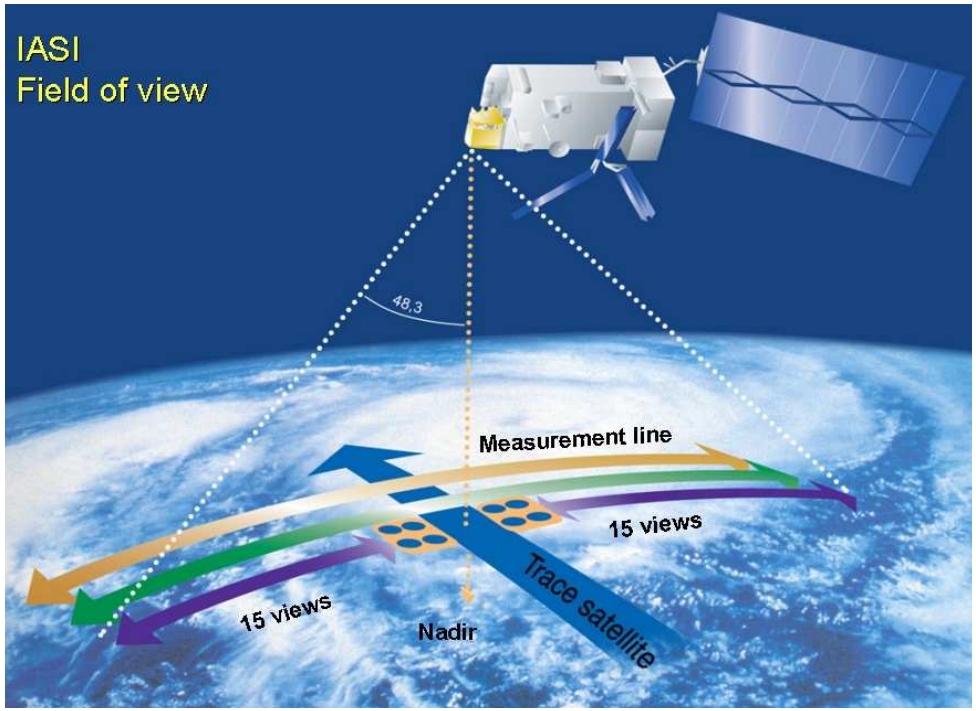
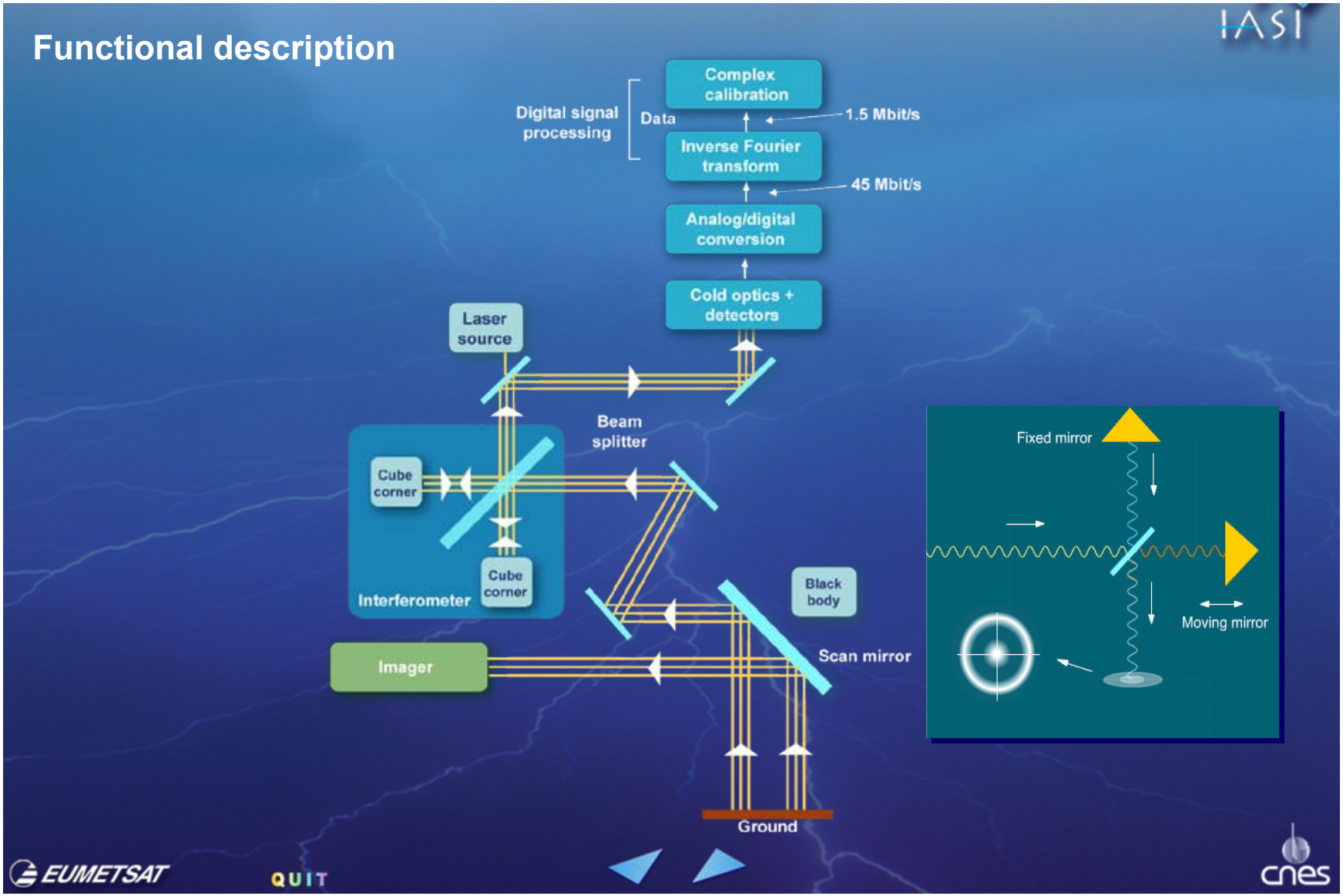
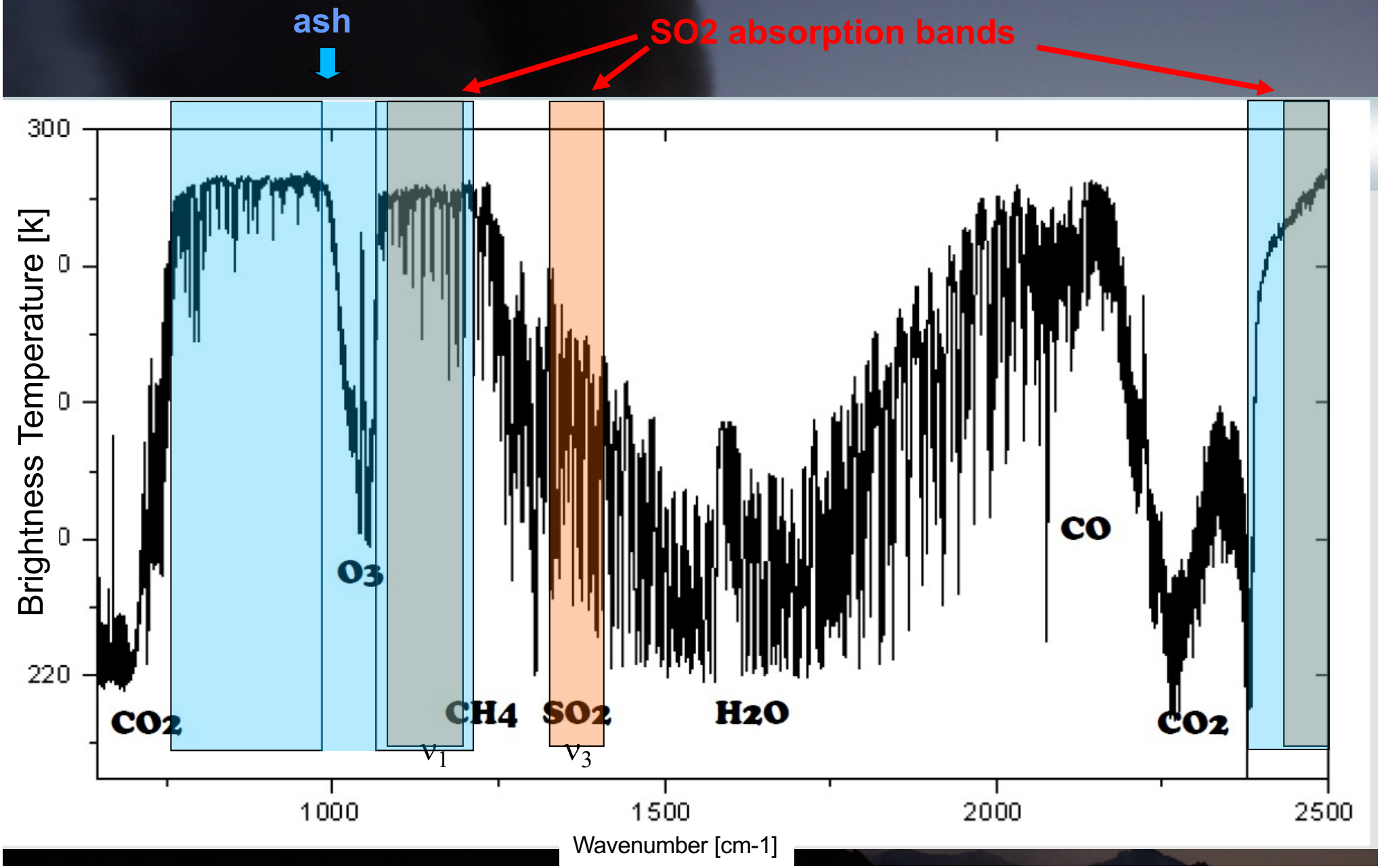


Image: CNES/EUMETSAT

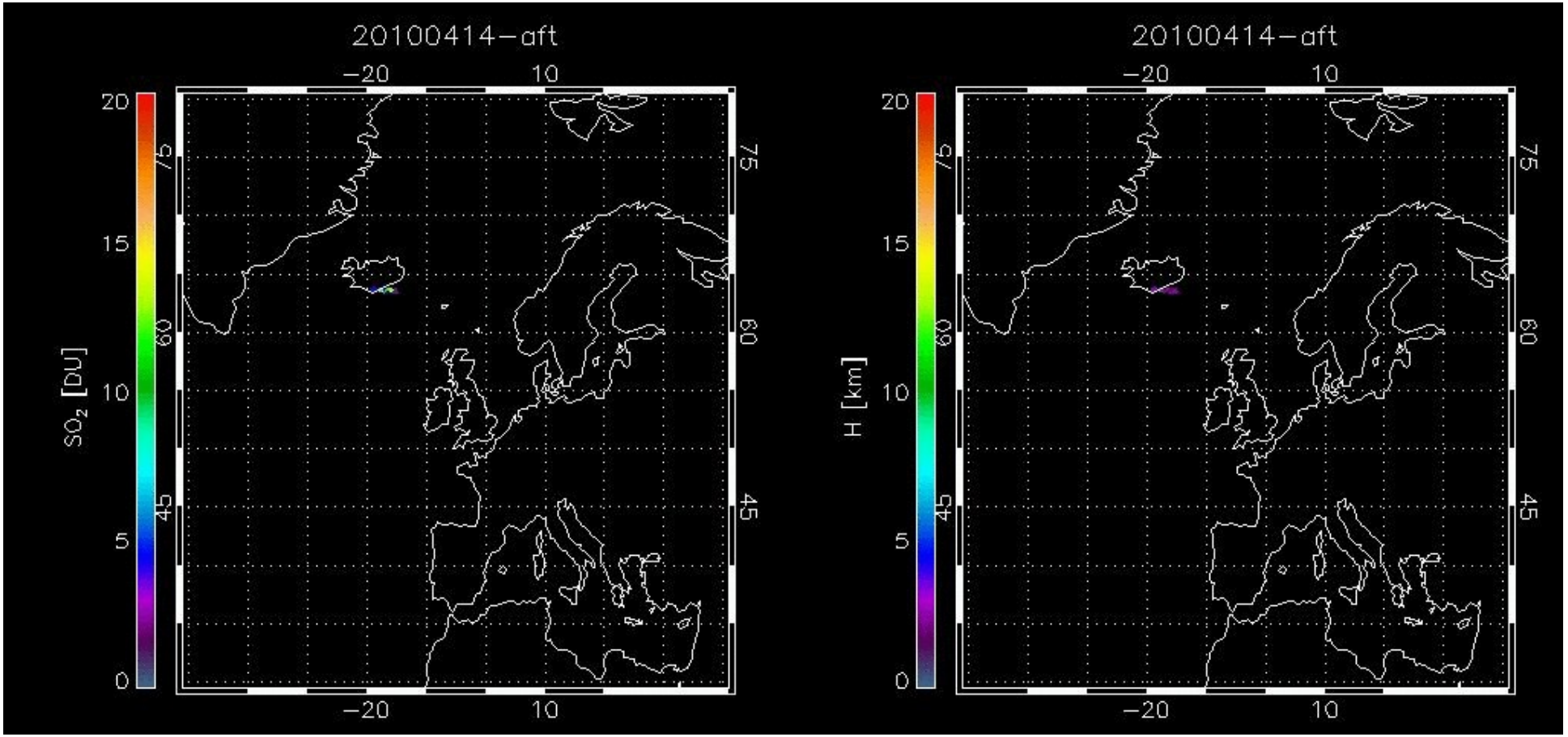
IASI On MetOp-A/B



IASI Spectrum, E Carboni et al. (AOPP)

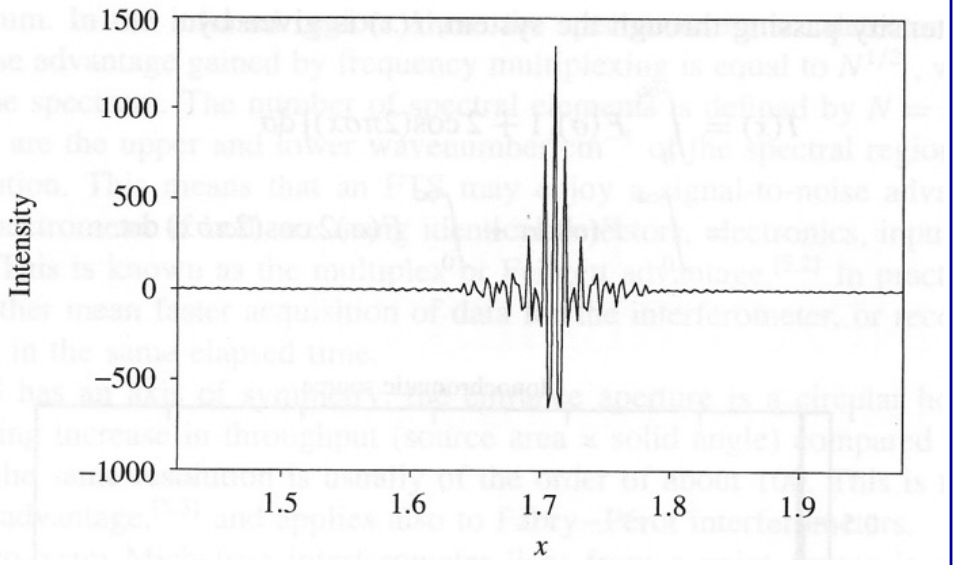


IASI SO_2 Retrieval Example
E. Carboni AOPP



Interferometers continued

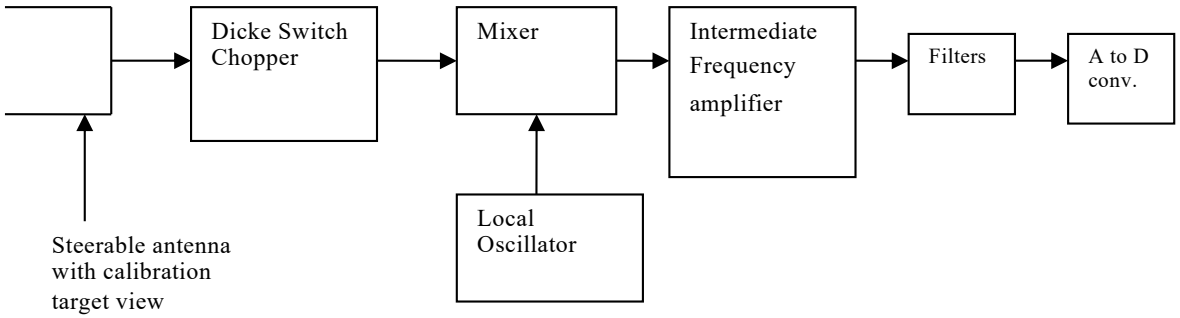
- **Advantages:**
  - **All the radiation from the scene is used the whole time, in grating instruments you need to scan the grating and so only sample a small fraction at any time.**
  - **Get whole spectrum, and with high-resolution instruments can see individual lines, you may find unexpected features or gas species so a spectrometer is a powerful survey instrument.**
- **Disadvantages**
  - **Need very precise optics. Especially the mirror drive since the mirror displacement controls the spectral resolution.**
  - **Need very linear response to flux from the detector since there is a lot of energy near zero path difference (the centre burst) which falls rapidly as the path difference increases.**
  - **Data rates are very high, need all of the interferogram even if you only want a small section of the spectrum.**



Example interferogram from Duxbury 1999

Microwave Sounding

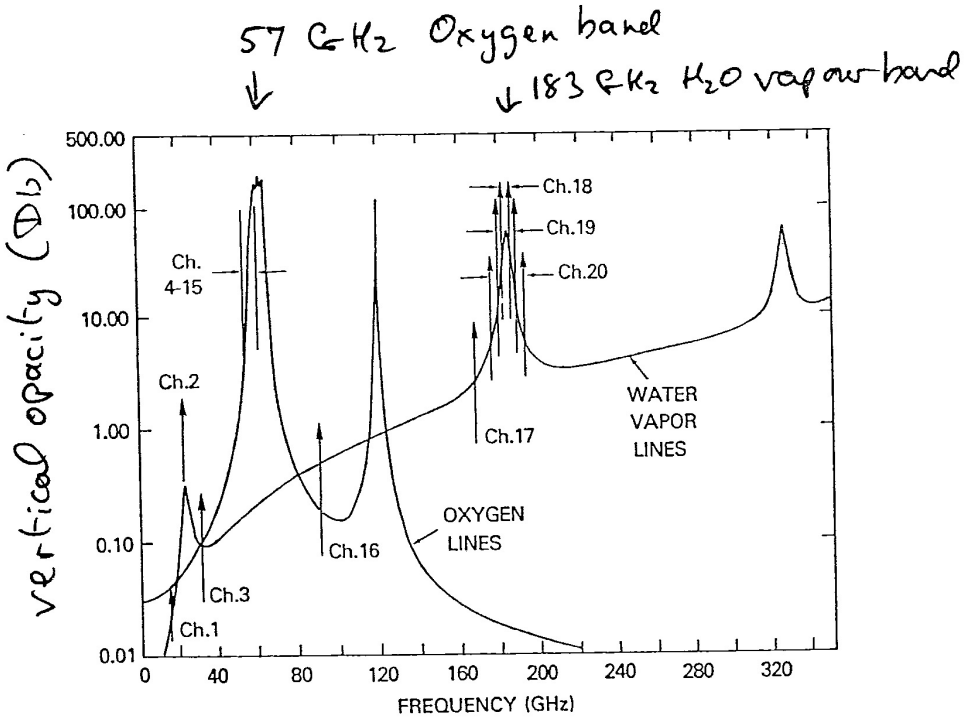
- **Block diagram:**



- **All microwave sounders rely on thermal emission from the atmosphere or surface.**
- **Advantages:**
  - **Water vapour clouds are transparent (λ>>drop size), but not rainfall so can be used to estimate rainfall volume in air, aerosol is also transparent.**
  - **Radio techniques (e.g. heterodyne detection (as above), can use one sideband or both for example) enable very high spectral resolution.**
  - **Much more sensitive detection is possible so cooling of detectors may not be necessary.**
  - **Planck function is proportional to temperature at long wavelengths so vertical sounding is made easier by linearity.**
- **Disadvantages:**
  - **Rayleigh criteria to resolve 2 point (1.22λ/D) requires a large antenna.**
  - **Ground surface is not very black in the microwave and this causes difficulties for vertical sounding due to lack of contrast. E.g. sea emissivity is ≈0.5**

Example Nadir Microwave instrument
AMSU (currently versions on e.g. Aqua and MetOp)

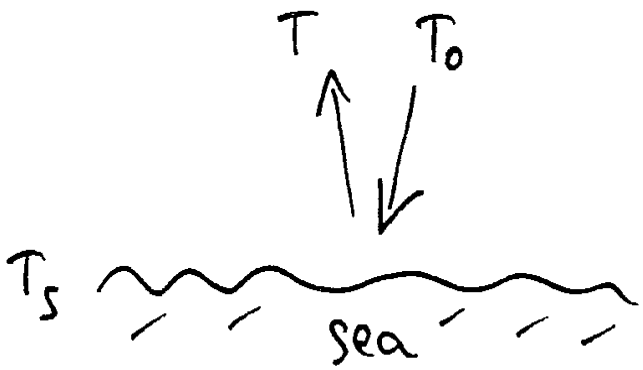
- **The current operational instruments include the Advanced Microwave Sounding Unit (AMSU).**
- **Various versions have been flown, figure shows channels for AMSU-A and B instruments.**
- **E.g. AMSU has channels at 50, 54, 55, 58GHz with 220MHz bandwidth for each channel. A 7.5° field of view gives 111km resolution at nadir.**
- **Very high, effectively monochromatic, spectral resolution gives 'perfect' 10km wide weighting functions.**
- **Can see through cloud when used with IR sounders.**



Microwave Imaging

- **The main microwave imager in use currently is the USAF SSM/I.**
- **It has channels at 19,22, 37, 86GHz, atmospheric window regions and are mainly used for ice and rain detection**

- **Excellent way to flag clear ocean since it has a surface emissivity of ≈0.5 giving an apparent temperature of 130K**

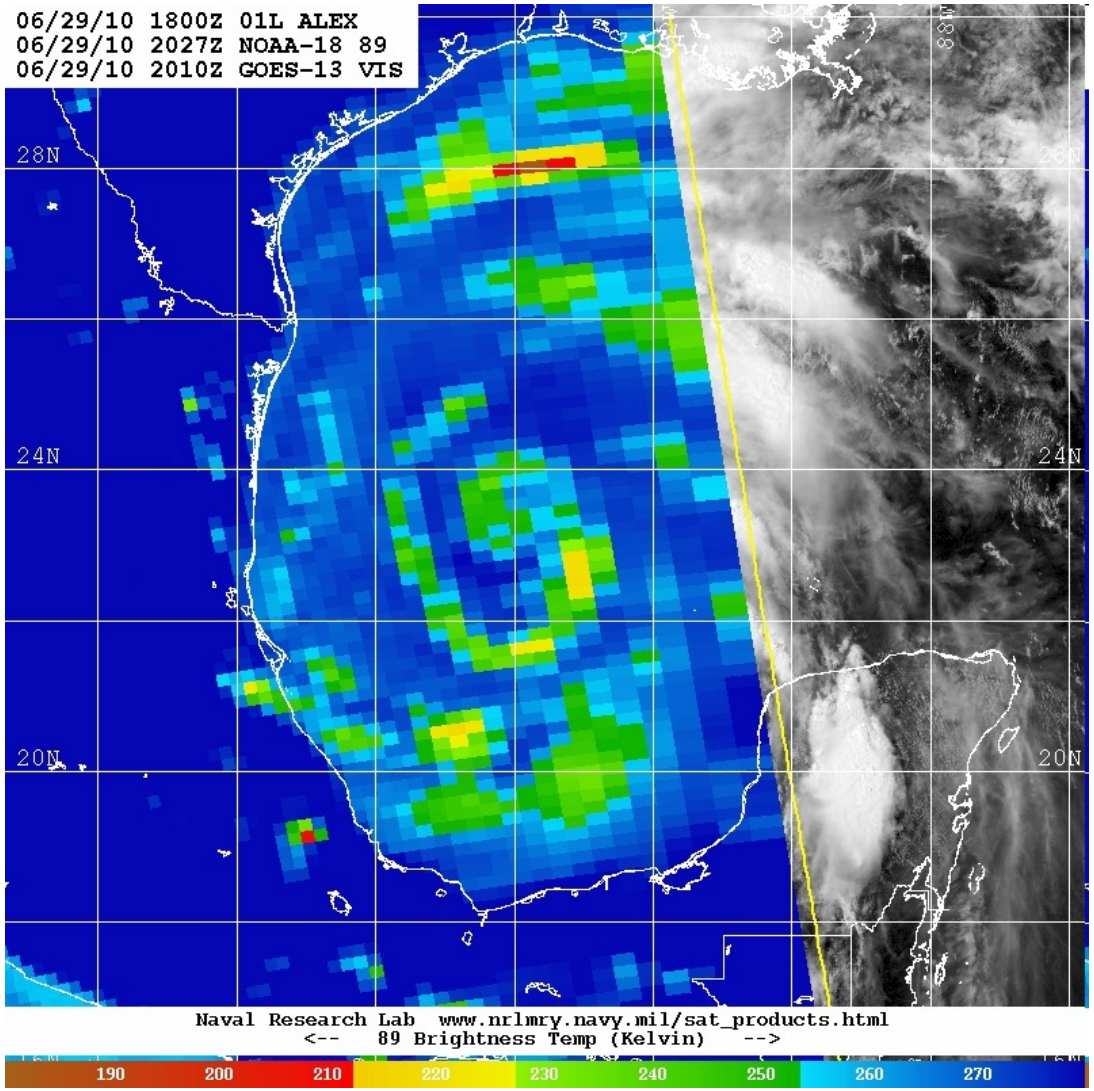


$$T = T_s(1-r) + T_0 r$$

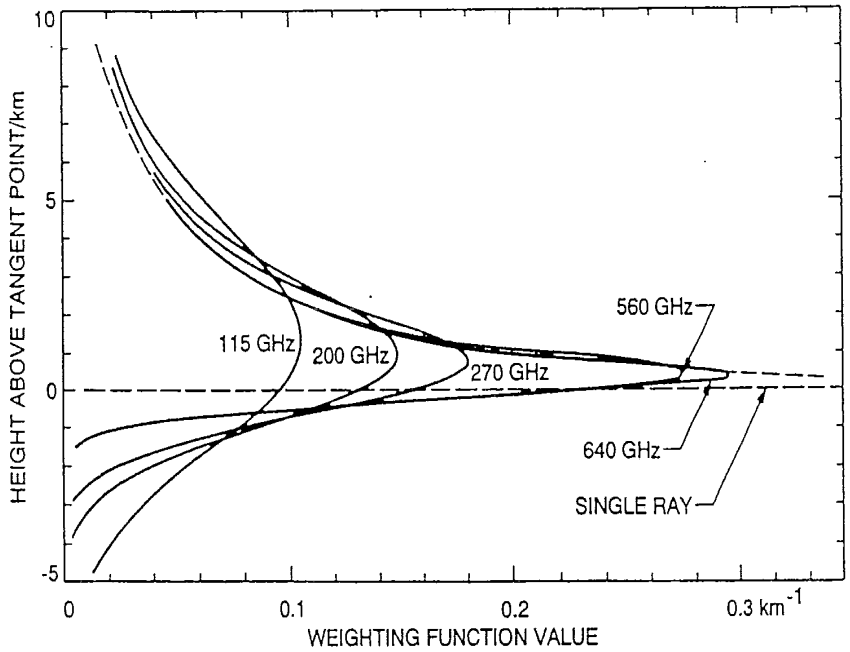
reflectivity

Space radiance = 3K

Microwave images 1



Microwave limb sounding
MLS on AURA

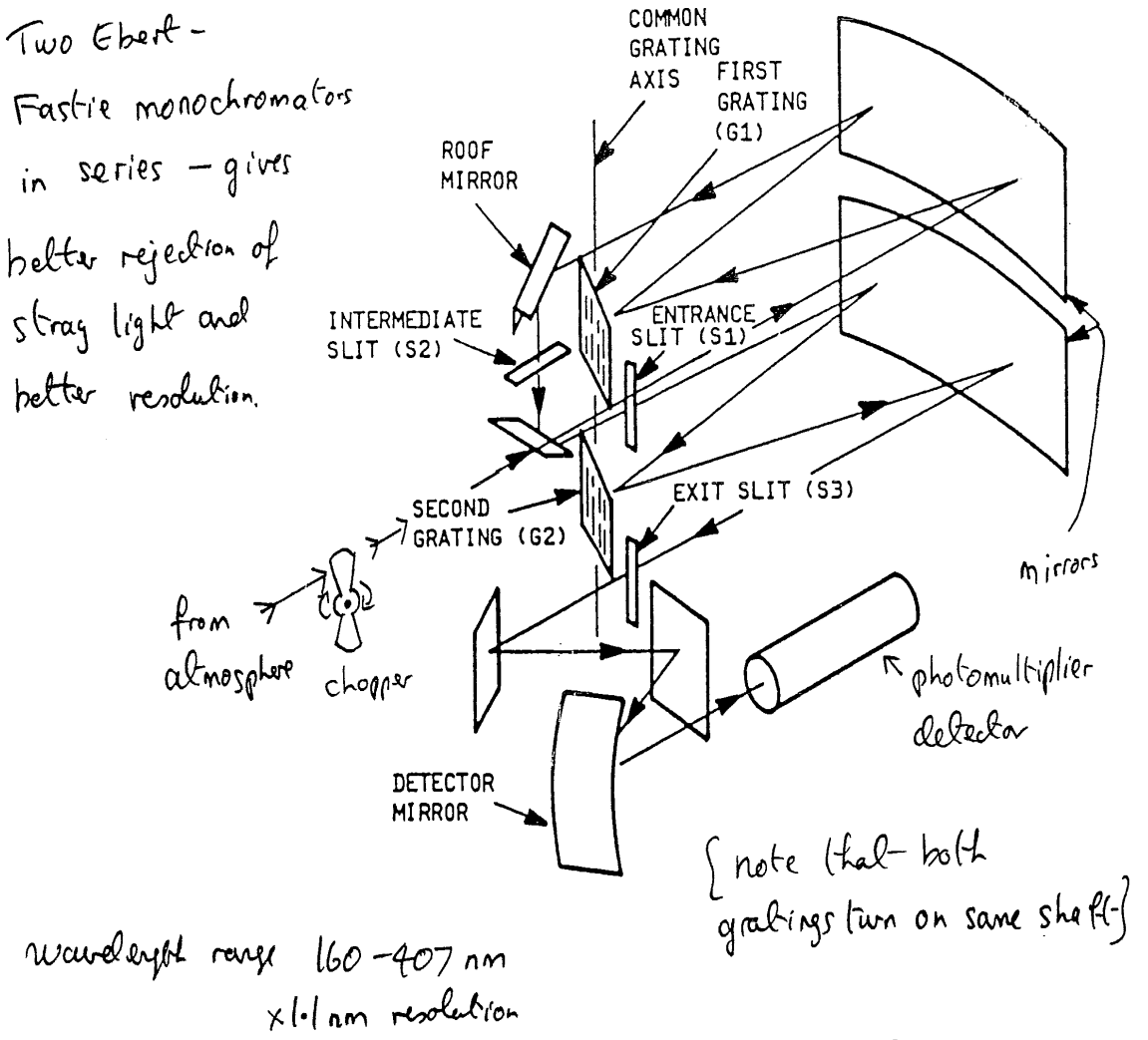


weighting functions for 1.6 m antenna in 600km orbit

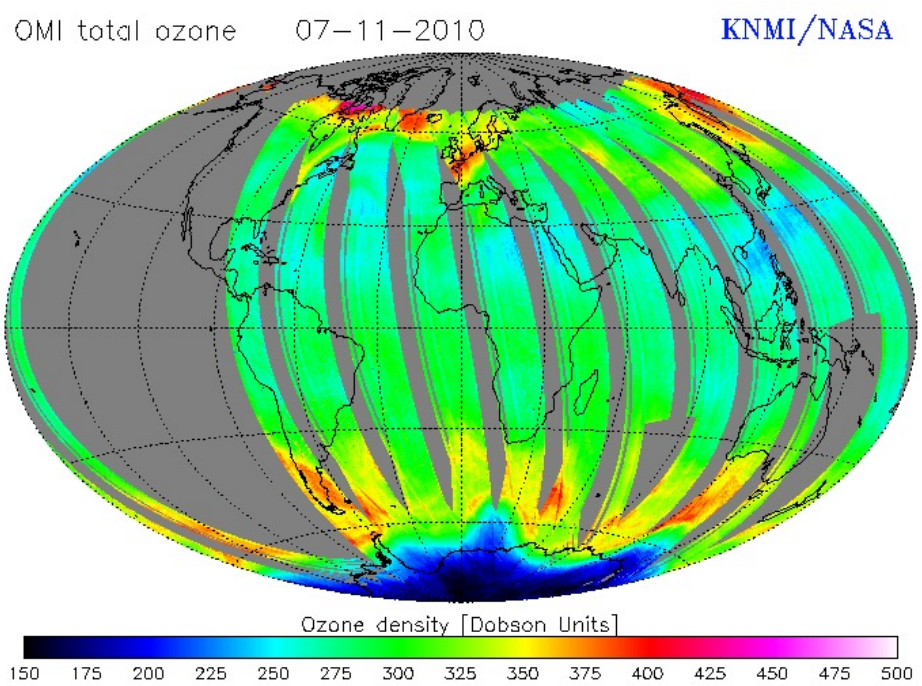
- **Microwave Limb Sounders have been flown on the Aura satellite.**
- **Typical configuration is a 1.6m antenna with channels in the 63GHz oxygen band to measure pressure, also has channels in the 183GHz and 205GHz regions to measure O_3, ClO, HCl, H_2O_2 and HNO_3**
- **Aura includes 2.5THz channels sensitive to lines of OH plus 600,200 and 120GHz.**
- **Again, very high spectral resolution with tens of MHz bandwidth, so essentially monochromatic.**

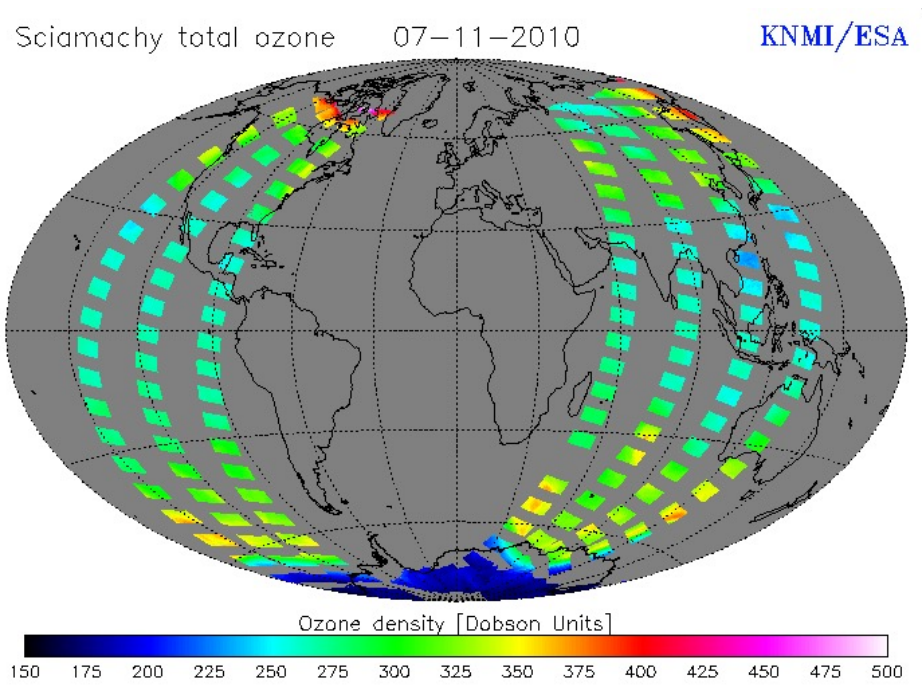
Finally, the UV Grating Spectrometer SBUV

- **Flown in various versions since 1970, now part of operational meteorological satellites.**
- **Nadir viewing measures backscattered UV.**
- **Gives total column of ozone and ozone layer.**
- **No scanning 11.3° diameter field of view (168km diameter ground track)**
- **In stepped mode it cycles round 12 wavelengths every 24 secs.**
- **Spectral calibration using on-board mercury lamp (spectral lines)**
- **Radiometric calibration requires input solar intensity to intensity reflected by the atmosphere. A solar diffuser is put into the beam with a known diffuse reflectance.**
- **However over several years the diffuser goes black as the UV carbonises any organic contaminants**



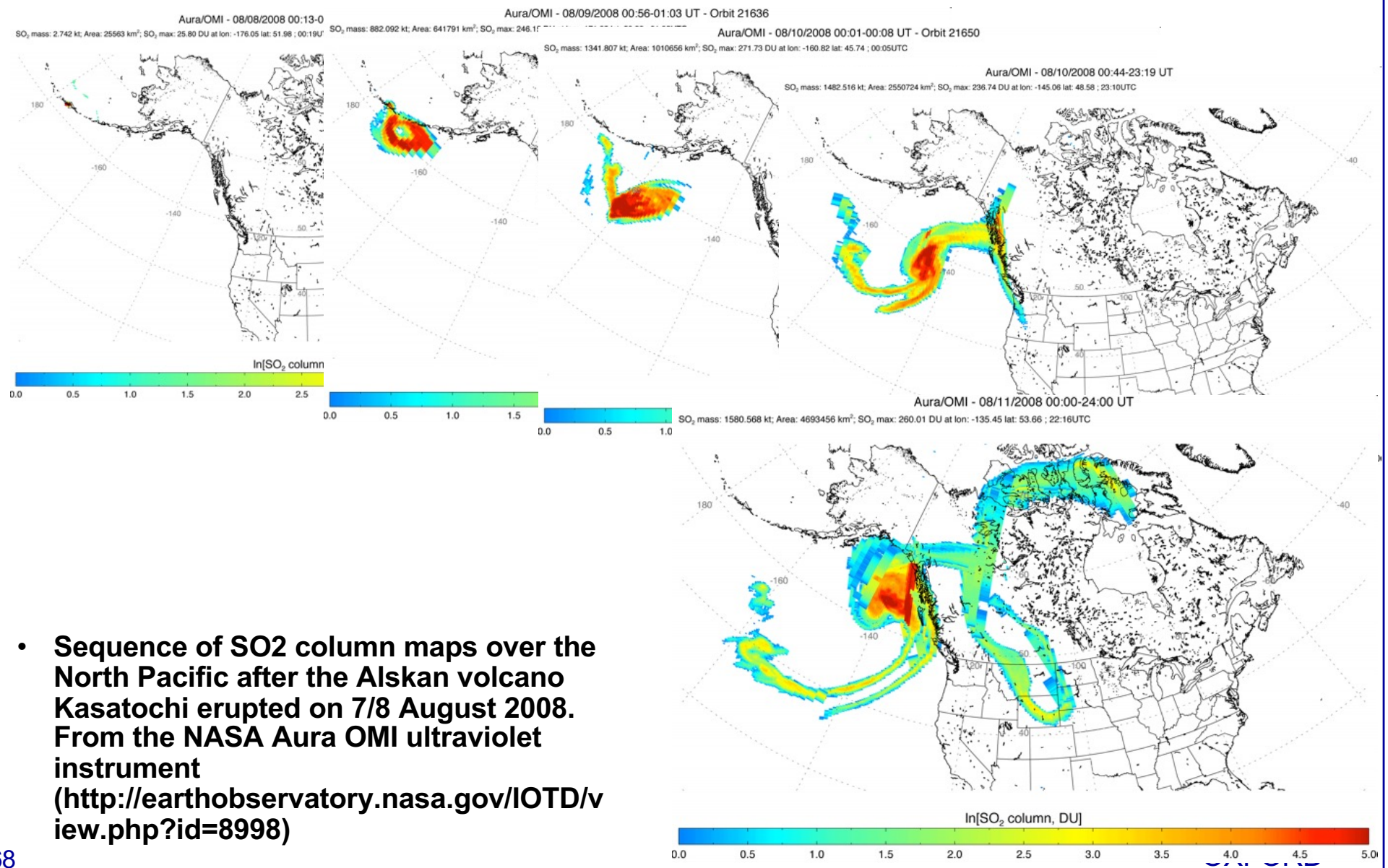
Example Ozone Column measurements





- **OMI on Aura, SCIAMACHY on Envisat**

SO2 image sequence



- **Sequence of SO2 column maps over the North Pacific after the Alskan volcano Kasatochi erupted on 7/8 August 2008. From the NASA Aura OMI ultraviolet instrument (http://earthobservatory.nasa.gov/IOTD/view.php?id=8998)**

Visible Light Imaging

- **Imaging in the visible is relatively cheap and simple to perform as the reflected sunlight signal is extremely strong.**
- **It has the disadvantage of only giving data during daytime (although the military do some night-time imaging with Moonlight). This means that the polar caps are invisible for weeks at mid-winter.**
- **Meteorological satellites carry imagers for viewing clouds. From geostationary orbit they are used to measure wind speed via cloud tracking.**
- **Visible light is sometimes used to reject pixels that contain clouds when doing infrared *retrievals*. This can be a bad idea because:**
  - **It only works in daytime**
  - **Ice and snow looks like cloud.**
  - **However it is useful for checking new retrieval codes.**
- **Other satellites take visible images, e.g. astronauts look out for interesting things to photograph, space tourists just because they can…**

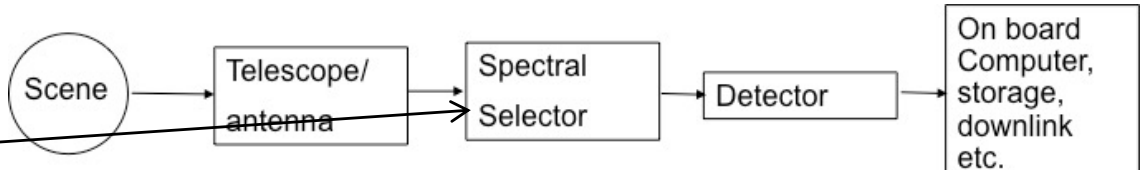
MERIS Phytoplankton

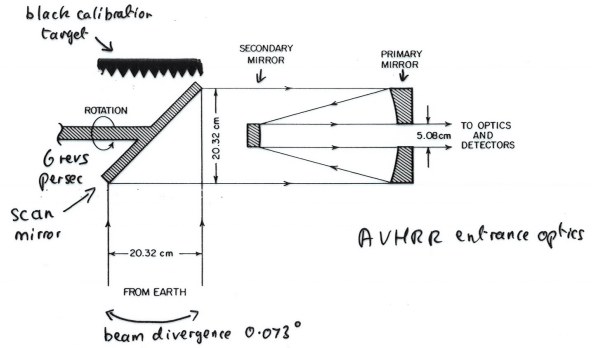


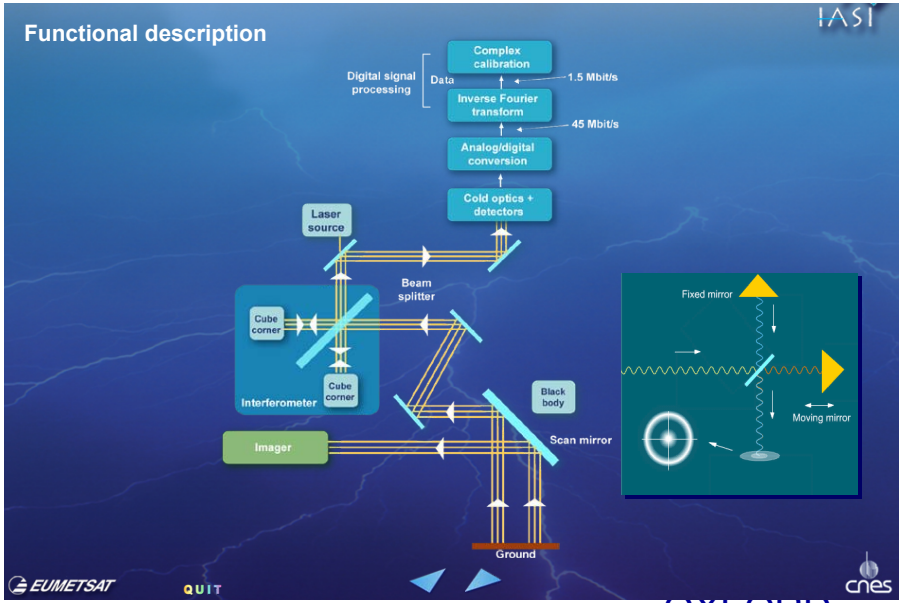
- **Phytoplankton in the North Sea imaged by the MERIS instrument on ESA's Envisat spacecraft (7/8/2008).**
- **Phytoplankton are tiny organisms that are important for fish and absorbing Carbon Dioxide.**
- **Image, ESA.**

Summary

- **Types of instrument**
- **Remember the block diagram!**
- **Spectral selection**
  - **Filter radiometer**
    - **"Simple", radiometrically accurate, lower data rate**
    - **Numerous examples (e.g. MODIS, SEVIRI etc.)**
  - **Spectrometer (FTIR, Grating etc.)**
    - **FTIR, wide wavelength range, programmable (high) resolution, excellent light utilisation. Drawbacks are high data rate, complex data product and calibration (interferogram rather than spectrum).**
    - **Grating, more limited wavelength range, often used in double grating config (to reduce scattered light) for UV measurements.**
  - **Microwave sounder/imagers**
    - **Very high spectral resolution possible with electronic filters, heterodyne techniques. Excellent Signal to noise possible**
    - **Need big antenna for moderate spatial resolution.**
  - **Cameras**
    - **Visible cameras, excellent spatial resolution (10s cm).**
    - **Limited night time coverage.**



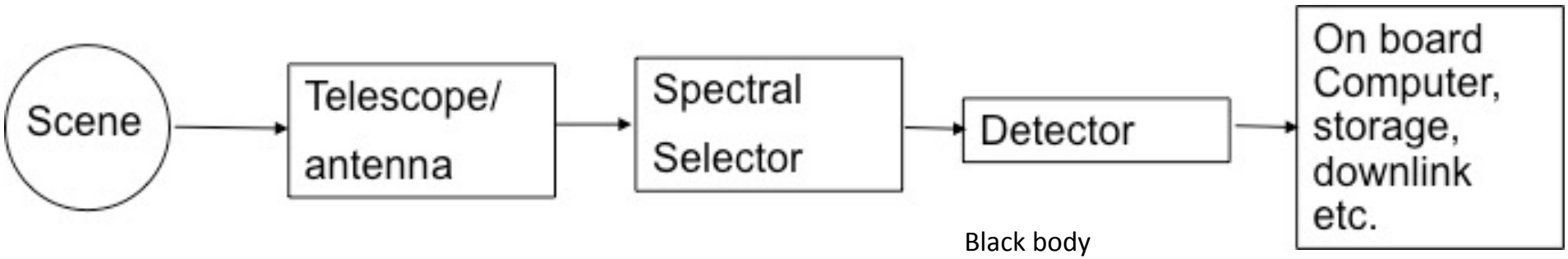


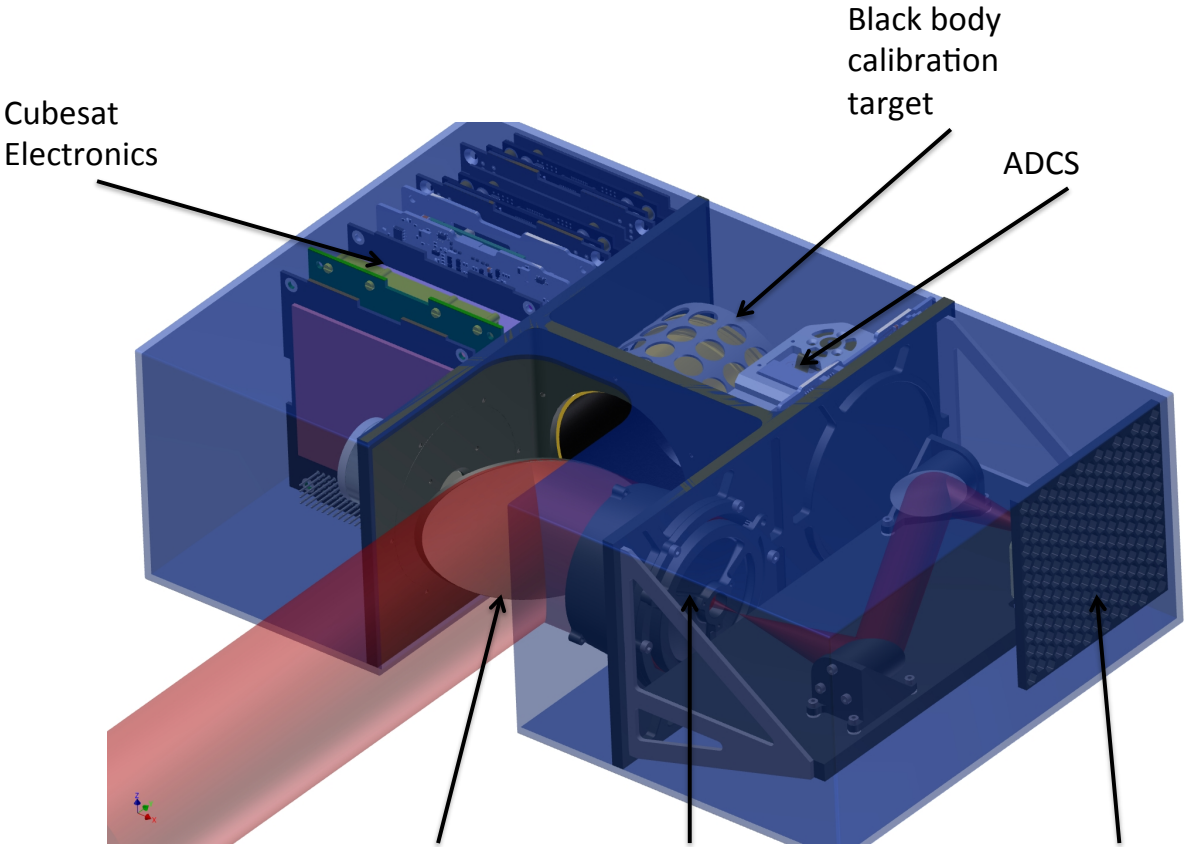


Leading into The Class……

- **Detectors, Noise, performance of optical systems**
- **Design an instrument for a Mars atmospheric science mission**

Detectors, performance modelling and instrument design





Detectors
(Not microwave)

- **Quantum Detectors**
  - **Photomultiplier vacuum tubes – 'Photoelectric effect' causes electrons to be ejected from metal and then amplified in a thermionic valve.**
  - **Photoconductive- photon excites semiconductor carriers and changes the conductivity of the detector element.**
  - **Photovoltaic – photon excites carriers in semiconductor and generates a current, similar to a solar cell.**
- **Thermal Detectors**
  - **Here the photons cause heating in the detector**
  - **Golay Cell, change in pressure of a gas volume is measured (slow response time)**
  - **Pyroelectric – temperature dependence of polarisation of a Ferro electronic material, sensitive to changes in temperature only.**
  - **Bolometer – resistance of a thermistor or other highly temperature dependent resistor measured, e.g. at a superconducting transition.**
- **Noise in detectors and the 1st amplifier stage.**
  - **Photon Noise – random arrival of photons, often try and cool the instrument internally to reduce incident flux.**
  - **Shot noise – amplifier noise due to current quantisation.**
  - **Johnson noise - Brownian motion introduces noise, can be reduced by cooling the detector and possibly electronics**
  - **1/f noise - noise is proportional to 1/frequency. This is a bit odd and is poorly understood. Chopping the signal is used to minimise this component of the noise.**

Detector Noise
Definitions and Calculations

- **Suppose the signal on the detector is integrated for a time Δt.**
- **The random noise will be proportional to** $\sqrt{\Delta t}$

- **Define the *Noise Equivalent Power* NEP for a detector. NEP usually has the units power (time)$^{1/2}$ so normally W Hz$^{-1/2}$.**
- **The NEP is the power in watts that will produce a signal equal to the rms noise. Smaller the better.**

- **This is for a detector of area *a***
  - **Normally the noise is proportional to** $\sqrt{a}$ **(since each element of detector area contains random independent noise sources).**
- **So we can define a 'quality factor' known as the D*:**

$$D^* = \frac{\sqrt{a}}{NEP}$$

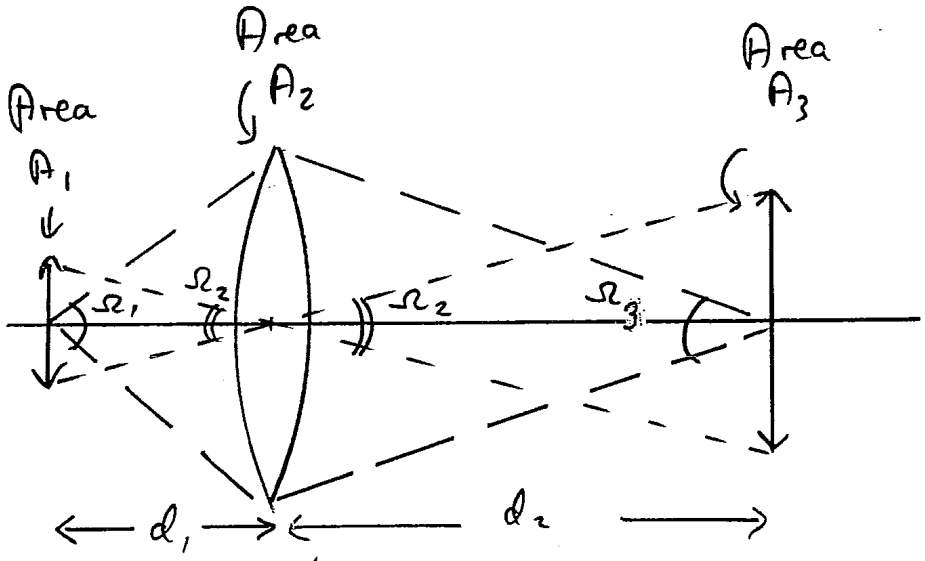
- **A high D* means a better detector (typical values e.g. 10^8 for uncooled pyroelectric detectors, 10^{11} for a cooled detector such as HgCdTe (MCT).**
- **D* will be constant for a material given fixed frequency, composition, construction, operating temperature etc.**

Determining instrument performance
Rules for perfect imaging optical systems (no losses etc)

- **Rule 1:**
  - **The radiance *L* is constant**
- **Rule 2:**
  - **The area x solid angle through the system is constant, usually written as *AΩ*.**

- **A_1, A_2, A_3 are areas**
- **Ω_1, Ω_2, Ω_3 are solid angles**
- **$A_1\,\Omega_1 = A_2\,\Omega_2 = A_3\,\Omega_3$**
- **=area of image x solid angle of the lens ($A_1\,\Omega_1$ or $A_3\,\Omega_3$) = area of lens x solid angle of either image ($A_2\,\Omega_2$).**

- **(Since the lens views the same solid angle both sides to focussed images)**



$$\Omega_1 = \frac{A_2}{d_1}$$

$$\Omega_2 = \frac{A_1}{d_1} \quad \text{eliminate } d_1$$

Hence:

$$\frac{A_2}{\Omega_1} = \frac{A_1}{\Omega_2} \quad \text{giving } A_1\Omega_1 = A_2\Omega_2$$

Similarly for $A_3\Omega_3 = A_2\Omega_2$

A recap on Solid Angles and some useful formula for optical systems

- **Solid angles can often cause some confusion…**
- **They are fundamental to understanding radiative transfer and optics problems so here's a quick recap and short cuts.**
- **The Planck function has units of spectral radiance:**
  - **$Wm^{-2}sr^{-1}m^{-1}$**
  - **Or $Wm^{-2}sr^{-1}\mu m^{-1}$ depending on your spectral unit.**
- **The usual definition is $\Omega = \frac{S}{r^2}$ so 4π in a complete solid angle.**
- **For most of our applications there are some very useful formulae that can simplify the calculation of solid angle, that we can then apply to calculate flux etc. from a radiance.**
- **The solid angle at the apex of a cone of semi-vertical angle θ is given by:**
  - **$\Omega = 2\pi(1 - \cos\theta)$**
  - **This very similar to a detector at focus of an optical system!**
- **Where the angle θ is small (and in radians) this can be further approximated to $\Omega \approx \pi\theta^2$**

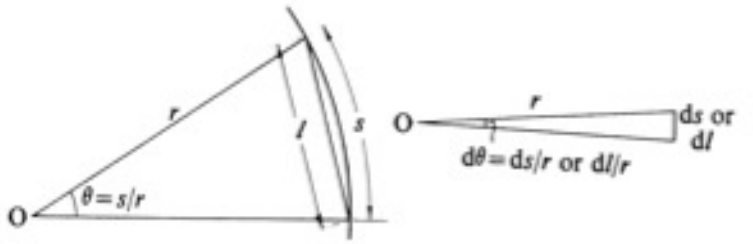
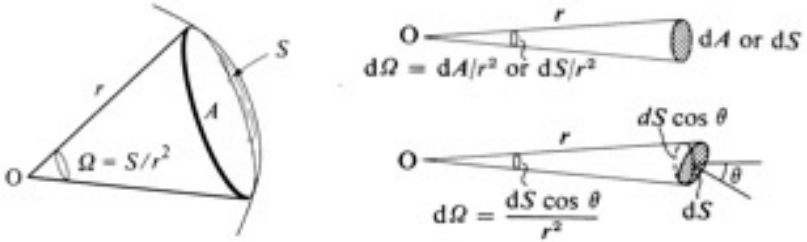
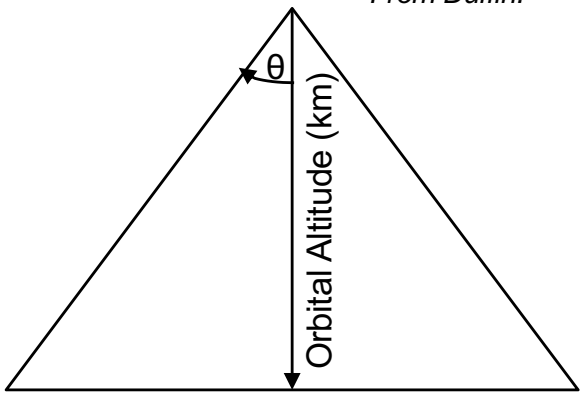


Figure A.1 Plane angles: θ and $d\theta$ are in radians.

$d\Omega = (dS \cos\theta)/r^2$

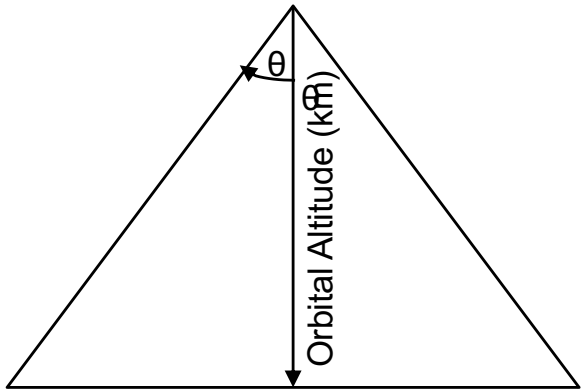


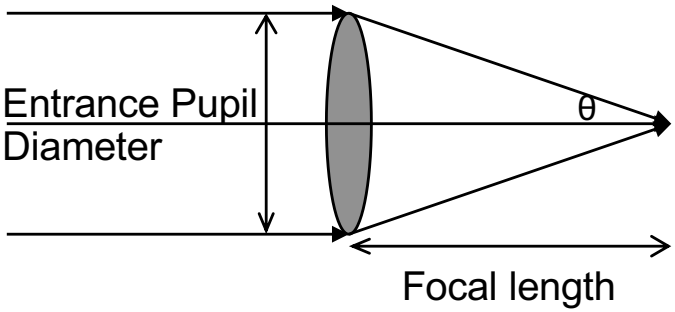
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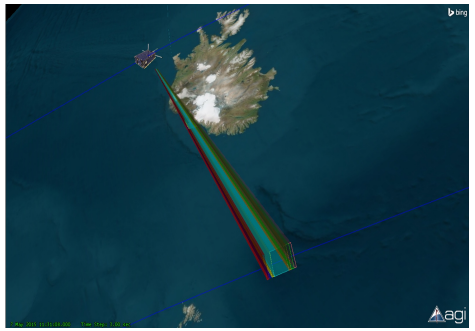


One final slide on Solid Angles and Fields of View

- **Since solid angle is related to half angle of a cone by $\Omega = 2\pi(1 - \cos\theta)$ there is direct connection between the focal ratio (f-number) and the solid angle of the field of view.**
- **The focal ratio, f/# is defined as (focal length)/(Entrance pupil diameter)**
- **The tangent of the half angle is therefore $\tan\theta = \frac{1}{2f\text{-}ratio}$**
- **This makes it straightforward to evaluate the solid angle subtended by an instrument to a scene or an pixel to the final focusing optic etc.**
- **From these and AΩ conservation (see later) it is possible to estimate the IFoV of an instrument to the scene.**
- **Very useful for quick checks!**
- **We will use these results later in the class example.**







Noise calculations continued…
(remember connection to Plank function units)

- **Power reaching the detector** $= LA\Omega\Delta\nu$ ($\Delta\nu$ is the frequency bandwidth)
- **Integrated signal** $= LA\Omega\Delta\nu\Delta t$ (Δt is the integrtation time)
- **Integrated noise** $= \frac{1}{D^*}\sqrt{a}\sqrt{\Delta t}$
- **Signal to noise ratio (SNR)** $= LA\Omega\Delta\nu D^* \frac{\sqrt{\Delta t}}{\sqrt{a}}$

- **However, we cannot just get the SNR as big as we want by making the detector area, *a*, very small.**
- **To see why, remember that the *AΩ* is conserved for an ideal optical system.**
- **This means that at the detector we must have, with *A* now the entrance aperture to the instrument, Ω the solid angle of the scene and Ω_d the solid angle at the detector :**

$$A\Omega = a\Omega_d$$

- **Eliminate *a* (since it can be made as small as needed) to give the SNR:**

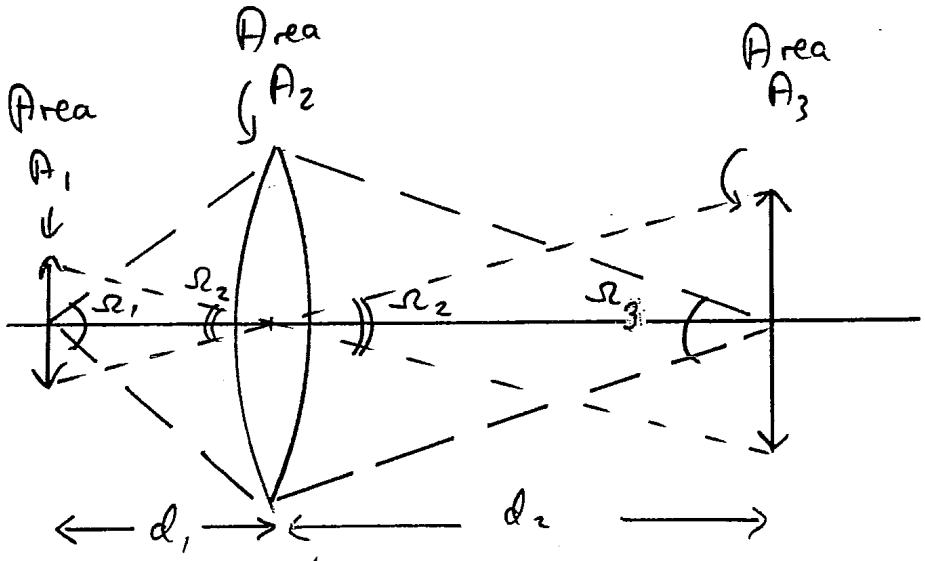
$$SNR = L(A\Omega\Omega_d\Delta t)^{\frac{1}{2}}\Delta\nu D^*$$

Recap: Determining instrument performance
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- **(Since the lens views the same solid angle both sides to focussed images)**



$$\Omega_1 = \frac{A_2}{d_1}$$

$$\Omega_2 = \frac{A_1}{d_1} \quad \text{eliminate } d_1$$

Hence:

$$\frac{A_2}{\Omega_1} = \frac{A_1}{\Omega_2} \quad \text{giving } A_1\Omega_1 = A_2\Omega_2$$

Similarly for $A_3\Omega_3 = A_2\Omega_2$

Noise Equivalent Radiance and Temperature

- **The Noise Equivalent Radiance, or *NER* is given by:**

$$NER = \frac{L}{SNR}$$

$$= \frac{1}{((A\Omega\Omega_d \Delta t)^{\frac{1}{2}} \Delta\nu D^*)}$$

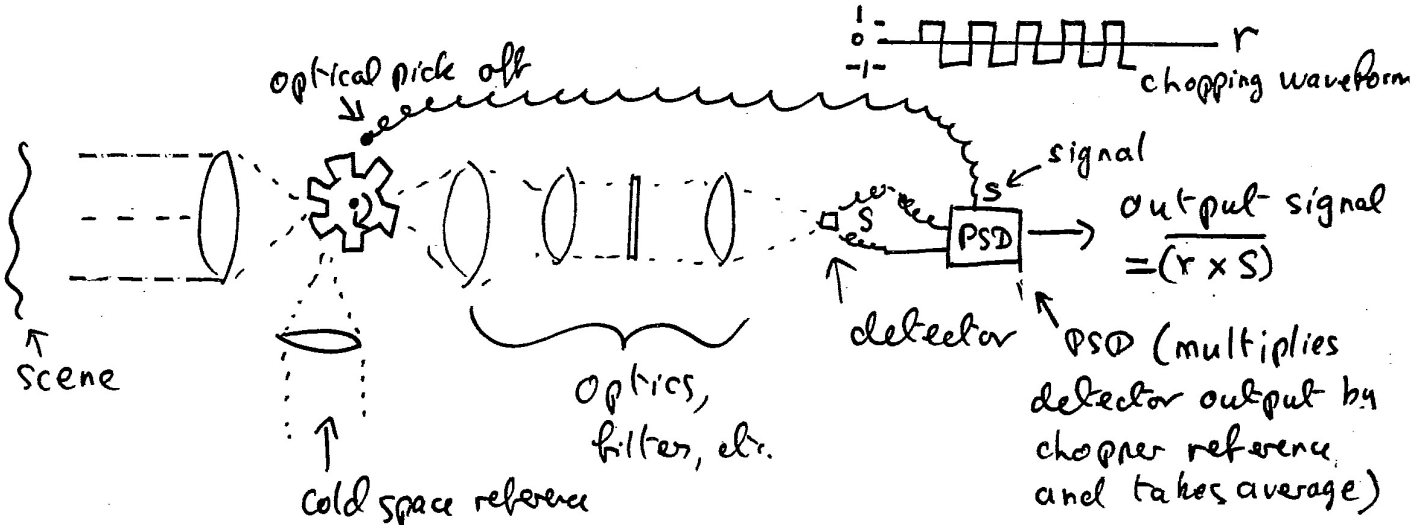
- **Normally we can't make Ω_d bigger than 3-4 if using a lens, probably less for mirrors.**
- **NER and SNR are terms applied to the whole instrument viewing a scene of Planck function *B***
- **Hence we can set *L=B***
- **Two more factors apply**
  - **the chopping factor $\sqrt{2}$ or bigger since the scene is only viewed for a fraction of the time.**
  - **Optical throughput due to losses in the optics etc.**
- **The radiance error ΔB will correspond to a scene temperature error ΔT, defining the *Noise Equivalent Temperature*:**

$$\Delta T = \Delta B \Big/ \frac{dB}{dT} \quad \text{with } \frac{dB}{dT} \text{ the Planck function derivative}$$

$$NET = NER \Big/ \frac{dB}{dT}$$

Chopping
(AP01 Again…)

- **Chopping the input signal to the instrument has been mentioned several times already.**
- **Most precision instruments employ a chopper, or chopping of some description:**
  - **1. To minimise the 1/*f* noise (see noise sources discussion) – typically ≈500Hz is desirable though it depends on the detector.**
  - **2. To avoid the effects of any drifts, fluctuations in the optical system.**
- **Because of point (2) we want to put the chopper as early in the optical chain as possible.**
- **The chopper will switch between the wanted scene and a constant reference.**
- **A phase sensitive detector, PSD, (e.g. lock-in amplifier) is then used to look for the same frequency and phase in the detector signal:**



Chopping (2)

- **Reference**
  - **Can be the chopper itself if it is painted black and stable in temperature.**
  - **Space or an internal blackbody if a reflective chopper.**
- **Mechanics**
  - **Choppers can take many forms**
  - **Rotating disks with holes or teeth**
  - **Tuning fork that oscillates (i.e. leaf spring with a flag on the end)**
  - **In the microwave an electronic switch can be used.**
  - **Liquid crystal display for visible light (no moving parts).**
  - **Scan mirror assembly, scanning from space to scene to blackbody**
- **Measurement techniques such as gas correlation and FTIR have chopping/modulation inherent in the technique.**
- **If you are very, very ,very careful with your thermal design you can avoid mechanical chopping and use an electronic modulator (e.g. Mars Climate Sounder) – but not ideal….**

Calibration

- **Instrument calibration is vital to performance (radiance in - volts out).**
- **Two types for space based instruments, pre-launch and post-launch.**

- **Pre-launch – instrument needs to be tested in a space-like environment, known as thermal vacuum testing. Can take several months in a vacuum chamber surrounded by cryogenic panels etc.**
- **Types of pre-launch calibration:**
  - **Radiometric - instrument views calibration, often blackbody, targets at different temperatures.**
  - **Spectral – map out the spectral response of the instrument using a monochromator**
  - **Geometric – map out the field of view of the instrument, calibrate the scan angle encoders.**
- **Post launch it is not normally possible to calibrate everything in flight.**
  - **Radiometric - need 2 measurements to obtain the gain and the offset. E.g. view space directly if possible to give the zero radiance level. In the IR view a warm blackbody cavity with a traceable temperature calibration. E.g. ATSR used 2 blackbody targets at different temperatures but not space. In the UV use the Sun or a standard lamp.**
  - **Can also use calibrated patches of ground e.g. AVHRR, SEVIRI**
  - **Geometric – normally impossible for bodies with an atmosphere**
  - **Spectral – mercury lamp in the UV, with a grating or FTIR (interferometer) use atmospheric line positions.**

Calibration from Volts to Radiance

- **Again, have a look at the AP01 Radiometer lab script.**

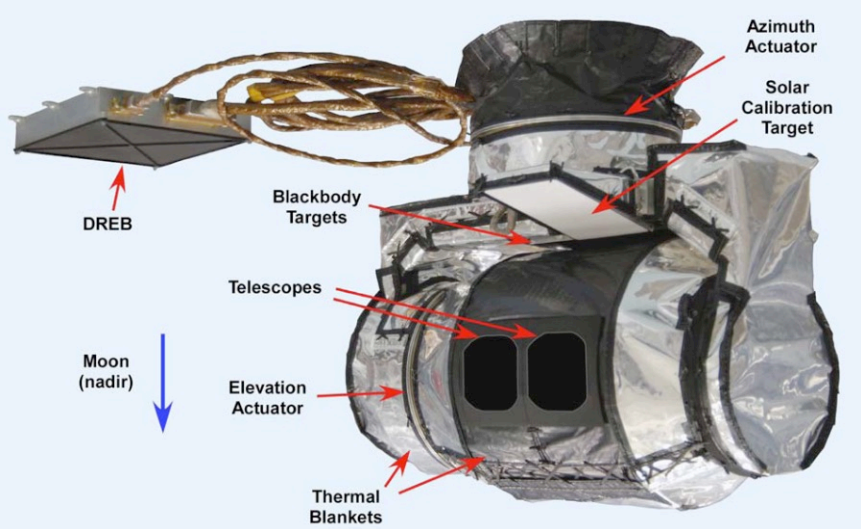
- **Assuming the spectral response is well known and you have a detector with a linear response to incident power then the radiance can be derived (to a good first approximation) as:**

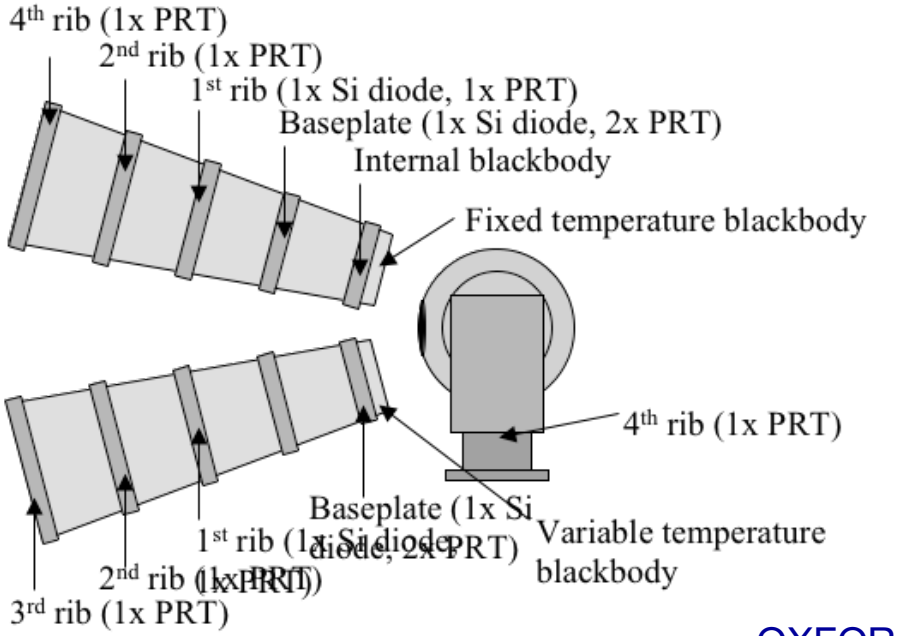
$$B_{Scene} = \left(B_{Hot} - B_{Cold}\right)\left(\frac{V_{scene} - V_{Cold}}{V_{Hot} - V_{Cold}}\right) + B_{Cold}$$

- **This assumes that all instrumental offsets have cancelled out, background variation is stable etc.**
- **There will be errors on all the measured terms (volts, radiance from calibration temperature sensors) so these need to be included in the final *measurement covariance matrix*, which will be covered in the inverse methods section.**
- **Does it work in a "real instrument"?**

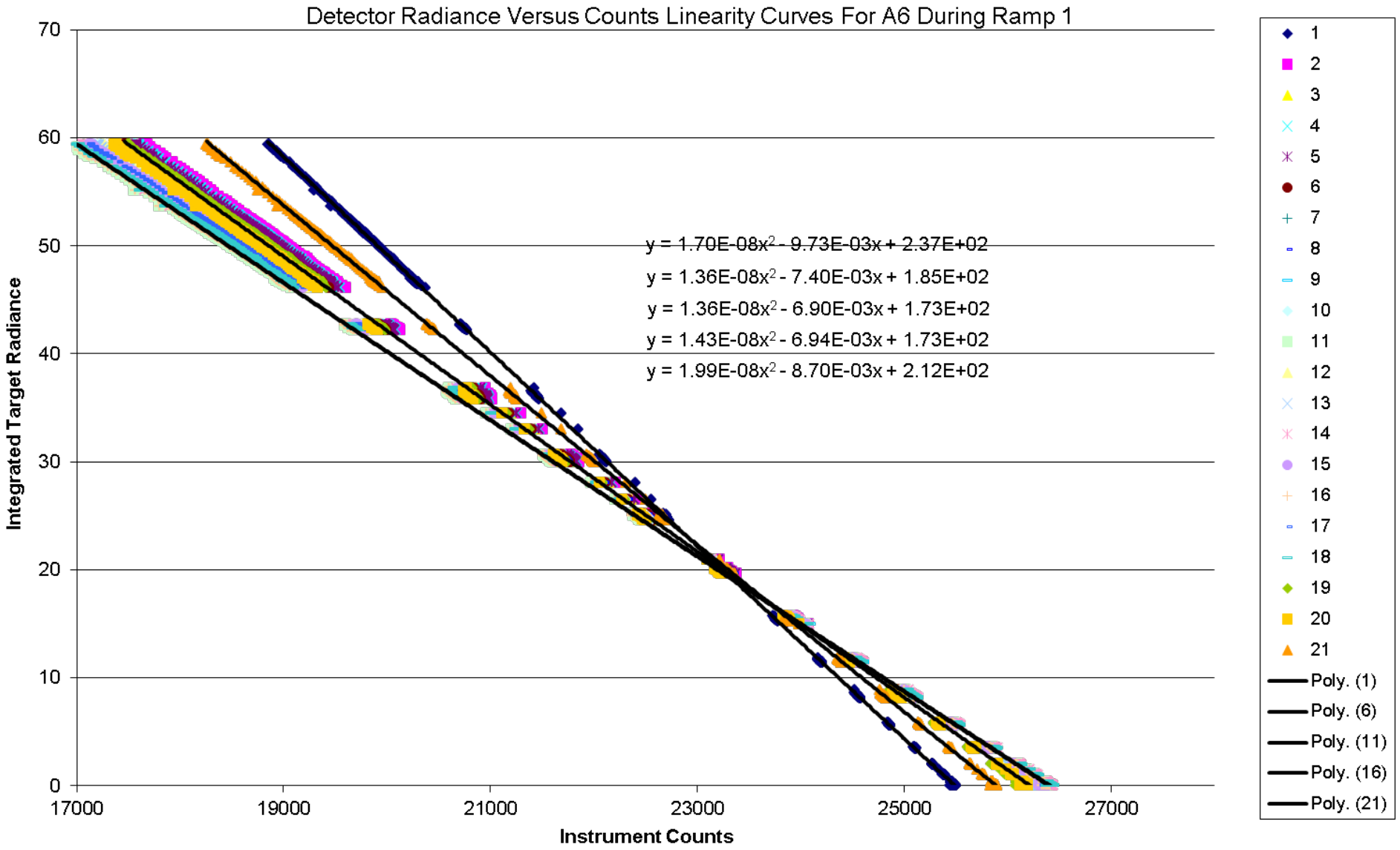
Example Ground Calibration Data, Diviner Lunar Radiometer Detector Linearity

- **Test Setup, environment thermal vacuum chamber at JPL:**
- **Diviner instrument (a 9 channel infrared radiometer to measure the surface temperature of the Moon) placed in front of two blackbody targets at two separate temperatures.**
- **Looking at the data from the test measurements, the results show how good you can get with a linear fit.**
- **(Images from Paige et al. 2010)**





Linearity of 21 detectors in the Diviner channel A6 (12.5 - 25 µm, from Thomas 2011)



OXFORD

Instrument Design Considerations

- **Big compromise - how do you spend your money to get the best overall data product? Modern instruments can take >5 years to design.**
- **Factors to take in to account:**
  - **Orbit, height inclination, equator crossing time.**
  - **Scanning**
  - **Pointing – how accurately can you point the line of sight? Very important for limb viewers**
  - **Field of view – small leaks from outside the field of view sometimes very undesirable (limb viewers again), diffraction effects (especially in the far-IR and microwave).**
  - **Detector type – do we need cooling (heavy and expensive) to achieve the SNR we need? Do we need to use chopping? Arrays or single element detectors?**
  - **Spectral resolution – range and resolution?**
  - **Measurement times - shorter times mean more noise**
  - **Data rates - can we get all the data back?**
  - **Onboard calibration – how much can be checked in flight? How stable is the calibration for e.g. measuring small scale climate changes?**
  - **Instrument control, on board computer etc – how flexible can we make it?**
  - **Redundancy and single point failures- can we work around failures after launch (happens a lot!)**
  - **Mass, power, size limits**

End of lectures

- **Next up the class example…**

INSTRUMENTATION

For these questions you may find it helpful to use a Python/MATLAB code or refer to the spreadsheets used in the class to help track each step in the calculations. Working through this question I found it useful to code a Planck function in wavenumbers to speed things up. Any queries or corrections please email me (neil.bowles@physics.ox.ac.uk)

1. **Instrument Noise Performance & Calibration**

   This question goes through some example noise and calibration calculations for an infrared radiometer, so you can see how detector performance contributes to the overall measurement sensitivity of an observation.

   Definitions and detector performance

   From the notes, define the term *Noise Equivalent Power* (NEP) and $D*$ as they relate to infrared detector performance. Why is the $D*$ (cm $\mathrm{Hz}^{1/2}\mathrm{W}^{-1}$) generally a more useful figure of merit for a thermal infrared detector than the NEP? [4]

   The Scene — measuring the sea surface temperature of the Earth

   Define the term *brightness temperature* of an object in the context of a measurement by an infrared remote sensing instrument such as a thermal camera. How might this be different to the temperature measured by a thermometer? How might you correct for this? [3]

   A satellite radiometer is designed to measure the sea surface temperature using an infrared atmospheric window at 833 cm^{-1}. The instrument has a scan mirror to switch between calibration and scene views. The scan mirror measurement sequence consists of viewing a 'cold' (273 K) blackbody calibration target, the 'scene' (i.e., nadir-view of the Earth), and a 'hot' (300 K) blackbody target, with 100 measurements of 1 s duration taken for each view. The figure below shows the recorded voltages from the instrument's detector electronics during a typical observation sequence.



Recorded voltages recorded during a typical calibration and observation sequence.

The data in the figure is saved in the file Radiometer_raw.csv on the course website. Using the time/voltage data in this file estimate the Noise Equivalent Radiance (NER) for 1 s measurement in the following way:

(a) Load the file into a data analysis tool of your choice (e.g. Python, MATLAB etc.)

(b) If a lower voltage represents a lower temperature, identify the 'cold', 'scene' and 'hot' blackbody in the data.

(c) From the data estimate the root mean square (RMS) error for each of the instrument's views.

(d) By considering the radiometer calibration equation from the notes and the two calibration scenes show that the Noise Equivalent Radiance for the detector is approximately 1.4×10^{-3} W m^{-2} sr^{-1} (cm^{-1})$^{-1}$

[6]

The estimate for the NER can be combined with information about the instrument's optics and detector area to estimate the NEP and the signal to noise ratio (SNR).

For this radiometer,the detector area is 1 mm^2 and the focal ratio (f-number) between the final focusing element and the detector is 2 (i.e., f/2). Using this data show that the etendue ($A\Omega$) for the instrument is approximately 1.9×10^{-7} m^2 sr. [6]

Assuming a spectral bandwidth of 1 cm^{-1} and a mean transmittance of energy from the scene to the detector through the instruments opticsof 0.6 show that the NEP is $\approx 1.6 \times 10^{-10}$ W Hz$^{-1/2}$ and estimate the $D*$ of the detector. [5]

Finally, estimate the SNR for the instrument when viewing a scene at 278 K for 1 second. [1]

[Total: 25]

In the next question we will take these results and see how they can be translated into a measurement of sea surface temperature with an associated accuracy. We will then consider what the implications might be for the science it is hoped the instruments observations can support.

2. **From Instrument Performance to Science**

The instrument described in question 1 has a 50 mm diameter telescope and is being operated in an 800 km altitude Sun Synchronous 3 pm orbit. Explain the terms 'Sun Synchronous' and '3 pm orbit'. [2]

Using this information and the instrument performance data from question 1 estimate the ground sample distance in km of the instrument when viewing the Earth in the nadir. Quantitatively, how does this compare with the 1 second integration time that the observations are taken at? [5]

From the notes, write down an equation that relates the scene radiance B_S to the observed voltage V_S in terms of the calibration target radiances B_H, B_C and voltages V_H and V_C. Hence show that the uncertainty in the scene radiance δB_S is related to the uncertainty in the measured voltages δV by:

$$\delta B_S = a\delta V_S + b\delta V_H - (a+b)\delta V_C$$

where the subscripts S, H and C refer to the scene, hot and cold measurements respectively, and derive expressions for a and b. [5]

Hence derive the scene radiance B_S and its associated error. [5]

Using the detector performance data from question 1, the calibration equation and the uncertainty equation above estimate the scene brightness temperature and its associated error based on the average values for each of the 100 s views [at the scene temperature and 833 cm^{-1}, $\mathrm{d}B/\mathrm{d}T \approx 1.6 \times 10^{-3}$] [3]

As described in question 1, one of the instruments main goals is to measure sea surface temperature, however for long term climate studies the measurement uncertainty required is typically $\sim$0.1 K. Comment on how well this instrument meets this requirement. [1]

Qualitatively, how might the following modifications to the instrument's design improve its performance and what would be the potential challenges (if any) to implementing them:

(a) Increasing the number of spectral channels?

(b) Modifications to the instrument's scanning pattern so that it looks off-nadir and then nadir as it moves along its orbit track?

(c) An increase or decrease in size of the telescope's aperture?

(d) A combination of the above?

[4]

[Total: 25]

ESTIMATION & CLIMATE

3. **Satellite Altimetry**

Estimate the average speed of a satellite orbiting the Earth at a mean altitude of 1,330 km. [3]

Give four factors that cause variations in the distance reported by a radar altimeter between a satellite and the ocean surface. Variations in this distance that are caused by steady ocean currents are known as the 'dynamic topography' η. Sketch how η is related to near-surface geostrophic currents in the Northern Hemisphere and state the balance of forces involved. Derive an expression for $\mathrm{d}\eta/\mathrm{d}t$ as the satellite travels along its orbit in terms of the satellite speed v_s, its orbital inclination θ and u_g and v_g, the zonal and meridional components of the near-surface geostrophic current. [8]

Three satellites, all at an altitude of 1,330 km and equipped with radar altimeters, pass almost simultaneously over a point in the open ocean at 20°N. Satellite A is travelling from north-west to south-east at an orbital inclination of -66° relative to the equator, satellite B travelling from south-west to north-east at an orbital inclination of 66° and satellite C is in a polar orbit travelling from south to north. Given $\mathrm{d}\eta/\mathrm{d}t = 21$, -7 and $-13\,\mathrm{mm\,s^{-1}}$ for satellites A, B and C, respectively, estimate u_g and v_g. [8]

If the altimeters measure η at 5 second intervals and the standard error of the difference between successive observations is 2 cm, compute the standard errors in your estimates of u_g and v_g arising from this instrument noise. How are these errors correlated? [6]

[Total: 25]

4. **Dynamics and predictability**

For a general dynamical system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ show that the direction of fastest instantaneous error growth is aligned with the eigenvector of $\mathbf{J} + \mathbf{J}^T$ with the most positive eigenvalue, where $\mathbf{J}$ is the Jacobian, $J_{ij} = \partial f_i / \partial x_j$. [4]

Error growth and adaptive observations

The Lorenz system is defined by the following equations

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= \rho x - y - xz \\ \dot{z} &= -\beta z + xy \end{aligned}$$

In the vicinity of the line $x = y = 0$, show that perturbations grow in at most one direction provided $(\sigma+\rho-z)^2 > 4\sigma$ (recall that the trace of a matrix is equal to the sum of its eigenvalues, while the determinant is equal to their product). [6]

If you had the opportunity to make a single observation (one linear combination of x, y and z) of the Lorenz system evolving in vicinity of the line $x = y = 0$ to minimise the error in a very short-term forecast, assuming your prior uncertainty is equal and uncorrelated in all three variables, what would it be? How does your prediction match up with the numerical simulation in the lectures? [8]

Climate change in a non-linear system

Tim Palmer's variant of the Lorenz system is defined thus

$$\begin{aligned} \dot{x} &= \sigma(y - x) + \alpha\cos(\theta) \\ \dot{y} &= \rho x - y - xz + \alpha\sin(\theta) \\ \dot{z} &= -\beta z + xy \end{aligned}$$

In the vicinity of the line $x = y = 0$ and for $\theta = -\pi/4$, find the velocity $\dot{\mathbf{x}}$ and acceleration $\ddot{\mathbf{x}}$ for both $\alpha = 0$ (standard Lorenz system) and $\alpha > 0$. Sketch trajectories in this region in the (x, y) plane for $z > \rho - 2\sigma + 1$ and hence explain why it is at least plausible that trajectories exit preferentially into the quadrant $x < 0, y < 0$ when $\alpha > 0$, $\sigma = 10$, $\rho = 28$ and $\beta = 8/3$ (don't worry if this doesn't look like a formal proof: we can really only understand the impact of external forcing on strongly non-linear systems through numerical simulation). Notice that the x-component of the forcing is positive while the x-component of the response, defined by the displacement of the centre of gravity of the attractor, is negative, and comment on the implications for recent weather events in Texas. [5]

[Total: 25]

[Total for Problem Set: 100]

[AD 20210228]