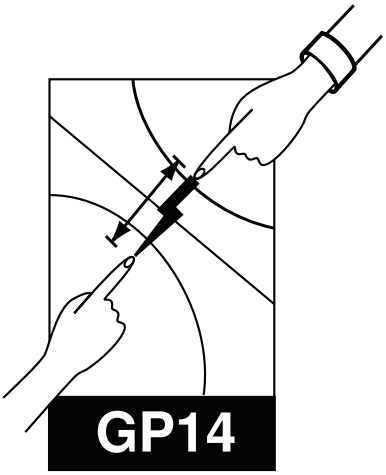
The electron: particle and wave



GP14

MJ, SMT revised July 2024

Introduction

This experiment is in three parts. The first two parts (sections 1 and 2) demonstrate that under different circumstances an electron can behave as a point particle, obeying classical mechanics, or as a quantum particle showing wave-like diffraction. The third part, Franck-Hertz scattering in Section 3), shows that electrons bound in atoms have discrete energy levels.

The three sections are independent and do not have to be done in the order presented in the script. As there is only one set of the Franck-Hertz equipment, one group should start with this in the morning so the other group may use it after lunch. Parts of this practical need the local room lights to be turned off and the switch is on the pillar opposite the apparatus.



Some of this apparatus involves high voltages and it is dangerous to operate it if any wires are disconnected. Do not unplug any wires when the power supplies are on. Do not unplug any wires when the power supplies are off, unless told to do so. If you notice that any wires are disconnected, consult a technician.

1 The classical electron and measurement of e/m

1.1 Theory

An electron of mass m and charge e moving with a velocity v in a magnetic field $\boldsymbol{B}$ will experience a force $\boldsymbol{F} = e\boldsymbol{v} \times \boldsymbol{B}$. If $\boldsymbol{B}$ is a constant, the classical equations of motion can be solved to give the trajectory of the electron which is a circle of radius R

$$R = \frac{mv}{eB}. \tag{1}$$

▶ What condition is required in the relative directions of B and v for the electron to move in a circle? If this condition is not met, what form does the trajectory take (B remains constant)?

In this experiment, the ratio e/m for the electron is measured using equation 1.

1.2 Apparatus

The apparatus, shown in figure 1, consists of an evacuated globe containing an electron gun with a variable voltage anode. The electron gun is essentially a heated wire that emits electrons thermionically. The anode is used to attract and collect the electrons. The electron beam can be focused by applying

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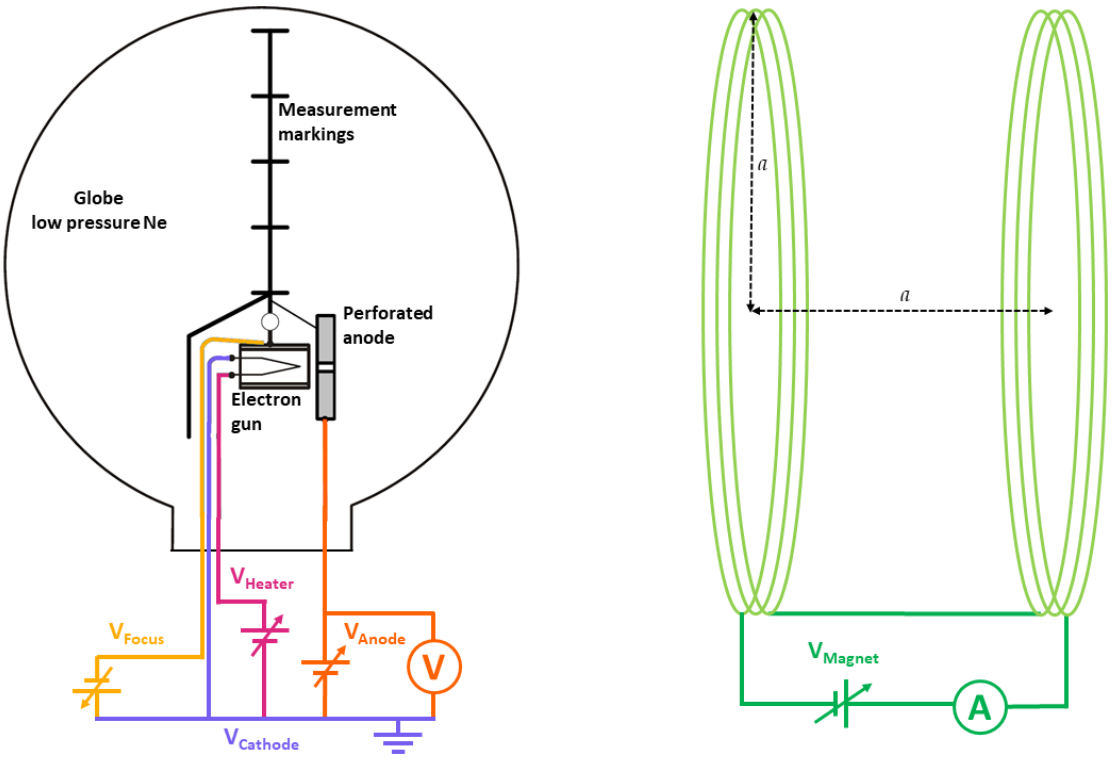


Figure 1: Schematic diagram of the e/m apparatus. Left: the globe containing the electron gun, anode and residual neon gas. The heater (pink), focus (yellow), and anode (orange) voltages can be varied using the appropriate dials on the power supply. Note that the focus voltage (applied to the Wehnelt cylinder) has the opposite polarity to the anode voltage. The cathode is grounded (purple). The anode voltage V is measured with the voltmeter separate to the power supply. Right: the Helmholtz coils of radius a (green). The current through the coils I is measured with the ammeter separate to the power supply. The voltage across the coils (the magnet voltage) can be varied using the dial on the power supply. The globe is located in the middle of the coils. The diagram is not to scale. Colours are simply used to highlight different parts of the circuitry, and do not match those in real life.

a voltage (of opposite polarity to the voltage on the anode) to the Wehnelt cylinder, which forms the outside of the electron gun. A magnetic field, provided by a pair of Helmholtz coils, is used to deflect the electrons. The apparatus is effectively a simple electron accelerator.

The velocity of the electrons is related to the anode voltage by

$$v^2 = \frac{2eV}{m}. \tag{2}$$

Combining equations 1 and 2 we get

$$\frac{e}{m} = \frac{2V}{(BR)^2}. \tag{3}$$

The Helmholtz coils are two circular coils of radius a placed a distance a apart. The field near the centre of this arrangement is highly uniform, with magnitude given by

$$B = \frac{m_0 NI}{a(\frac{5}{4})^{\frac{3}{2}}} \tag{4}$$

where m_0 is the permeability of free space $= 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$, N is the number of turns and I is the coil current. For the given coils $a = 0.147\,\mathrm{m}$ and $N = 124$.

This thin-walled, evacuated tube could implode if not treated with care. Do not subject the tube to mechanical stresses, or exceed the operating parameters defined below.



In your lab book, draw the magnetic field at the centre of the system and slightly outside the centre line.

Look at the apparatus and check you have identified the heater, anode, focus, and magnet (Helmholtz coil) power supplies and measuring equipment based on figure 1. If in any doubt consult a technician. Make sure all controls are turned to zero (counter clockwise) before turning on the power supply.

Switch on the heater control to about 6 V and wait about a minute. Do not exceed 8 V as this will cause damage to the tube. You should see the red glow from the heater.

Increase the anode voltage, but do not exceed 300 V. The horizontal electron beam should now be visible.

Adjust the focus voltage to sharpen the beam.

Turn up the magnet voltage until you see the circular discharge inside the tube. Readjust the focus voltage if necessary.

To get a good measurement of the magnet current I and the accelerating voltage V use the separate meters provided, not the meters on the power supplies.

▶ Give a qualitative account of how the electron's trajectory changes as V and B are varied. How is it possible to 'see' the electron beam? Does it depend on the pressure or composition of the residual gas in the globe?

1.3 Measurements and analysis

Using equation 3 the e/m ratio can be measured for a fixed radius R from the measured Helmholtz current I and anode voltage. You can use the black curtain as a cover to see the beam clearly if needed.

Select I so the radius of the circular path coincides with the corresponding marking of the beam tube (20 mm per division). Note the current.

Decrease the anode voltage V in steps of 20 V to 200 V. In each case reset the coil current I so the radius stays the same. Record these values.

Deduce a value for e/m, for which you will need to calculate the magnetic field. Don't forget to estimate the error in the measurement.

Turn all controls back to zero before switching off.

▶ Does it matter which method is used to work out e/m? Should the origin be included as a 'data point'?

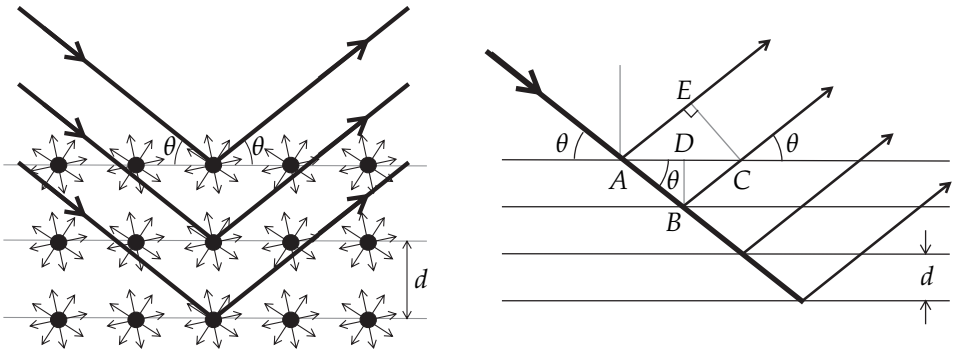
2 Electron diffraction

2.1 Background

Quantum mechanics predicts that under certain circumstances electrons will exhibit wave-like effects. In fact, in 1926, a few years before quantum mechanics was firmly established, de Broglie postulated that the wavelength associated with an electron is $\lambda = \frac{h}{p}$ where h is Planck's constant and p the momentum of the electron. For a non-relativistic electron of kinetic energy $E = \frac{1}{2}mv^2$, the momentum is given by $p = \sqrt{2mE} = mv$. Check that an electron with energy of 1 keV corresponds to $\lambda \approx 4 \times 10^{-11}$ m, the typical spacing between atoms in a solid. This suggests that a regular crystalline solid could act as a diffraction grating for electrons, provided that it is thin enough to allow appreciable transmission.

Figure 2 shows the geometry of the plane and beams. The beam is scattered by all the atoms, and constructive interference only occurs when the angle of incidence equals the angle of reflection. The waves from neighbouring planes interfere, and the condition for constructive interference is $\exp(ikD) = 1$ where $k = \frac{2\pi}{\lambda}$ is the wave-vector of the electron's de Broglie wave and D is the path difference. The scattered beam emerges at an angle 2θ with respect to the incident beam. In figure 2b, $D = AB + BC - AE$ and $D = 2d\sin\theta$. Thus the condition for a maximum in the scattered intensity is given by the Bragg law:

$$2d\sin\theta = n\lambda \quad \text{where } n \text{ is a positive integer such that} \sin\theta \leq 1 \tag{5}$$



(a) Sketch of the physical processes involved

(b) Detail of the geometry. D is the path difference given by $D = AB + BC - AE$ and $D = 2d\sin\theta$ which permits derivation of Bragg's law.

Figure 2: Electron beam scattering from parallel atomic planes a distance d apart. The scattered beam is at an angle 2θ with respect to the incident beam.

2.2 Experimental procedure

The apparatus is an evacuated glass envelope containing an electron gun. The beam passes through a thin graphite screen which consists of many fine crystallites oriented in all directions. Each crystallite oriented to satisfy the Bragg relation gives a diffracted beam. The emerging electrons strike a phosphor on the glass screen, and the electron current density is observed by the illumination produced.

To switch on the tube:

Set all switches off, all knobs fully anticlockwise and plug into the mains.

Switch on the mains to warm up the heater, and wait one minute.

Switch the supply for the EHT ON and turn up the COARSE control slowly. Notice the voltage changes slowly, and do not raise the EHT supply above 5.5 kV.

The EHT meter indicates the accelerating potential for the electrons (V). As EHT approaches 3kV, two rings appear on the screen and their diameter changes as the voltage rises. Measure the diameter of the rings, and calculate θ from $D_r \approx 4L\theta$ (valid for small θ) where D_r is the ring diameter and L is the distance the electrons travel across the tube, $\approx 13.5\,\text{cm}$.

The rings are not very bright, so a dark box and curtain are provided to reduce ambient illumination. An electron accelerated by a voltage V gains kinetic energy $E = eV$. Use this and the information above to show that D_r is proportional to $V^{-0.5}$ and find an expression for the constant of proportionality in terms of d, L, h, e and m (assuming that $n = 1$)[1].

For voltages of 3.8 kV upwards take enough readings of the two diameters to be able to plot graphs of D_r against $V^{-0.5}$. From the measured slopes obtain the values of d and associated errors.

▶ Explain with a diagram why the diffraction pattern is in the form of rings. Why are only two rings observed?

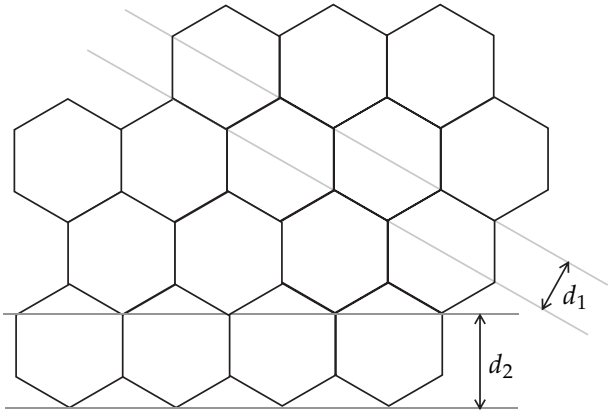


Figure 3: Two dimensional representation of the graphite structure showing two different sets of equivalent planes of carbon atoms. You will need to derive the geometrical relationship between d_1 and d_2.

Graphite has a hexagonal structure (see figure 3). The carbon–carbon distance within the layers d_{C-C} (i.e., the length of the edges of the hexagons) is 0.142 nm. Find the geometrical relationship between d_{C-C}, d_1 and d_2 and the corresponding values of d_1 and d_2.

▶ Is it consistent with the measured values?

▌ Ramp down and switch off the high voltage after use.

[1] Planck's constant $h = 6.63 \times 10^{-34}\,\text{J s}$, electronic charge $e = 1.6 \times 10^{-19}\,\text{C}$, electron mass $m = 9.11 \times 10^{-31}\,\text{kg}$.

3 The Franck-Hertz experiment



The PHYWE Franck-Hertz apparatus is shared between two groups. If you are assigned to use it in the morning, you must complete your use data-taking by 1:00PM. Afternoon people should not expect it to be available before 1:30PM but can nevertheless start reading this section before lunch and proceed as far as the visual demonstration using your Leybold (LD) kit (up to Sec. 3.2.1).

In 1914, James Franck and Gustav Hertz performed an experiment which demonstrated the existence of excited states in mercury atoms at discrete energies. Later, this was taken as evidence for Bohr's theory of the atom, which predicted that electrons occupied quantised energy states, and they were awarded the Nobel Prize in 1925. Here, you will repeat the Franck-Hertz experiment with mercury (section 3.3) but first we will look at neon, which readily exhibits similar discrete atomic structure.

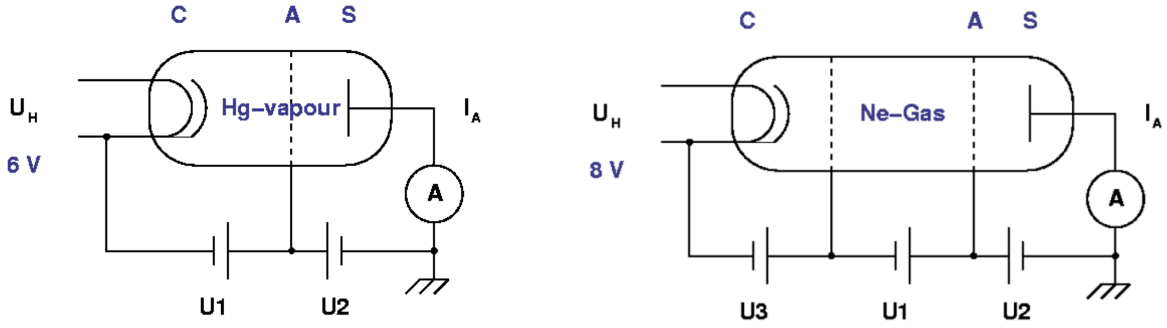


Figure 4: Diagram of the mercury and neon Franck-Hertz tubes, showing the cathode C, accelerating electrode A and signal-collector S. The accelerating/braking potentials are U_1 and U_2 and in the neon case, an extra boost can be provided by U_3. The measured current, I_A, is also indicated.
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3.1 Franck Hertz tubes

In these experiments, the gas is at a low pressure (1000 Pa $\approx$ 10 mbar) within glass bulbs. Electrons are generated by thermionic emission from a heated filament and accelerated through the low-pressure gas towards a grid held at a positive voltage. Due to their momentum, most electrons pass through this grid to reach a charge-collector which measures the accumulating charge as a current. The charge collector is held at a small negative voltage with respect to the grid. This creates a potential barrier that means only electrons of sufficient momentum contribute to the collected charge. Schematic diagrams of the tubes are shown in Fig. 4. The mercury and neon tubes have a similar design though the neon version sometimes needs an additional accelerating voltage.

3.1.1 Phenomenology

During their passage through the low pressure gas, electrons may undergo a range of interactions.

1. Electrons can be elastically scattered (electron energy diminished and direction altered, atom recoils but total kinetic energy conserved). Elastic scattering is (by far) the most likely interaction especially as low electron energy (< 2eV).
2. Electrons can scatter inelastically (kinetic energy transferred to internal [potential] energy as an atomic electron is raised to a higher atomic shell. Because of the quantised nature of atomic structure, there is a threshold for inelastic scattering called the *first excitation energy*.
3. With sufficient energy transfer, an atomic electron may be ejected from the atom entirely. This is called ionisation. Free electrons recombine with positive ions resulting in a cascade of visible photons. If this process occurs in a runaway manner, the Franck-Hertz becomes a discharge tube. In this case computer interface will report that the tube has *ignited*.

In a Franck-Hertz experiment, the inelastic scatter is detected via a drop in the number of electrons able to surmount the potential barrier in front of the signal-collector (S). Electrons become unable to pass the potential barrier having lost energy to the gas. Famously, James Franck and Gustav Hertz showed that the drop in the electron flux occurred at discrete values of accelerating voltage.

3.1.2 Apparatus

This experiment uses a commercial educational kit provided by PHYWE as well as a demonstrator from Leybold. They consist of the Franck-Hertz bulbs in protective structures and a control unit. The PHYWE unit interfaces to the desktop PC. The PHYWE data-acquisition software is called measure and an icon should be available on the desktop computer.

measure allows users to define the heater voltage, the braking potential and the control voltage (neon only). It can automatically sweep though a range of values of the accelerating voltage, U_1. The values of I_A will be sampled during the sweep and measure displays the result on a graphical interface. The software allows you to print these plots for transfer to logbooks, or you can export the data to a text file and read into Python for further analysis.

3.2 Neon

3.2.1 Visual demonstration

Identify a Leybold control unit (brand logo **LD**) and its neon tube. Switch it on (power button: rear left) and adjust the settings as follows: $U_1 \simeq 1.7\,\mathrm{V}$; $U_3 = 0.0\,\mathrm{V}$. Then switch the LED display to show U_2, select manual operation (MAN.) and turn the large black knob to scan U_2 over its range[2], carefully observing the bulb throughout.

> With suitable annotations, sketch what you see in the tube. Explain the evidence for quantised energy levels in neon. By considering the amount of energy provided to the electrons by the electric field, roughly estimate the first excitation energy of neon.

> With the use of a handheld spectroscope, or simply colour-matching to a published electromagnetic spectrum, estimate the wavelength of any light you observe.

▶ Does your wavelength measurement match your estimate of the first excitation energy?

Returning U_2 to 0 V and switch off the Leybold (**LD**) unit. You will continue with the PHYWE apparatus, which benefits from a computer interface for more precise data analysis.

3.2.2 Data analysis

Check that the two cables from the smaller neon apparatus are connected to the **PHYWE** control unit. Switch it on and press the 'Function' button until the letter 'PC' are shown on the LED display. Log on to the adjacent desktop computer and open the measure application, which should automatically detect the the DAQ interface to the control unit.

Press the new measurement icon. Set $U_2 = 6.8\,\mathrm{V}$, $U_3 = 4.0\,\mathrm{V}$ and $U_H = 4.8\,\mathrm{V}$. If necessary, adjust these values and rerun until the set of peaks on the graphical output is shown without saturation of the charge-collector's dynamic range. As with the Leybold bulb, visible light is emitted though it is fainter here due to a choice of lower heater current to avoid saturation. Record your optimal settings.

▶ Why does collected current depend on voltage? Why is there a regular pattern? Why is there a general upwards trend? How do U_H and U_2 affect the form of the data?

[2]Note: The Leybold apparatus uses the label U_2 for the principal acceleration voltage, PHYWE labels this U_1.

Use the built-in peak analysis software to find and record the voltage values corresponding to the peaks in these data. Plot the inter-peak distance (ΔE) versus peak number and estimate the first excitation energy of neon. Print your data and add it to your logbook.

It is recommended to define the inter-peak distance as,

$$\Delta E_n = E_n - E_{n+1}$$

with the first peak (just below 20 eV in the neon case) labelled $n = 0$.

▶ **The ground-state electronic configuration of neon (atomic number $Z = 10$) is $1s^2 2s^2 2p^6$. It is possible to promote a $2p$ electron to achieve either the $1s^2 2s^2 2p^5 3s$ and $1s^2 2s^2 2p^5 3p$ excited state. Thus it is expected that ΔE will increase with peak number.**

Justify this statement; and explain the light output from the neon tubes.

Conclude this part by saving your measurement and exporting the data to a textfile in case you later wish to replot the data. Switch off the control unit and unplug the two black cables.

3.3 Mercury

Ensure the black cables from the [large] mercury apparatus are connected to the Franck-Hertz control unit and switch it on. Press the Function button such that manual (man.) is indicated and then press 'Oven on/off', which should flash as the mercury apparatus is brought up to temperature. Press Function one more time so that 'PC' is shown on the LED display, then continue reading.

Mercury vapour is particularly useful to demonstrate inelastic collisions for two reasons. Firstly, the large relative atomic mass of mercury ensures that the energy lost by the electrons in elastic collisions (approximately $4m_e E/M_{\mathrm{Hg}}$) is very small. It is therefore possible to work at relatively high pressures and have many electron-atom collisions without contaminating the electron flow with a cloud of slow electrons formed in successive elastic collisions.

Secondly, the cross section for an inelastic electron scattering is relatively large so with suitably high number density of mercury atoms a large effect is expected (once the energy threshold is reached). Mercury is liquid at room temperature so the Franck-Hertz bulb must be heated to produced a vapour of sufficient pressure to achieve a necessary number density of target atoms.



The Franck-Hertz oven is very hot. Do not attempt to touch the apparatus.

3.3.1 Procedure

In the measure application, click the little red menu icon ('New measurement') to brings up the main control panel. Set $U_2 = 1.0\,\mathrm{V}$, $U_H = 5.8\,\mathrm{V}$ and set the temperature to 175 °C, and press 'Continue'.

In a few seconds, the graphical interface will display the data. Several peaks should be visible before the signal saturates. Most importantly, the peaks should vary smoothly. If discontinuities or spiky plots result, discard and rerun. Once settled, with the suggested values of U_2 and U_H, multiple peaks and a maximum current of $\sim 30\,\mathrm{nA}$ is expected. If curious, adjust U_2 and U_H to investigate their effect.

Deduce the average ΔE between peaks. This can done with the built-in peak-analysis tool, or on exported data with Python (a code snippet is provided below). Plot ΔE versus peak number.

3.3.2 Interpretation

The atomic number of mercury is A=80 so the outer-most ground-state electrons fill the 6s shell (full configuration: $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^{10}\,4s^2\,4p^6\,4d^{10}\,4f^{14}\,5s^2\,5p^6\,5d^{10}\,6s^2$). Above this, the energy levels that excited states are shown in Fig. 5. The first excitation energy corresponds to the $6s\,^1S_0 - 6p\,^3P_0$ transition. Note that, once promoted into the $6p\,^3P_0$ state, the electron cannot readily de-excite as single photon emmission is forbidden for this state. Therefore, the excited $6p\,^3P_0$ state is metastable with a lifetime of many seconds. In contrast, the $6p\,^3P_1$ state readily excites then decays via photon emission after 120 ns.

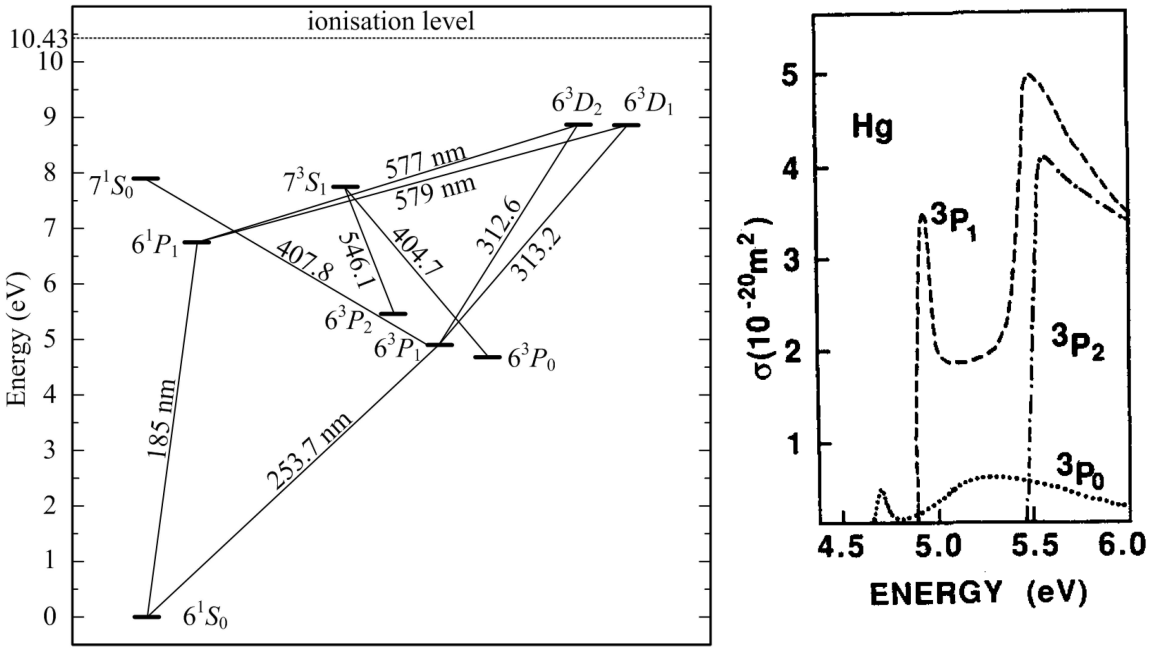


Figure 5: Left: energy level diagram for mercury.. Right: cross section of e-Hg inelastic scattering.

It is also instructive to consider the mean free path of an inelastic scatter. It is related to the number density, N, and the cross section, σ, which is shown in Fig. 5 (right). The mean free path is calculated,

$$\lambda = \frac{1}{N\sigma} = \frac{k_B T}{p\sigma}\,, \qquad p = 8.7 \times 10^{\alpha}\ \text{Pascals}\ \ \text{where}\ \alpha = 9 - 3110/T$$

where k_B is Boltzmann's constant ($1.38 \times 10^{-23}\,\text{JK}^{-1}$) and temperature, T, is expressed in Kelvin. The expression for the vapour pressure, p, is empirical and is specific to mercury.

▶ With reference to the energy level diagram and mean free path, can you explain your data and estimate a value for the first excitation energy ($6\,^1S_0 - 6\,^3P_0$)?

Darkening the surroundings and watch the mercury bulb during another ramp. Some faint bluish light may be visible.

▶ What is the origin of these photons? Do you see any structure?

Print your plots and enter them in your logbook with your conclusions. A photograph of the tube is a nice addition if any emission is seen.

▶ How would you expect the plot of current (I) versus acceleration voltage (U_1) change if this experiment is run at a (i) lower temperature (ii) higher temperature?

At the end, switch off the control unit and reconnect the neon apparatus and log-out of the PC.

Optional

3.3.3 Investigate the dependence on temperature

Take measurement at 190 °C, 160 °C and 145 °C. It is desirable, bot not vital, to take thedata with the same values of U_2 and U_H as before. For analysis, at-least three clean, smooth peaks should be recorded. In the case of discontinuous or spiky data, rerun!

Perform a ΔE analysis for each dataset, either with the built-in peak-analysis or using Python. On the same axes, graph ΔE versus peak number for each temperatures. Explain any trend with temperature. With justification, update your estimate of the first excitation energy ($6\,^1S_0 - 6\,^3P_0$). Check for light emission and explain when it becomes more evident.

Listing 1: A Python snippet for reading textfile data exported from a PHYWE measurement

```
import numpy as np
import pandas as pd
from scipy.signal import find_peaks, savgol_filter
# Read the data from the text file skipping the first two rows
data = pd.read_csv('file.txt', sep='\t', header=None, names=['x', 'y'], skiprows=2)
# Possible data cleaning to remove indistinct or satuated data
data = data[(data['x'] >= 14) & (data['x'] <= 43)]
# Smooth the data using Savgol filter
smoothed       = savgol_filter(data['y'], 70, 3)
# Find peaks in the smoothed data
peaks, _    = find_peaks(smoothed, prominence=0.2, distance = 100)
# Calculate distances between consecutive peaks
peak_distances    = np.diff(data['x'].iloc[peaks])
```