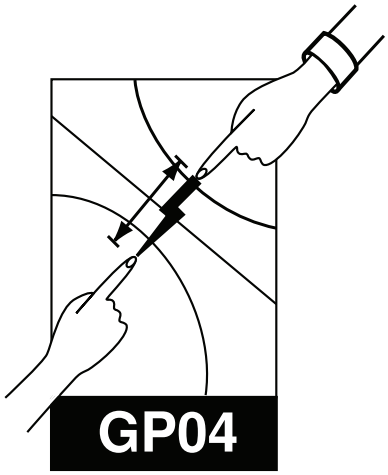
Driven harmonic motion



revised PGH/PGR/LS/KLA/AJW August 2022

GP04



Handle the Driven Harmonic Motion apparatus carefully. It is delicate, and if it breaks, recoiling cords and dropping weights could present a hazard. The mechanical oscillator could cause injury. Avoid obstructing the moving arm.

1 Introduction

Harmonic oscillators are found everywhere. The dynamics of any stable system oscillating around its equilibrium configuration — be it a 0.3-nm-long water molecule vibrating due to thermal excitation, or a 300-m-tall skyscraper swaying in the wind — reduces to harmonic motion for sufficiently small oscillations. Understanding how a harmonic oscillator responds to damping and driving forces[1], and the universal equations describing this response, is an essential part of your first-year physics course.

The aim of this experiment is to investigate and test the theory of the simple harmonic oscillator. You will study free oscillations, damped oscillations, and oscillations driven over a wide range of frequencies. You will measure the oscillator's effective spring constant, its natural frequency, and its damping constant independently, allowing you to make quantitative comparisons of theory and experiment.

The system you will study is a rotational oscillator (also known as *torsion pendulum*), consisting of an aluminium disc mounted onto a pulley wheel whose position is stabilised by two opposing springs. A rotational oscillator is a vibrating system that experiences a restoring torque given by $\tau = -\kappa\theta$ when the rotational displacement from its equilibrium position is θ. The torsion constant κ (kappa) is analogous to the spring constant k for the traditional translational oscillator (for which the restoring force F is proportional to the linear displacement x), while the role of the mass is taken by the rotational inertia I of the rotating element (see the theory in Section 2). In our setup, the aluminium disc serves as the rotating element, while the springs exert restoring torques on the oscillating disc.

2 Theory

The following section lays out the mathematics describing free, damped, and driven harmonic motion. If you are doing GP04 in Michaelmas term, much of this theory may be unfamiliar to you. If so, don't worry: the most important equations that you will test today have been placed in boxes for clarity.

[1]The Millennium Bridge is a relatively large harmonic oscillator whose lowest resonant frequency unfortunately happened to coincide with human walking pace. When driven by the synchronous footfall of several hundred pedestrians, the bridge experienced driven oscillations so large that the bridge had to be temporarily closed and refitted with viscous dampers costing several million pounds. Sadly, these dampers were rendered futile by the bridge's complete destruction in 2009 at the hands of flying Death Eaters, presumably incensed by the disregard for driven harmonic motion shown by the Millennium Bridge's designers.

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2.1 Linear harmonic oscillator

The equation of motion of an undriven, damped harmonic oscillator with mass m and position $x(t)$ attached to a spring of stiffness k can be derived from Newton's Second Law:

$$m\ddot{x} = -kx - B\dot{x}. \tag{1}$$

The first term on the right is the restoring force, the second is the damping force. If the support point of the spring is moved according to the function $\tilde{x}(t)$, then the restoring force is $k[x - \tilde{x}(t)]$ instead of kx, yielding the equation for driven (or forced) motion:

$$m\ddot{x} + B\dot{x} + kx = F_D(t), \tag{2}$$

where $F_D(t) = k\tilde{x}(t)$ is the driving force. The most usual case of the driven oscillator is for a sinusoidal displacement, so that the driving force can be written in the form $F_D(t) = F_0 \cos(\omega t)$.

2.2 Rotational harmonic oscillator (torsion pendulum)

The rotational equivalent of the linear oscillator equation is obtained by replacing the linear variables for straight-line motion in Eq. (2) with their corresponding rotational variables: displacement x becomes angular displacement θ, velocity $\dot{x}$ becomes angular velocity $\dot{\theta}$, acceleration $\ddot{x}$ becomes angular acceleration $\ddot{\theta}$, force F becomes torque τ, and mass m becomes moment of inertia I. The rotational analogue of Eq. (2) reads

$$I\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + \kappa\theta = \tau_D(t), \tag{3}$$

where I is the rotational inertia of the disc, κ is the effective torsional spring constant of the spring(s), and $\tau_D(t)$ is the driving torque.

Eq. (3) is an inhomogeneous linear second-order differential equation. The general solution consists of the sum of two parts: the *particular integral* or steady-state solution, which depends only on the magnitude and frequency of the driving torque; and the *complementary function* or transient solution, which has two arbitrary constants determined from the initial conditions.

We will now examine how to solve Eq. (3) for free, damped, and driven oscillations.

2.2.1 Free oscillations

Consider undriven, undamped oscillations, for which both $B = 0$ and $\tau_D = 0$ in Eq. (3). The most general solution is of the form $\theta(t) = \theta_0 \cos(\omega_0 t + \delta)$, where the natural frequency of oscillation ω_0 is

$$\boxed{\omega_0 = \sqrt{\frac{\kappa}{I}}.} \tag{4}$$

The period is related in the usual way to the free oscillation angular frequency via $T = 2\pi/\omega_0$.

For our specific case, we can estimate the moment of inertia of the solid disc oscillating about its axis of rotational symmetry using the equation

$$\boxed{I = \frac{1}{2}MR^2,} \tag{5}$$

where M is the disc mass and R the disc radius. The torsional spring constant is determined by applying a known torque ($\tau = rF$) a distance r from the centre of the disc and measuring the resulting angular displacement (θ) through which the disc turns. It is calculated as shown in Eq. (6):

$$\boxed{\kappa = \frac{\tau}{\theta}.} \tag{6}$$

2.2.2 Damped oscillations

Now consider oscillations that are undriven ($\tau_D = 0$, but damped ($B > 0$). These oscillations are described by the complementary function of Eq. (3), i.e. the solutions of the differential equation

$$\frac{d^2\theta}{dt^2} + \frac{B}{I}\frac{d\theta}{dt} + \frac{\kappa}{I}\theta = 0, \tag{7}$$

where we can define the damping coefficient as $\gamma = B/I$ and where $\kappa/I = \omega_0^2$.

At this point, it is useful to introduce the complex equivalent of Eq. (3), which reads

$$I\frac{d^2\tilde{\theta}}{dt^2} + B\frac{d\tilde{\theta}}{dt} + \kappa\tilde{\theta} = \tilde{\tau}_D(t), \tag{8}$$

where $\tilde{\theta}$ and $\tilde{\tau}_D$ are complex numbers and where θ and τ_D are their real parts. The corresponding homogeneous equation, which describes undriven oscillations, is

$$\frac{d^2\tilde{\theta}}{dt^2} + \gamma\frac{d\tilde{\theta}}{dt} + \omega_0^2\tilde{\theta} = 0. \tag{9}$$

Introducing these complex formulae is a mathematical convenience that makes the differential equations considerably easier to solve. Eq. (9) is solved by the function $\tilde{\theta} = Ae^{i\omega t}$, which can be seen by direct substitution:

$$(-\omega^2 + i\gamma\omega + \omega_0^2)Ae^{i\omega t} = 0. \tag{10}$$

So

$$\omega = \frac{i\gamma}{2} \pm \left(\omega_0^2 - \frac{\gamma^2}{4}\right)^{\frac{1}{2}} \tag{11}$$

$$= \frac{i\gamma}{2} \pm \omega_\gamma \tag{12}$$

where

$$\boxed{\omega_\gamma = \omega_0\sqrt{1 - \frac{\gamma^2}{4\omega_0^2}}} \tag{13}$$

is the *damped frequency* [i.e., the real part of the 'complex frequency' in Eq. (11)]. Eq. (13) tells us that the more strongly damped a harmonic oscillator is, the smaller its oscillation frequency.

Piecing everything together, we find that the most general solution of Eq. (9) can be written as

$$\tilde{\theta} = e^{-\frac{\gamma t}{2}}(Ce^{i\omega_\gamma t} + De^{-i\omega_\gamma t}). \tag{14}$$

The angular displacement $\theta = \Re(\tilde{\theta})$ and can thus be expressed as

$$\boxed{\theta(t) = Ee^{-\frac{\gamma t}{2}}\cos(\omega_\gamma t + \delta)} \tag{15}$$

with an amplitude E and a phase δ. Note that, because of the $e^{-\frac{\gamma t}{2}}$ factor, this contribution to the angular displacement dies away in a characteristic time $2\gamma^{-1}$. In the limit that $\gamma \to 0$, we recover pure, undamped sinosoidal oscillation, as expected.

By examining Eq. (15), we can derive alternative expressions for the *quality factor* Q of the oscillator. Conventionally, the quality factor is defined as the (dimensionless) ratio of the natural frequency to the damping coefficient:

$$\boxed{Q = \frac{\omega_\gamma}{\gamma}.} \tag{16}$$

Now consider the number of oscillations $n(t)$ that the oscillator has executed by time t, given by $n(t) = \omega_\gamma t/2\pi$. In terms of n, the exponent in Eq. (15) becomes $n\pi\gamma/\omega_\gamma \equiv n\pi/Q$. That is,

$$\boxed{\theta(t) = Ee^{-n(t)\pi/Q}\cos(\omega_\gamma t + \delta).} \tag{17}$$

The number of full oscillations required for the oscillation amplitude to decay to $1/e$ of its original value, $n_{1/e}$, therefore satisfies $n_{1/e}\pi/Q = 1$. In other words, the quality factor can be expressed as

$$\boxed{Q = \pi n_{1/e}.} \tag{18}$$

This provides an intuitive way of thinking about the physical significance of Q: the quality factor expresses the *number of oscillations* required for a damped oscillator's motion to be significantly diminished.

Note that if the quality factor Q is sufficiently high, it is not uncommon to define Q as ω_0/γ, since the difference from its 'true' value is too small to be significant. With this assumption, the damped frequency given by Eq. (13) is approximately

$$\boxed{\omega_\gamma = \omega_0\sqrt{1 - \frac{1}{4Q^2}}.} \tag{19}$$

2.2.3 Driven oscillations

Now consider an oscillator subjected to a sinusoidal driving torque of the form $\tau_D(t) = \tau_0\cos(\omega t)$. The *complex* driving torque in Eq. (8) is therefore $\tilde{\tau}_D(t) = \tau_0 e^{i\omega t}$. To solve Eq. (8), we can once again try an exponential solution of the form $\tilde{\theta} = Ae^{i\omega t}$, with A being a complex number. Substituting this ansatz into Eq. (8) gives

$$(-\omega^2 + i\gamma\omega + \omega_0^2)A = \omega_0^2\frac{\tau_0}{\kappa}. \tag{20}$$

We then massage Eq. (20) into a more palatable form:

$$A = \frac{\tau_0}{\kappa} \frac{\omega_0^2}{(\omega_0^2 - \omega^2) + i\gamma\omega} \tag{21}$$

$$= \frac{\tau_0}{\kappa} \frac{\omega_0^2 \left[(\omega_0^2 - \omega^2) - i\gamma\omega\right]}{(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2} \tag{22}$$

$$= \frac{\tau_0}{\kappa} \frac{\omega_0^2}{\left[(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2\right]^{\frac{1}{2}}} e^{-i\phi}. \tag{23}$$

The step between Eqs. (22) and (23) involves a *polar transformation*, with ϕ representing the *phase lag* of the response (i.e., the phase shift between the pendulum and the external oscillator), defined via

$$\boxed{\tan\phi = \frac{\omega\gamma}{\omega_0^2 - \omega^2}.} \tag{24}$$

Note this equation is ambiguous as to the quadrant ϕ is in, so *take care when inverting it*; the correct phase varies from 0 at zero driving frequency to $+\pi$ at infinite driving frequency.

Meanwhile, the *amplitude* of the response is

$$\boxed{\theta_0 = |A| = \frac{\tau_0}{\kappa} \frac{\omega_0^2}{\left[(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2\right]^{\frac{1}{2}}}.} \tag{25}$$

Thus $A = \theta_0 e^{-i\phi}$, and the angular displacement $\theta = \Re(\tilde{\theta})$ is given by

$$\theta = \theta_0 \cos(\omega t - \phi) \tag{26}$$

with θ_0 given by Eq. (25) and ϕ by Eq. (24).

In terms of the quality factor $Q = \frac{\omega_0}{\gamma}$, the maximum angular displacement at a given frequency can be written as:

$$\boxed{\theta_0 = \frac{\tau_0}{\kappa} \frac{\omega_0^2}{\left[(\omega_0^2 - \omega^2)^2 + (\frac{\omega\omega_0}{Q})^2\right]^{\frac{1}{2}}}.} \tag{27}$$

When $\omega = 0$, we obtain the *static angular displacement* $\theta_s = \tau_0/\kappa$, which is consistent with Eq. (6).

We can now make an important observation. If $\omega_\gamma \approx \omega_0$ [see Eq. (13)], the quality factor Q can be measured in two distinct ways: either directly from the decay period of the *damped* oscillations, without the drive [Eq. (17)]; or by examining the amplitude of the *driven* oscillations, i.e., the shape of the resonance peak, at constant driving torque [Eq. (27)]. You will test both methods in this experiment.

3 The apparatus



Remember that you are responsible for your own safety when working in the laboratory. Always talk to a demonstrator or technician if you are concerned that anything may be unsafe.

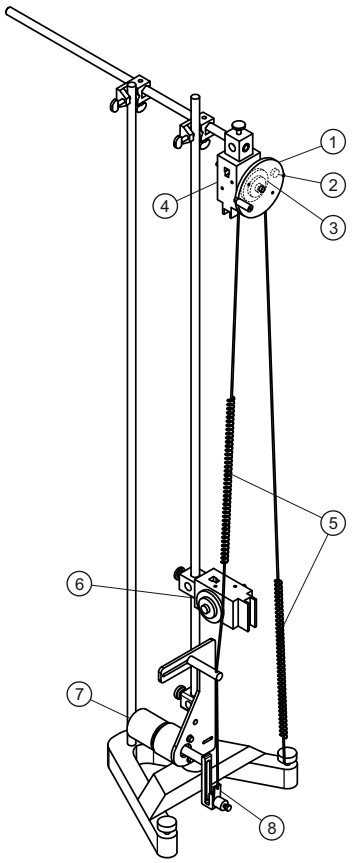


Figure 1: The Driven Harmonic Motion Analyser apparatus. The numbered components comprise: (1) an aluminium disc, (2) a magnet, (3) a pulley wheel, (4) the Disc Rotatory Motion Sensor, (5) two springs, (6) the Driver Rotatory Motion Sensor, (7) a motor, (8) a lever.

The apparatus (Figure 1) consists of an aluminium disc attached to a pulley with a string wrapped around it, whose ends are connected to two springs. The disc is mounted onto a rotatory motion sensor for monitoring, referred to here as the Disc Rotatory Motion Sensor. Behind the disc there is a magnet, the position of which can be modified with respect to the disc to introduce different damping.[2] The disc can be spun manually to produce oscillations. Alternatively, one end of the string can be driven by a motor via a lever of adjustable length, effectively impressing a sinusoidal motion to the attachment point of one spring (see Theory section). The motor's motion is monitored by a second sensor, the Driver Rotatory Motion Sensor. The Disc Rotatory Motion Sensor is connected to channel P1, the Driver Rotatory Motion Sensor to P2. Their parameters and settings can be read using the manufacturer's software — PASCO Capstone — which also controls the motor. **Note**: the connecting strings should be spun *clockwise* on the 3 cm wheel of the driver and on the 5 cm wheel of the disc.

- Familiarise yourself with the different aspects of the equipment.

[2]Damping is induced by producing *eddy currents* in the aluminium disc — this is the same principle used in train brakes.

3.1 Software controller

Frequency, angular displacement, and all other variables are recorded in PASCO Capstone.

Open the "PASCO Capstone" software. An experiment with the required settings has already been created for you. It can be opened from File > OpenExperiment. Open the project called `GP04.cap`, which is located in the `GP04` folder in the `C:` Local Disk. The first thing to do after opening this file is to save it to your `H:` Drive.

Before starting the measurements, try to become familiar with the software. In Figure 2 below, a representative screenshot of PASCO is shown. Various tools for you to try are circled in red. Actions you can take include the following:

- Circle 1 indicates the button to create a new page. Once it is selected, you will be able to choose to show a table, a graph, a table + a graph, and so on.
- Different 'frames' can be included using the bar on the right (circle 2). For example, you can include a digits display which you will need in Section 4.1.
- Many tools are available at the top of the graph. Circle 3 indicates the "Coordinate Tool" that you will need in Section 4.3 to display the coordinates of specific data points. Circle 5 shows the button allowing you to display more than one plot on the same graph.
- On the left is the "Signal Generator" (circle 4) that controls the application of the sinusoidal force. You will use this feature in Section 5.1.
- On the bottom bar, you have the "Record" and "Stop" buttons, and access to the recording conditions (circle 6) that will need to be changed later.

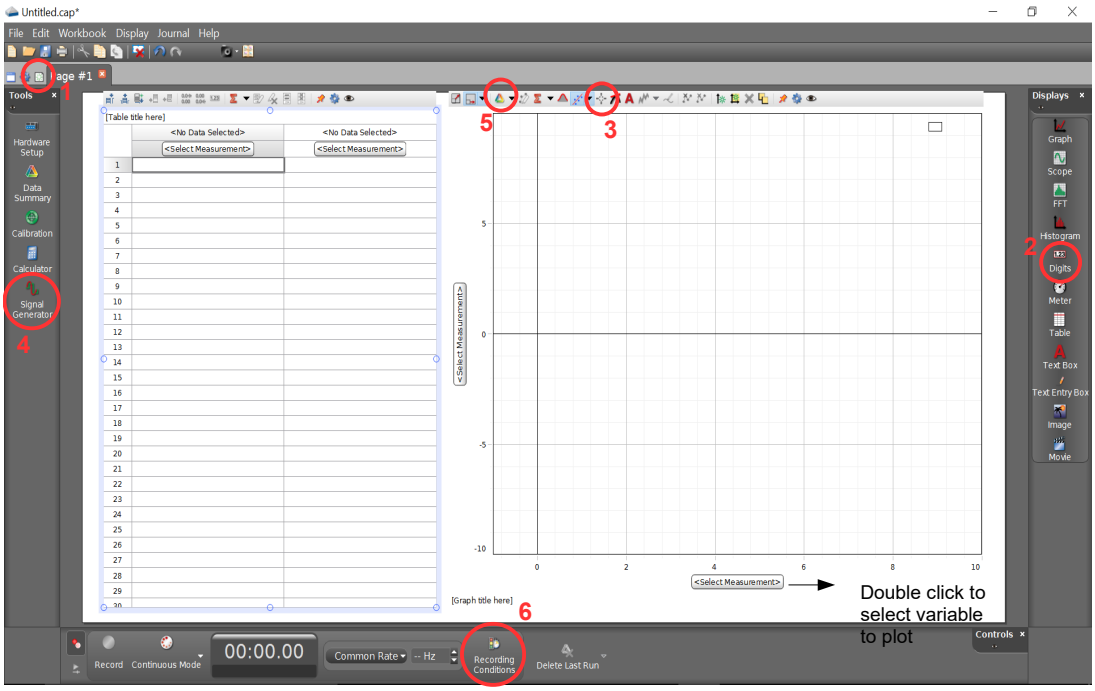


Figure 2: A screenshot of the PASCO Capstone software, showing a table and a graph yet to be populated with data. See the text above for explanations of the different features indicated.

4 Undriven oscillations: comparing experiment and theory



Before starting the data collection you should become familiar with the procedures to estimate and propagate experimental errors, as described in, for example, AD02 — *Estimating Experimental Errors*[3]. For derived quantities, it is good practice to calculate values and errors at the same time. Subsequently, only significant digits should be quoted or tabled (one digit only for the error) — for example, 0.2796 ± 0.0017 should be denoted 0.280(2).

4.1 Spring constant

For this part of the experiment, you will need at least one brass weight and a plastic hook. These can be found in the blue box near the apparatus.

In Capstone, create a digits display (tool on the right side of the screen) showing the angle of the Disc Rotatory Motion Sensor. Start recording. Hang the hooked mass at the top of one of the springs and measure the resulting angle through which the disc rotates. Then stop recording. Remove the mass and measure the radius of the groove of the large pulley. Calculate the torsional spring constant using Eq. (6).

If more than one brass weight is available, take more than one measurement and obtain κ from a fit to reduce uncertainties.

4.2 Disc rotational inertia

Remove the disc and measure the mass and radius of the disc. Calculate the rotational inertia of the disc using Eq. (5).

Using the torsional spring constant and the disc rotational inertia, calculate the theoretical period and the resonant frequency of the oscillator (ignoring friction), including experimental uncertainties.

4.3 Period of free oscillations without damping

Create a graph of angular velocity of the disc vs. time. You can do this either clicking twice on the tool on the right or adding a new page and selecting the option table + graph. Crank the magnet all the way back. Click "Record", displace the disc manually about 90 degrees and let it oscillate. Stop recording once it has stopped.

Measure the period between peaks using the "Coordinate Tool" on the graph. It is located on the toolbar on top of the graph. A cursor showing the coordinates will be drawn on the screen. Move it to the maximum of a peak and record the time in seconds. Do the same for more points and calculate the period between different peaks to minimise the error.

▶ Comment on the accuracy of the period obtained, and compare this result with the predicted value from previous section.

4.4 Period and decay of free oscillations with damping

The three-armed magnet spacer tool has been designed to provide you with three different widths (3 mm, 4 mm and, 6 mm) which are needed when adjusting the distance between the aluminium disc and the magnet. To adjust the distance, place the required thickness of arm between the disc and

[3] `www-teaching.physics.ox.ac.uk/practical_course/scripts/srv/local/rscripts/trunk/Admin/AD02/AD02.pdf`

magnet, then use the thumb screw to turn the magnet until the correct distance is achieved. Using the magnet spacer tool, set the spacing between the magnet and disc to 3 mm and measure the amplitude of the damped oscillations versus to time. Plot Angle in channel 1 vs. time.

Rotate the disc about 360 degrees and hold it. Start recording and let go of the disc. Stop recording after the disc stops. Repeat the process a couple more times for the 3 mm spacer to get some statistics.

Once you have measured the quality factor a few times (as explained belowin Sec. 4.5), repeat the whole process for the 4 mm and 6 mm spacers.

4.5 Determining the Quality Factor

After each curve has been collected, we want to use it to calculate the quality factor Q. There are two ways of doing this, which you are free to choose between:

The first method uses the definition $Q = \omega_\gamma/\gamma$. The data in PASCO can be directly fitted to Eq. (15). In the icon bar above the graph there is an option to fit curves to the measured data (select the damped sine function). The fitted curve will display on the graph, along with the curve fit parameters, which correspond to the damping parameter $\gamma/2$ and the damped frequency ω_γ. Using Eq. (15), you can now derive a value for the quality factor with errors.

Alternatively:

The second approach uses the definition of Q found in Eq. (17). From your data, create a table containing the zero-to-peak oscillation amplitude versus the oscillation number, and save it as a comma-separated value (`.csv`) file. Read this into your favourite fitting software (e.g. Python) and fit a decaying exponential to the resulting curve per Eq. (17). Look back at the script for DA01 for help on doing this fitting if required.

Whichever method you choose, make sure you obtain errors on your measurements of Q.

▶ Are the decay data consistent with the exponential function? For each magnet-disc spacing, do you find that repeated measurements of Q are consistent with each other? How should each set of measurements be combined to derive a single best value of Q for each spacing?

Finally, measure the damped frequency ω_γ for each magnet-disc spacing, using either the peak-to-peak separation (as you did in Section 4.3) or using PASCO's damped sine fit.

▶ Comment on your results in light of Eq. (13). Is the difference between ω_γ and ω_0 significant?

5 Driven oscillations: comparing experiment and theory

5.1 Acquiring amplitude data

In this part of the experiment, we will program the PASCO Capstone software to drive the oscillator with a periodic driving torque whose frequency slowly increases with time. This is achieved by supplying the drive motor with a time-varying voltage $V(t)$ that is set to rise linearly over the course of a few minutes by the PASCO Capstone's Signal Generator module. The frequency at which the motor shaft rotates is, in turn, linearly related to the applied voltage via $\nu(t) = mV(t) + b$, where $m \approx 0.210\,\text{Hz}\,\text{V}^{-1}$ and $b \approx -0.125\,\text{Hz}$[4]. The overall effect is to give a drive frequency $\nu(t)$ that increases linearly over time. You can adjust the range of frequencies over which you sweep — and the rate at which you do so — by adjusting the Signal Generator parameters; recommended values are given in the table below[5].

If necessary, the drive *amplitude* can be changed by adjusting the position of the rod attached to the motor shaft, as illustrated in Figure 3.

Create a graph of "Angle Amp" vs. "freq".

Set the Signal Generator (left side of screen) on Auto, to perform an automatic positive ramp of the voltage $V(t)$ – the practical result will be a scan of the drive frequency $\nu(t)$. The range and speed of the ramp can be controlled by the Signal Generator parameters, while the frequency of data acquisition and the measurement start/stop conditions are set at the bottom of the main window. The correct parameters should already be set in `GP04.cap`, but a recommended set of Signal Generator and Recording parameters is given here below for reference:

Signal Generator parameters (left of screen)

Parameter	Value	Explanation
Waveform	Positive ramp	
Sweep	Off	
Activation	Auto (tick)	Ensures the motor starts as soon as you press "Record".
Phase Shift	0	
Voltage Limit (V_L)	15.0 V	Hard upper limit on the applied voltage
Voltage Offset (V_0)	1.5 V	The voltage varies according to $V(t) = V_0 + 2Aft$; this is the starting voltage, $V(0)$.
Frequency (f)	0.002 Hz (or 0.001 Hz)	The total duration of the ramp is $t_R = 1/2f$; however, the stopping condition may stop acquisition before t_R. **Note:** This is the frequency of the voltage supplied to the motor, *not* the drive frequency of the motor $\nu(t)$.
Amplitude (A)	7.0 V (or 5.7 V)	The maximum voltage is $V(t_R) = V_0$ + A; the increase in voltage per unit time is therefore $2Af$.

[4]This frequency is calculated by the user parameter *calc_freq* in the Capstone software.

[5]Given the recommended parameters of V_0 = 1.5 V, V_s = 7.0 V, f = 0.002 Hz, A = 7.0 V, the total sweep time $t_s = (V_s - V_0)/(2Af)$ = 196 s. Using the conversion $\nu = mV + b$, the start drive frequency is ν_0 = 0.19 Hz, and the final drive frequency is ν_s = 1.35 Hz. Note the data acquisition stops at time t_s before the total ramp time of $t_R = 1/2f$ = 250 s can elapse.

Recording conditions (bottom of screen)

Parameter	Value
Acquisition frequency	Common rate, 25 Hz
Recording condition	Stopping
Starting condition	None
Stopping condition	Output voltage rises above 7.0 V (V_s)

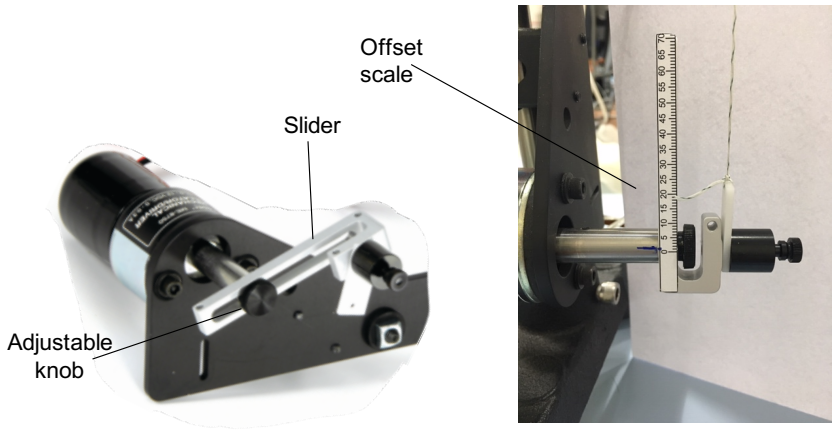


Figure 3: Driving rod offset setting, front and side views. The scale indicates the offset between the motor shaft and the string puller, which determines the maximum amplitude of the pulling displacement.

Set the driving rod offset to 5 mm (see Figure 3 — you can experiment with other settings later, but do not exceed 20 mm).

Start recording. Data collection will automatically stop when the frequency rises above about 1.3 Hz. Once it has stopped, export your data to CSV.

Now change the distance of the magnet to the disc to 4 mm and 6 mm and repeat the measurement.

You can plot more than one run in the same plot to compare them.

▶ Examine the resonance curves for different amounts of damping. How does increasing the damping affect the shape of the curve (width, maximum amplitude, frequency of the maximum)?

▶ Is the resonant frequency with the lowest damping the same as the theoretical frequency? Calculate the percent difference.

▶ Why is the resonance curve asymmetrical about the resonant frequency?

5.2 Fitting the amplitude data

We now want to fit the resonance curves obtained in the previous section.

The amplitude is fitted with a peak function, given by

$$y(x) = \frac{k}{[(x_c^2 - x^2)^2 + x^2 w^2]^{1/2}} \tag{28}$$

which you can compare with Eqs. (25) and (27).

Here, k is a constant, and the interpretation of the parameters x_c and w depends on the form in which you enter your data. If you enter angular frequencies ω for x, then x_c will be ω_0, and w will be γ. Alternatively, if you enter frequencies $f = \omega/(2\pi)$, then x_c and w will also be scaled by 2π.

The Python codes `GP04_Amp_Fit.py` and `GP04_Amp_Fit.ipynb` for use in Jupiter Notebook have been written to determine values for ω_0, γ and the drive amplitude y_0.

The data exported from Section 5.1 to CSV can't be straightforwardly imported into Python — unfortunately it is common for experimental data to be generated in an unhelpful format — so before proceeding let's prepare it. Open the CSV file you have created in Section 5.1. To help Python interpret the data, remove the first row, so that only one 'header' line remains. You will also find blank spaces in the two columns of interest [`'Angle Amp (units)'` and `'freq (Hz)'`] that should be removed before you start processing the data.

If you are having trouble getting your CSV data into a Python-readable format, see Appendix A for detailed instructions.

Load up the data into Jupiter Notebook or Python from the `GP04` folder in the `S:` shared drive, using the same approach as in Section 4.4. Don't forget to save the script with a different name in your home area before going any further.

Run the code to fit the amplitude data and plot your data as points with the fit as a line. Note that if the fit fails to converge on your first attempt, you may need to tinker with the initial guesses for the fitting parameters. Look at the standard error in the fit parameters. Also examine the residuals; they can be calculated by finding the difference between the collected data values and the fitted amplitude curve. You can experiment with fitting just a subset the data to see if the fit improves.

Now compare the results of your fits with Eq. (27), and identify the three parameters of the fit: the pre-factor τ_0/κ (which can in principle be calculated from the spring constant and the lever displacement), the undamped frequency ω_0, and the quality factor Q.

▶ Comment on the accuracy of the fit. Compare the value of Q you have just obtained with that obtained in Section 4.5. Do they agree within error?

5.3 Measuring the phase

Create a graph of Disc Angular Velocity and Driver Angular Velocity vs. time (you can plot both together on the same axis; click on the y-axis label to open up the context menu). Using the same conditions as in the previous section and with the magnet at 6 mm from the disc, start a frequency scan.

Measure the phase difference between these oscillations at low frequency, at resonance frequency, and at high frequency. **Note**: double-check that your string is looped the correct way around both the driver and disc pulley wheels before carrying out this measurement!

Calculate the limit of Eq. (24) for the three cases, with $\omega \to \omega_0$, $\omega \to 0$, and $\omega \to \infty$.

▶ Do these measured phase differences agree with the ones calculated from theory?

▶ Now hand-write a brief summary of the experiment in your logbook. State the aims of the experiment, the method used, quantitative results, and whether you consider that the experiment supports the theory. Discuss the reasons for any discrepancies between theory and experiment. This should take no more than about half a page. S

▶ Discuss your results with a demonstrator as part of your assessment.

A Processing data exported from PASCO Capstone

PASCO Capstone exports far more data than you need to analyse the resonance peaks in Section 5. Of the raw data it exports, you only require the 'Angle Amp' and 'freq' variables. Furthermore, due to the way these variables are internally calculated by Capstone, you will notice that their columns contain many blank entries. The raw CSV files therefore cannot be processed as-is by Python; the data must be manipulated beforehand. We can do this one of two ways:

1. By importing the raw data first, and then reformatting it by issuing commands within Python.

2. By reformatting the data first, and then importing this processed data into Python.

This section contains suggestions on how to carry out both methods.

A.1 Internally reformatting in Python

First, import the data in its entirety into Python. The codes `GP04_Amp_Fit.py` and `GP04_Amp_Fit.ipynb` already contain commands of the form

```
df = pd.read_csv("filename.csv")
```

needed to do this. However, we also need to tell our script where the data actually start. You will notice that there are two 'header' lines preceding the data in the raw CSV file, the second of which contains the names of the variables appearing in each column. Python doesn't know this, so we must tell it by amending the `read_csv` command to

```
df = pd.read_csv("filename.csv", header = 1)
```

This tells the `read_csv` command that row 1 contains the variable names, and rows 2 and above contain the data proper; everything before row 1 is ignored.

The next step is to extract the two relevant columns from the dataframe `df` we just created. The two columns we need are named '`freq (Hz)`' and '`Angle Amp (units)`'. By issuing the commands

```
freq = df.loc[:, 'freq (Hz)']
Amp  = df.loc[:, 'Angle Amp (units)']
```

we extract the frequency and amplitude data and assign them to the arrays `freq` and `Amp`.

The final step is to deal with the gaps in the data. If you print out either `freq` or `Amp`, you will notice that the vast majority of the entries are `NaN` ('Not a Number'), which are no good for our analysis. To strip out these missing data points, you can issue the commands[6]

```
import numpy as np
freq = freq[~np.isnan(freq)]
Amp = Amp[~np.isnan(Amp)]
```

The remainder of the code can now be used as-is to plot and analyse the resonance curve described by `freq` and `Amp`.

[6] Warning: copy-and-pasting these commands into your Python script could fail due to the '~' character; type the commands into your code manually.

A.2 Externally reformatting in Excel

If you prefer a more 'graphical' approach, you might prefer to tidy up the data using Excel before importing it into Python. This can be done as follows.

First, open the data in Excel.

Remove the first, useless reader line (containing '`Run #1`' or similar) by right-clicking on the `1` next to the first row and selecting Delete.

Next, remove all columns except the ones named '`Angle Amp (units)`' and '`freq (Hz)`'. To do so, left-click the Select All button in the top-left corner of the spreadsheet to highlight the entire dataset. Then, Ctrl-click the columns named `Angle Amp (units)` and `freq (Hz)` to deselect them. Right-click anywhere in the remaining highlighted region to bring up the context menu and press Delete.

We now need to remove the blank spaces between the data entries in the remaining two columns. To do so, select these columns by shift-clicking the columns names `A` and `B`. Then, go to Find and Select 〉 Go to Special ... 〉 Blanks 〉 Ok. This will highlight only the empty cells in columns `A` and `B`. Right-click anywhere in the highlighted region to bring up the context menu and press Delete. When you delete these cells, make sure you select `Shift cells up`.

One last optional step is to rename the columns to '`A`' and '`f`', as this is the default behaviour expected by `GP04_Amp_Fit.py` and `GP04_Amp_Fit.ipynb` are expecting.

Finally, save your formatted CSV file somewhere it can be imported into your Python script.