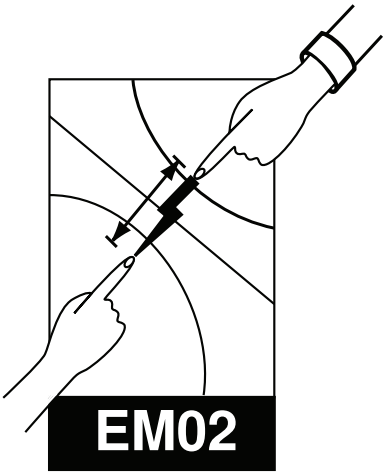
Self and mutual inductance and Faraday's law of induction



JL after JWH October 2018

1 Aims of the experiment

In this experiment we will study various aspects of Faraday's Law of magnetic induction. A quantitative test of the Law is made in one experimental configuration. In particular we will:

- Study the voltage induced across an inductor as a function of drive frequency.
- Examine the influence of core material on the self-inductance (air vs. ferrite) of a coil.
- Study self and mutual inductance in a system with full flux coupling.
- Study self and mutual inductance in a system with weak flux coupling.
- Compare two measurements of the mutual inductance of two weakly coupled coils, one determined using Faraday's Law and one determined via a direct calculation of the flux linked between the two coils.

2 Introduction

Consider a plane loop of wire of area A. If a magnetic field $\mathbf{B}$ passes normally through the loop we may say that the magnetic flux ϕ cutting the loop is given by $\phi = \mathbf{B} \cdot \mathbf{A}$. If the magnetic flux through the loop changes, a voltmeter connected across the loop shows that an emf. V is generated while the flux is changing. Faraday's law draws together the results of such experiments and equates the emf generated in the loop to the time derivative of the magnetic flux ϕ passing through the circuit.

$$V = -\frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{1}$$

Note: In general the flux ϕ is the surface integral of the magnetic induction through the loop

$$\phi = \int_{area} \mathbf{B} \cdot \mathrm{d}\mathbf{A} \qquad \text{where } \mathrm{d}\mathbf{A} \text{ is an element of the area of the loop.}$$

We will not study here the various dynamo effects in which the variation of flux coupled to a circuit is achieved by mechanical movement. Rather we will use time-varying currents to generate time-varying magnetic flux. The term *self inductance* is applied to the voltages induced in one circuit by time-varying currents flowing in the same circuit . *Mutual inductance* refers to the voltages induced in one circuit by a time-varying magnetic flux caused by currents flowing in another nearby circuit.

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The self inductance, L, expressed in Henrys, of a circuit may be defined as being numerically equal to the total flux, Λ, linking the circuit ($\Lambda = n\phi$ for a well linked system, where n is the number of turns) as the result of the passage through it of a current of one ampere. Equivalently using Faraday's Law, a voltage $V = -L\frac{\mathrm{d}I}{\mathrm{d}t}$ is *generated* by the inductance when the current through the circuit changes, if the self inductance remains constant.

We may alternatively say that an external voltage $V = +L\frac{\mathrm{d}I}{\mathrm{d}t}$ must be applied to the inductor to change the current at the rate given by $\frac{\mathrm{d}I}{\mathrm{d}t}$.

Paralleling the definition of L, the mutual inductance M_a of two circuits, a and b, may be defined as the flux coupling circuit a when unit current flows in circuit b. The mutual inductance is symmetrical in source and subject and $M_{ab} = M_{ba}$.

2.1 Air-cored coils

In few cases only can the magnetic flux produced by unit current be calculated and expressed by a simple formula. The difficulty is shown in Figure 1a where the field produced in a short solenoid is sketched. Not all the field produced by currents in each turn passes through all the other turns. The calculation of the self inductance of such a coil is a tedious exercise in elliptic integrals. So also is the calculation of the mutual inductance of two such coupled and coaxial air-filled coils.

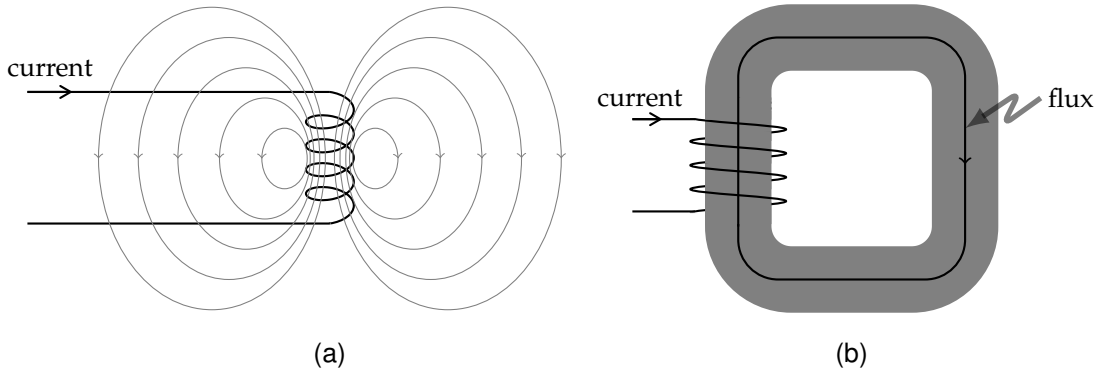


Figure 1: (a) Field produced by a short solenoid. (b) Coil wound onto a magnetic core, showing retention of flux within the central region of the coil.

2.2 Coils on an ideal magnetic core

The self and the mutual inductances of a coil system may however be related readily (even if not calculated absolutely) if it can be arranged that all the flux is retained within the central region of every coil. Each turn then encompasses the same flux. (See Figure 1b.) This can be achieved to a good approximation by winding all coils onto a magnetic core of high permeability. It can then be shown that it is energetically preferable for the flux to remain within the core. Each turn of each coil wound on the core then encompasses the same flux and is coupled equally. The magnitude of the flux in the core still has to be calculated, but the same flux ϕ now couples to each turn of each coil.

In such a situation we might anticipate the following results:

(a) If we were to double the number of 'primary' turns of the coil producing the field, while maintaining the same current, the magnetic fields within the system should be doubled.

(b) If we were to double the number of 'secondary' turns sensing the magnetic flux we would expect that the induced voltage would be doubled.

Either action (a) OR action (b) should double the magnitude of a mutual inductance. Doubling the

number of turns in EACH coil would quadruple the mutual inductance. In the case of a self inductance where the one coil serves both functions, doubling the number of turns should quadruple the self inductance. These matters are susceptible to experimental test.

2.3 A test of Faraday's Law

A quantitative test of Faraday's Law could be envisaged along the following lines: Take a search coil whose number of turns and area per turn are known. Place the search coil in an air-cored solenoid whose internal field per unit current has been measured or calculated. Sweep the current in the solenoid at a known rate while measuring the emf induced in the search coil. Compare the measured emf with that predicted by Faraday's Law. Such a test comprises part of this experiment.

3 Apparatus



Remember that you are responsible for your own safety when working in the laboratory. Always talk to a demonstrator or technician if you are concerned that anything may be unsafe.

Inductors of several forms are provided on circuit-boards. Some inductors are air-cored, whereas others are wound on industry-standard ferrite cores. Time-varying currents are produced by a signal generator and series resistor. The signal generator is a source of a known voltage V. We require known currents. This we achieve by connecting a resistor in series with the voltage generator, making sure that the impedance R of the resistor dominates the impedance of the whole circuit, Figure 2. To a good approximation the output current then has magnitude V/R and is in phase with the voltage V. The applied current and the voltages produced are monitored and measured with a two beam oscilloscope. A calibrated Hall effect meter is provided to measure the magnetic induction produced by currents flowing within a coil.

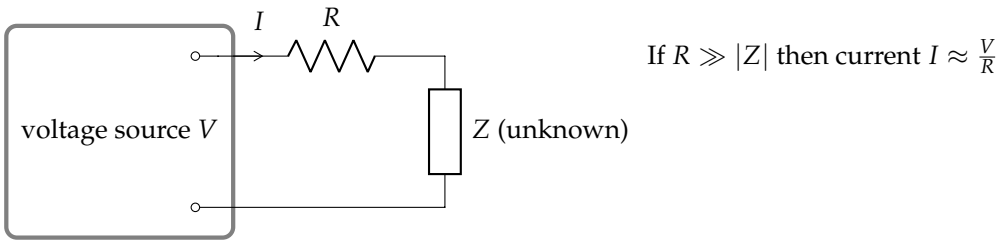


Figure 2: Basic circuit used in this experiment.

Set the function generator to provide a symmetrical triangular sawtooth waveform 20 volts peak-to-peak at a fundamental frequency of 1 kHz, as shown in table 1.

Set the oscilloscope as shown in table 2.

We need to observe the timing of the voltages at channel 1 or at channel 2 relative to the signal generator so we use an external trigger from the signal generator. Internal triggering from one or other input signal would not provide the necessary temporal link to the signal generator.

Experiment with observing the output of the function generator on CH1 and on CH2. Vary the frequency, sweep time, output voltage, waveform and Y gains until you are familiar with the operation of the scope and the signal generator. (Note that the 'A.C. coupling' of an oscilloscope connects a capacitor in series with the input signal. This acts to remove the D.C. component of the the signal which can be useful in certain situations e.g. when the D.C offset on the circuit is large compared the

control	initial setting
FUNCTION	triangular wave
ATTENUATOR (dB)	0
FREQUENCY	adjust for 1.000 Hz in display
DUTY CYCLE	50 percent
AMPLITUDE	adjust for 20 V pk-pk on scope
D.C. OFFSET	adjust for 0 V offset
TTL O/P	to scope EXT. TRIG
50 Ω O/P	to circuit

Table 1: Function generator settings.

Input/control	initial connection/setting
CH1 and CH2	5 VOLTS/DIV, DC COUPLED
SWEEP TIME/DIV	0.25 ms
TRIGGERING SOURCE	EXT
LEVEL	0 V, positive rising-edge
EXT TRIG	from TTL/CMOS output of function generator

Table 2: Oscilloscope settings.

A.C. signal. It is not needed in this experiment and can produce serious distortion of low frequency signals, so set the coupling to "D.C" throughout.

▶ Ask a demonstrator to confirm your understanding of the measuring equipment before proceeding.

4 Experiments

4.1 The inductor as a circuit element - ferrite core coils

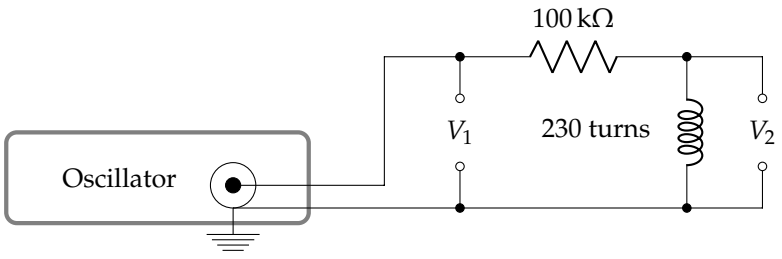


Figure 3: Inductor circuit.

Set up the circuit of Figure 3 using the 230 turn ferrite cored coil on the larger circuit-board.

It is convenient to be able to approximate the current I through the inductor by the expression $I = \frac{V}{100k}$. This will be valid if $V_2 \ll V_1$. With V_1 typically 20 V peak-to-peak, our approximation for I will be valid

to 5% accuracy if $V_2 < 1\,\text{V}$ peak-to-peak. This limitation will be maintained for our components for "frequencies" up to 5 kHz.

Drive a current through the coil using a triangular waveform of 1 kHz fundamental. Display the current through the coil (approximately $\propto V_1$) and the voltage appearing across the coil, V_2, on the scope.

Make detailed sketches of the V_1 and V_2 waveforms in your lab notebook when the circuit is driven at 1 kHz. Also examine and sketch the waveforms when the circuit is driven with frequencies of 20 Hz and 5 kHz.

▶ You should see that the shape of these waveforms is a strong function of driving frequency. Explain their shapes in terms of the resistance and the self-inductance of the coil, where $V_2 = rI + L\frac{\mathrm{d}I}{\mathrm{d}t}$.

Use the relation $V_2 = rI + L\frac{\mathrm{d}I}{\mathrm{d}t}$ to measure the self-inductance of the coil at 1 kHz and at 100 Hz.

Suggest why the coil has been damped with a 33 kΩ resistor. For comparison use the other 230 turn coil, which is mounted on a separate wooden board, at 1 kHz. Sketch the waveforms in both cases. Every real electrical circuit has capacitance, inductance and resistance. How might this account for the increased oscillations seen when no damping resistor is present? Does this remind you of a mechanical system you may have studied?

▶ Discuss your results with a demonstrator.

4.2 The inductor as a circuit element - air filled coils

Two 0.5 H air-cored coils are provided; one with a damping resistor and one without.

Set up the same circuit as for section 4.1 with each coil.

▶ Explain why the waveforms at 20 Hz and at 1 kHz are so different from those with the ferrite core coil.

You will have seen that the resistance of an air-cored copper coil is a major component of its impedance at audio frequencies. Only when one works with a superconducting solenoid magnet does one see only a simple inductive response from an air-cored coil at low frequencies.

4.3 Self and mutual inductance within a fully coupled system

One of your ferrite cores has three windings with 30, 60 and 120 turns. By appropriate connection of the windings in series, seven different turn numbers may be achieved.

Measure the self inductance of the coil system at 1 kHz as a function of the number of coil turns.

Determine, by plotting your data on log-log paper, the relation between the self inductance and the number of turns. Ask a demonstrator if you have not used log-log graph paper previously. It is always good practice to plot each data point onto the graph paper as you go along i.e. as soon as you calculate the inductance value. This enables you to best assess the likely quality of your measurements as quickly as possible, enabling you to repeat them if they look suspicious.

Deduce the mutual inductance by passing a sawtooth drive current through the 60 turn coil while measuring the voltage induced in the 30 and the 120 turn coils. Measure thereby $M_{60,30}$ and $M_{60,120}$. Reverse the connections to measure $M_{30,60}$ and $M_{120,60}$ etc. Also measure $M_{30,120}$ and $M_{120,30}$.

▶ Discuss your results and their quantitative relation to the values of self inductance, L_{30}, L_{60} and L_{120}.

You should have found that for totally coupled coils, p and s, in which all flux produced by any one turn passes through all other turns (achieved here by a common magnetic core of high permeability):

$$L \propto n^2 \tag{2}$$

$$M_{ps} = M_{sp} \tag{3}$$

$$M_{ps} \propto n_p n_s \tag{4}$$

$$(M_{ps})^2 = L_p L_s \tag{5}$$

$$\tag{6}$$

4.4 The non-ideal coupling of two coils

For this purpose use the 'rectangular' Unilab coil comprising a centre-tapped winding of 2400 turns. This provides us with two coils each of 1200 turns. Note that the second coil (after the centre tap) is physically wound around the first coil so it has a larger average area per turn.

Measure the self inductance of each 1200 turn coil without the insertion of any magnetic core. Note one of the coils , presumably the one with the larger area per turn, has a larger self inductance. In the situation in which the path of the predominant flux is not defined by a magnetic core, the self inductances, L, of coils surrounding a fixed location can no longer be compared merely by knowing their number of turns.

A crude magnetic core may now be inserted in to the coil by using the rectangular pieces of high permeability ferrite available in a plastic bag. Note how the self inductance of either coil increases by a significant factor as a continuous core is produced. Finally note the once the core is complete and a dominant fixed and unchanging path is provided for the flux, the self inductances of the two coils are found to be the same (as was found earlier for coils strongly coupled by a shared magnetic core - see Figure 1).

4.5 The self and mutual inductance of weakly coupled coils

In this experiment use the damped 0.5 H Unilab (coil 1) together with the small coil (coil 2) whose bobbin is mounted on a wooden mandrel. The small coil may be inserted within the larger coil.

1. Measure the self inductance L_1 and L_2 of each coil separately. Coil 2 has a very small self inductance, so to get a good measurement V_2 it helps to measure it at a high frequency e.g. 50 kHz (why?). (Remember to lower the frequency again afterwards).
2. Measure as before the mutual inductances M_{12} and M_{21} of the two coils, holding the smaller coil in the centre of the larger coil in such a position as to maximise the flux overlap and maximise the mutual inductance. To what extent can you demonstrate the expected result that $M_{12} = M_{21}$, the mutual inductance M? An accurate value of M is needed for the following section, best done when the measured voltage is that on the small coil. Again, the mutual inductance is low, so try to measure the inductive voltage at the highest practical frequency, taking into account the effect on the shape of the waveform of the damping resistor on coil 1. You should aim for the induced voltage to be greater than ± 2.5 mV. Estimate your likely random error (due to errors in reading from the oscilloscope scale), and calibration (systematic) errors.

3. Show from your measurements that $M^2 \ll L_1 L_2$. Why would you expect this to be the case?

4.6 A quantitative test of Faraday's Law

Continue to work with the combination of small coil (coil 2) inserted within the 0.5 H coil 1.

Through your measurement of the mutual inductance of the two coils you have already measured the voltage induced in coil 2 as the current in coil 1 is changed:

$$V_2 = M\frac{\mathrm{d}I_1}{\mathrm{d}t} \tag{7}$$

Faraday's Law states that this voltage, V_2 will equal the rate of change of magnetic flux passing through coil 2. We have used this relationship to measure all of the inductances up until now in this experiment. However, we can also measure the mutual inductance directly if we can determine the *flux linked* by coil 2 per unit current flowing in coil 1. To do this we need to find, for a given current, the magnetic field produced within the bore of the outer coil (coil 1) and multiply this by the area presented by turns of the inner coil (coil 2) through which the magnetic field passes.

It is convenient now to regard coil 1 as a solenoid magnet within which coil 2 is placed. A small d.c. power supply able to pass up to 180 mA through coil 1 will be found near the booking computer. A 'Hall effect meter' or 'Magnetic Flux Density Unit' enables the resulting magnetic induction B within the bore of coil 1 to be measured. Note how B varies with axial and transverse position within the bore of coil 1. Measure the highest value of B which is to be found at the centre of the bore coil 1, the position that will have been chosen for coil 2 when measuring their mutual inductance M. By reading the value from the power supply LED display, make a note of the current, I_1, required to produce this magnetic field.

From the fundamental definition of *mutual* inductance, the relationship between the flux linked by coil 2, ϕ_2, and the current through coil 1, I_1, is

$$\phi_2 = MI_1 \tag{8}$$

where ϕ_2 is the area of coil 2 multiplied by the magnetic field from coil 1 i.e.

$$\phi_2 = B_1 A_2. \tag{9}$$

So, combining these equations, we can write

$$M = \phi_2 / I_1 = B_1 A_2 / I_1. \tag{10}$$

Coil 2 comprises 6 layers of insulated copper wire wound on to the green plastic bobbin. Each layer has 21 turns. The measured diameters d_n of the turns comprising the 6 layers are as follows: 18.5 mm, 19.5 mm, 20.5 mm, 21.5 mm, 22.5 mm and 23.5 mm.

Work out thereby the total area A_2 of the turns of coil 2 through which the field B_1 passes, where

$$A_2 = 21\pi \sum_{n=1}^{6} \frac{{d_n}^2}{4} \tag{11}$$

You are now in a position to compare your measurement of the mutual inductance using Faraday's law (Equation 7), and the mutual inductance found measuring the magnetic fields, currents and areas

directly (Equation 10). Comment on the agreement between the two values and discuss sources of systematic and random errors.

▶ If a demonstrator advises that you don't have time to complete the section below, you should now hand-write a brief summary of the experiment in your logbook. State the aims of the experiment, the method used, quantitative results, and your conclusions, in not more than about half a page. Show this to a demonstrator as part of your assessment.

Optional

4.7 Magnetic circuit in the presence of an air gap and determination of the permeability of the ferrite core

A demonstrator will advise you if there is time for you to attempt this section. The technical discussion here is covered in second year electromagnetism.

The essence of this section of the experiment is to introduce air-gaps into the magnetic circuit of the ferrite core. The *magnetic reluctance* of the circuit is thereby increased in a calculable way. The variation of the inductance as the gap increases may then be used to deduce the magnetic permeability of the material of the core. The introduction of air-gaps in this way is used commercially to tune the characteristics of a core to the requirements of a particular application.

Consider a ferrite core of high permeability whose magnetic circuit has an average area of cross-section A and a length L_c. A relatively small air-gap of thickness T is introduced as shown in Figure 4. Note that the gap appears twice in the magnetic circuit. The coil has n turns.

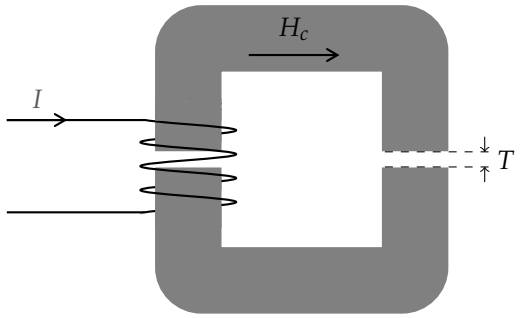


Figure 4: Ferrite core of showing an air gap of thickness T.

A new magnetic field vector $\mathbf{H}$ is introduced with magnitudes H_c in the core and H_g in the air gap. The familiar magnetic induction $\mathbf{B}$ is related to $\mathbf{H}$ by the relations $B_c = \mu_0\mu_r H_c$ in the core and $B_g = \mu_0 H_g$ in the air gap.

$\mathbf{H}$ is determined by the equation

$$nI = \int_{loop} \mathbf{H} \cdot \mathrm{d}\mathbf{l} \tag{12}$$

where the loop encloses a total current nI.

In our case with the loop taken through the centre line of the core this becomes

$$nI = \frac{B_c}{\mu_0\mu_r} L_c + \frac{B_g}{\mu_0} 2T \tag{13}$$

The magnetic induction $\mathbf{B}$ has zero divergence, $\mathrm{div}\,\mathbf{B} = 0$, reflecting the fact that lines of $\mathbf{B}$ never start or stop. At the face of a gap the lines of magnetic induction pass normally across the interface from core

to gap. We must therefore have $B_c = B_g$, and

$$nI = \frac{B_c}{\mu_0}\left(\frac{L_c}{\mu_r} + 2T\right) \tag{14}$$

or

$$B_c = \frac{nI\mu_0}{L_c/\mu_r + 2T} \tag{15}$$

In our case with a coil of n turns around a core of area A, the flux ϕ through the coil is given by

$$\phi = AB_c = \frac{nIA\mu_0}{L_c/\mu_r + 2T} \tag{16}$$

and the self inductance L follows as

$$L = \frac{n\phi}{I} = \frac{n^2 A\mu_0}{L_c/\mu_r + 2T} \tag{17}$$

Inverting we find a linear relation between $1/L$ and the air-gap:

$$\frac{1}{L} = \frac{1}{n^2 A\mu_0}\left(\frac{L_c}{\mu_r} + 2T\right) \tag{18}$$

If $1/L$ is plotted against air-gap T the anticipated straight line could be extrapolated to the non-physical negative T at which $1/L$ would go to zero enabling μ_r to be deduced.

> Cut small pieces of plastic or paper, or use those already at your bench, and measure their thickness with a micrometer. Insert equal numbers of pieces on the left and right sides of the core to raise the upper half uniformly. Maintaining firm finger pressure on the upper half of the core, measure L at 1 kHz with 0, 1, 2, 3, 5 and 10 pieces inserted on each side.
>
> Plot $1/L$ against T. Draw a straight line passing through the $T = 0$ point where all the circuit is in ferrite and taking the gradient from the next 2 or 3 points. Find the non-physical T intercept at which $1/L$ goes to zero. Determine μ_r taking L_c to be 70 mm.

You should find a value for μ_r near to 1000 confirming our expectation throughout the experiment of a large value for μ_r and justifying our initial assumption that to all practical purposes all the flux will stay within the core.

▶ Suggest reasons why your plot departs from linearity at larger values of T, the inductance becoming larger than anticipated.

(If you use a resistance meter, you will find the ferrite has quite a high resistivity, typically with a resistance of 10 kΩ between points 1 cm apart. The combination of high permeability and high resistivity produces a compact inductor with low eddy-current loss.)

▶ Now hand-write a brief summary of the experiment in your logbook. State the aims of the experiment, the method used, quantitative results, and your conclusions, in not more than about half a page. Show this to a demonstrator as part of your assessment.