Electromagnetism

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Revision lecture problems

■ *Section A* (shorter questions)

1. **[2022 Q1]** An electric field with the following form is generated by a static distribution of charge,

$$\mathbf{E} = A\frac{\mathrm{e}^{-kr}}{r}\hat{\mathbf{r}},$$

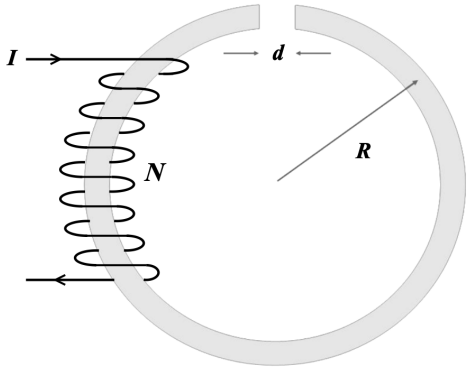
where A and k are constants.

(a) Find the charge distribution and plot it. [4]

(b) Find the total charge. [2]

2. **[2014 Q3]** Derive the condition that relates the **B**-fields in two media separated by a boundary. [2]

A circular ring of radius R is formed of soft-iron of relative permeability μ_r, with a small air gap of extent d.



Around the ring is a coil of N turns of wire through which a current I passes. Calculate B_g, the flux density in the gap. [4]

A ferromagnet of the same geometry (without the coil) is prepared with large magnetisation of magnitude M, directed around the ring. What is B_g in this case? Sketch the B and H field lines inside the magnet material and in the gap.

3. **[2011 Q2]** Use Maxwell's equations to show that

$$\frac{\partial U}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{j} \cdot \mathbf{E}$$

where $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$ is the Poynting vector and $U = \frac{\epsilon_0}{2}\mathbf{E}^2 + \frac{1}{2\mu_0}\mathbf{B}^2$. Explain the meaning of this equation.

[You may use the identity $\nabla \cdot (\mathbf{E} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{B})$.] [8]

4. **[2015 Q6]**

Electromagnetic waves of wavelength λ in air are incident normally onto a glass surface of refractive index n_1, covered by a thin coating of a transparent dielectric material of refractive index n_2 and thickness $\lambda/(4n_2)$. Determine the value of n_2 in terms of n_1 that reduces the reflected intensity to zero. Such an arrangement is used in 'non-reflective' coatings for spectacles. Suggest reasons for why such coatings might not be completely effective.

5. **[2012 Q4]** Write down the wave equations for $\mathbf{E}$ and $\mathbf{H}$ propagating in a highly conducting lossless medium, and show that high frequency waves propagate only a short distance δ, the skin depth. Estimate the impedance $|\mathbf{E}|/|\mathbf{H}|$ for copper at a frequency of 1 MHz and compare it with the free space impedance. [7]

[The conductivity of copper is $6 \times 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$.]

■ *Section B* (longer questions)

6. **[2011 Q7]** Consider three concentric spherical conductors with radii $r = a$, b and c, where $a < b < c$. Under what conditions can the electric field, $\mathbf{E}$, for this configuration be constructed from a potential Φ that satisfies Laplace's equation, $\nabla^2\Phi = 0$? What are the boundary conditions for $\mathbf{E}$? [4]

Assuming spherical symmetry, show that Φ takes the form

$$\Phi = A + \frac{B}{r}\,. \qquad [4]$$

[Hint: the Laplacian in spherical polar coordinates takes the form

$$\nabla^2 F = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial F}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial F}{\partial\phi}\right) + \frac{1}{r^2\sin^2\phi}\frac{\partial^2 F}{\partial\theta^2} \quad .\,]$$

The potentials of the three shells are held at values $\Phi = 0$ (at $r = a$), $\Phi = \Phi_0$ (at $r = b$) and $\Phi = 0$ (at $r = c$). Find the solution to Φ for all r and plot the amplitude of $\mathbf{E}$ as a function of r. [7]

Find the charge density on the conductors of radius a and b. Calculate the total electrostatic energy of the system contained between a and c. [5]

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