1 CP4 June 2025 report

Total number of candidates: 171
Average mark: 64.3
Top mark: 93
Standard deviation: 15.3
Median: 66
Number passing ($\geq 40\%$): 160
Number failing ($< 40\%$): 11

Q1 (3.4 ± 0.8 out of 4)
Limits of a function of one variable (L'Hopital's rule and/or Taylor series). Straightforward for most students.

Q2 (4.1 ± 1.9 out of 6)
Wave packets, group velocity and dispersion. A significant minority of students struggled to express the wavelengths of the envelope and carrier wave in terms of the given wave numbers.

Q3 (3.1 ± 1.6 out of 5)
Differentials of functions of two variables. A number of students did not realise that they could isolate the requested differentials by taking linear combinations of the starting differentials in part (a); others did not realise that they should equate the coefficients in front of the appropriate differential in part (b).

Q4 (4.7 ± 1.7 out of 7)
Marginal distributions and expectation value. Many students used the wrong limits of integration when calculating the marginal distribution of Y in part (b).

Q5 (4.8 ± 2.0 out of 7)
Surface integral for a conic section. The most common mistake was the neglect of the surface curvature when calculating the area element.

Q6 (3.9 ± 1.8 out of 6)
Line integrals and conservative fields. A small fraction of the students realised that one of the terms could be written as the gradient of a scalar and

quite a few struggled to express this term correctly in Cartesian coordinates.

Q7 (1.9 ± 1.5 out of 5)
Taylor series of a function of two variables and the nature of its stationary points. Many students skipped this question and others seemed to be confused about how to apply the Taylor expansion.

Q8 (13.4 ± 4.2 out of 20; attempted by 161 out of 171)
Jacobian determinant, surface and volume integrals and the divergence theorem applied to a torus. Part (a) required calculation of the Jacobian determinant for a given set of curvilinear (toroidal) coordinates. Most students had no trouble, though a few made mistakes in differentiation or in the subsequent algebraic manipulations for the determinant. Part (b) was a calculation of the surface area and volume of a torus, and most students had no difficulty with this. Parts (c-d) requested students to state and verify the divergence theorem for a given vector field. Obtaining the volume integral was straightforward, but quite a few students struggled with decomposing the surface integral and calculating the different contributions.

Q9 (13.5 ± 4.0 out of 20; attempted by 144 out of 171)
Transmission and reflection at the interface between connected strings of different mass density. Parts (a) and (b) were textbook derivations of the wave equation, boundary conditions between connected strings, reflection and transmission coefficients and corresponding powers. Points were most commonly lost for not giving an adequate justification for the 'no-kink' boundary condition and for omitting the wave number ratio that should appear in the ratio of transmitted to incident powers. Part (c) was an extension to include a point mass at the join; the biggest challenge here was confusion over how to treat the complex expression for the reflection coefficient and to convert it to a power.

Q10 (12.3 ± 4.2 out of 20; attempted by 168 out of 171)
Normal modes for a pair of coupled oscillators. Parts (a)-(c) were standard calculations of the normal mode frequencies and eigenvectors, and part (d) was a calculation of the position of the oscillator masses as a function of time. Part (e) required students to write down the normal coordinates for the system, express the kinetic energy in terms of them and determine how energy was distributed between the normal modes. This proved the most

challenging part of the problem, with many students unable to write down the normal coordinates.

Q11 (10.6 ± 5.6 out of 20; attempted by 37 out of 171)
Correct use of the gradient, divergence and curl operators and application of Stokes' Theorem. Part (a) was a derivation of the chain rule for the curl operator and part (b) tested its application, as well as the chain rule for the gradient operator. Parts (c) and (d) probed the use of Stokes' Theorem and integration over a simple (time-dependent) 2D surface. Few students attempted this problem. Of those that did, parts (a) and (b) proved fairly straightforward, while parts (c) and (d) proved more challenging. A number of students did not see that one could fruitfully apply Stokes' Theorem and that one should impose $(\partial\omega/\partial t)_{\mathbf{r}} = 0$. There was a typo in the vector identity provided for this problem, and this affected the detailed form of the final solutions to (c) and (d). For the handful of students who did not spot this typo, full points were awarded for correct application of the incorrect vector identity.