*Key lessons:*

- A summary of the methods to solve 1st order ODEs:
  - Can you write the ODE as $P(x,y)\mathrm{d}x + Q(x,y)\mathrm{d}y = 0$? If yes,
    - Can you find a potential function $\Phi$: Is $\partial P/\partial y = \partial Q/\partial x$? If yes, the ODE is exact. Find $\Phi$ from integration and set it constant. If no, find integrating factor $\Lambda$,
    - Is $\frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$ a function of $x$ only? If yes $\Lambda = \exp\left(\int \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)\mathrm{d}x\right)$, if no,
    - Is $\frac{1}{P}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$ a function of $y$ only? If yes $\Lambda = \exp\left(\int \frac{1}{P}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)\mathrm{d}y\right)$, if no,
    - Guess $\Lambda$.
    - Potential function can be found from $\partial\Phi/\partial x = \Lambda P$ and $\partial\Phi/\partial y = \Lambda Q$. Find $\Phi$ and set constant.
  - Can you write the ODE as $y' = f(x,y)$? If yes,
    - Can you write it as $y' = f(x)/g(y)$? If yes, the equation is separable, solve by integration $\int f(x)\mathrm{d}x = \int g(y)\mathrm{d}y + C$.
    - Can you write it as $y' = f(ax+by)$? If yes, the equation is almost separable. Substitute $z(x) = ax + by(x)$, use $(\mathrm{d}z/\mathrm{d}x) = a + b(\mathrm{d}y/\mathrm{d}x) = a + bf(z)$ and solve for $z$, substitute back.
    - Can you write the ODE as $y' = f(x/y)$? If yes, the equation is uniform homogeneous. Substitute $z(x) = y/x$. Then $\mathrm{d}z/\mathrm{d}x = (f(z) - z)/x$ and integrate.
    - Can you write the ODE as $y' = f\left(\frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2}\right)$? If yes, the equation is almost homogeneous. Substitute $X = x + u$ and $Y = y + v$ and solve $a_1 u + b_1 v = c_1$ and $a_2 u + b_2 v = c_2$. Integrate $\frac{\mathrm{d}Y}{\mathrm{d}X} = f\left(\left(a_1 + b_1\frac{Y}{X}\right)/\left(a_2 + b_2\frac{Y}{X}\right)\right)$, which is homogeneous.
  - Is the ODE linear, $y' + a(t)y = f(t)$?
    - Either solve using an integrating factor. The integrating factor is $\Lambda(t) = \exp\left(\int a(t)\,\mathrm{d}t\right)$ and the solution $y(t) = (1/\Lambda(t))\left(\int \Lambda(t) f(t)\mathrm{d}t + C\right)$.
    - Or use $y(t) = y_\mathrm{p}(t) + y_\mathrm{h}(t)$, with the solution to the linear homogenous equation $y_\mathrm{h} = C\exp\left(-\int a(t)\mathrm{d}t\right)$. Find a particular solution using the method of variation of constants. which yields $y_\mathrm{p} = \exp\left(-\int a(t)\mathrm{d}t\right)\int f(t)\exp\left(\int a(t')\mathrm{d}t'\right)\mathrm{d}t$.
  - Are there any singular solutions?
    - Is the differential equation autonomous ($y' = f(y)$ only)? If yes, there will be a special solution $y(x) = C$.
    - Have we divided by a function of the variables? If yes, the divisor equal zero can correspond to a solution. Check whether it is.
    - If the special solution is not already included in the general solution, it is a singular solution.