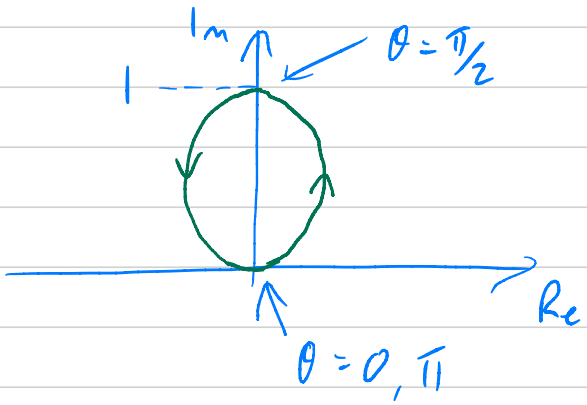
1/ $\sin\theta\, e^{i\theta} = \sin\theta(\cos\theta + i\sin\theta)$

$0 \to \pi$:



1

Near 0: $\sin\theta\, e^{i\theta} \approx \theta(1 + i\theta)$

$\Rightarrow x \approx \theta$, $y \approx \theta^2$, parabola $y = x^2$ 2

$\theta: \pi \to 2\pi$

Note $\frac{1}{2}\sin 2\theta + i\sin^2\theta$

so both real & imaginary parts repeat the same path as for $0 \to \pi$. 1

(Also shows max/min real values are $\pm \frac{1}{2}$)

2/ $2\sin ix - 3i\cos ix = 3i$

$$2\cdot\frac{1}{2i}\left(e^{-x}-e^{x}\right) - 3i\cdot\frac{1}{2}\left(e^{-x}+e^{x}\right) = 3i$$

$$\left(e^{-x}-e^{x}\right) + \frac{3}{2}\left(e^{-x}+e^{x}\right) = -3$$

$$2\left(e^{-x}-e^{x}\right) + 3\left(e^{-x}+e^{x}\right) + 6 = 0$$

$$e^{x} + 5e^{-x} + 6 = 0$$

Let $u = e^{x} \rightsquigarrow u^2 + 6u + 5 = 0$

$$(u+5)(u+1) = 0 \rightsquigarrow u = -1, -5$$

$e^{x} = -1$ 2

$$x = \ln(-1) = \ln(e^{i\pi})$$

$$= i\pi(2n+1)$$

$e^{x} = -5$

$$x = \ln(5e^{i\pi})$$

$$= \ln 5 + (2n+1)i\pi$$ 3

3a/ Let $c_{ij} = a_{ik} b_{kj}$, & note $a_{ij} = b_{ij} = 0 \quad \forall i > j$

For $i > j$, consider the sum over k

for $k < i$, $a_{ik} = 0$

for $k \geq i > j$, $b_{kj} = 0$

$\Rightarrow c_{ij} = 0 \quad \forall i > j$ & hence AB is upper triangular

2

Diagonal elements are given by $c_{ii} = a_{ik} b_{ki}$

In this sum over k: $k < i \rightsquigarrow a_{ik} = 0$

$k > i \rightsquigarrow b_{ki} = 0$

so only $k = i$ contributes $\Rightarrow$ diagonal is $(a_{11} b_{11}, \dots a_{nn} b_{nn})$

2

3b/ Let $A = \begin{pmatrix} x & y \\ 0 & z \end{pmatrix} \rightsquigarrow A^3 = \begin{pmatrix} x & y \\ 0 & z \end{pmatrix} \begin{pmatrix} x^2 & xy + yz \\ 0 & z^2 \end{pmatrix}$

$$= \begin{pmatrix} x^3 & y(x^2 + xz + z^2) \\ 0 & z^3 \end{pmatrix}$$

$\Rightarrow x = 3, z = 4 \rightsquigarrow y(9 + 12 + 16) = -74$

$\Rightarrow y = -2$ and $A = \begin{pmatrix} 3 & -2 \\ 0 & 4 \end{pmatrix}$

2

4/ Augmented matrix : $\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 2 & 2 & \lambda & 2 \\ 1 & -\lambda & 0 & 1 \end{array}\right)$

$r2 \to r2 - 2r1$: $\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 0 & \lambda+2 & 0 \\ 1 & -\lambda & 0 & 1 \end{array}\right)$

$r3 \to r3 - r1$: $\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 0 & \lambda+2 & 0 \\ 0 & -\lambda-1 & 1 & 0 \end{array}\right)$

$r2 \leftrightarrow r3$: $\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -\lambda-1 & 1 & 0 \\ 0 & 0 & \lambda+2 & 0 \end{array}\right)$

i.e.
$$x + y - z = 1 \quad (1)$$
$$-(\lambda+1)y + z = 0 \quad (2)$$
$$(\lambda+2)z = 0 \quad (3)$$

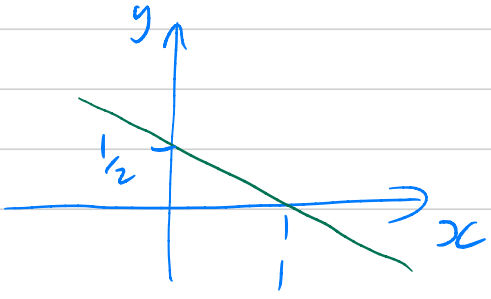
2

Cases:

- $\lambda = -2$: Line solns with z as a free parameter

(2) $\rightsquigarrow$ $y + z = 0$

(1) $\rightsquigarrow$ $y = \frac{1-x}{2}$

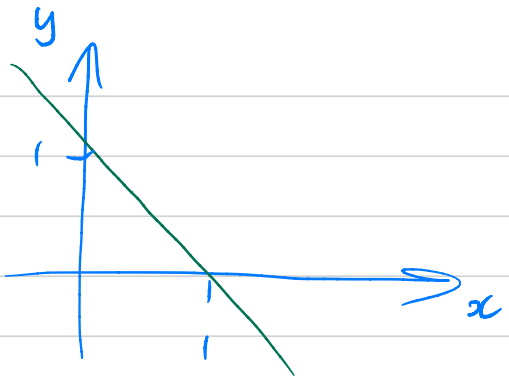


2

- $\lambda = -1$

  ② & ③ both give $z = 0$

  ① $\rightsquigarrow$ $y = 1 - x$



- O/wise there is a point sol^{n}:

  ③ $\Rightarrow z = 0$, ② $\Rightarrow y = 0$, ① $\Rightarrow x = 1$

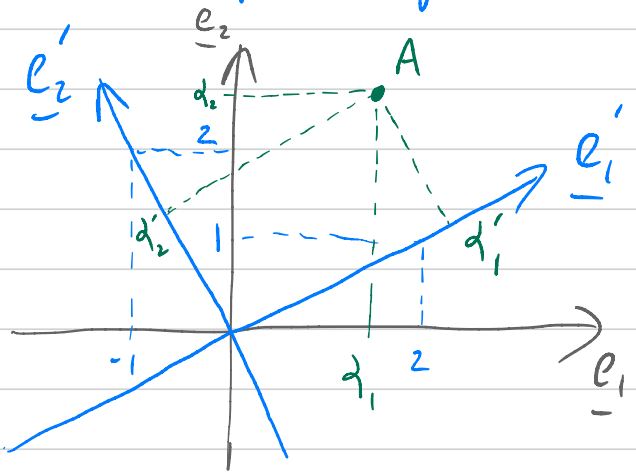
  ie $(1, 0, 0)$. 2

All sol^{n}s are independent of λ & none have both x & y simultaneously negative. 1

5a/ P^{-1} maps primed coords onto unprimed ones,

eg $P^{-1}\underline{e}_1' = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ 1

$\rightsquigarrow$ columns of P^{-1} give the new axes in coords 2.



2

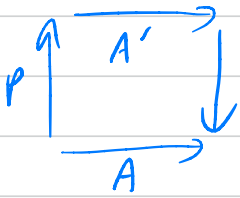
Point A is given by:

- (a_1, a_2) in $\{\underline{e}_1, \underline{e}_2\}$
- (a_1', a_2') in $\{\underline{e}_1', \underline{e}_2'\}$

1

b/ In ' coords, this is a reflection about $\underline{e}_1'$, ie $A' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(note $\underline{e}_1' \cdot \underline{e}_2' = 0$)



$A = P^{-1}A'P$

$= \frac{1}{5}\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$

2

$\det(A) = -1$: Reflection + rotation

$= \det(P^{-1})\det(A')\det(P)$

$= 1 \times -1 \times 1$

2

6/

Case 1: $\lambda = 0$ is an eigenvalue of AB.

Then $\det(AB) = \prod \lambda_i = 0$

But $\det(BA) = \det(AB) = 0$ so 0 is also an eigenvalue of BA.

2

Case 2: $\lambda \neq 0$ is an eigenvalue of AB

Then there is a non-zero vector v s.t. $ABv = \lambda v$

Let $w = Bv$ (Check: $Aw = ABv = \lambda v \neq 0 \Rightarrow w \neq 0$)

Then $BAw = BABv = B\lambda v = \lambda Bv = \lambda w$

i.e. λ is an eigenvalue of BA as well, with eigenvector w.

2

7/ $\frac{dy}{dx} = \frac{4x+2y-10}{2x-y-3} = \frac{4X+2Y+4a+2b-10}{2X-Y+2a-b-3}$

Put $x = X+a$, $y = Y+b$

Choosing $a=2$, $b=1$ gives $\frac{dY}{dX} = \frac{4X+2Y}{2X-Y}$

Now sub. $Y = vX \Rightarrow v + X\frac{dv}{dX} = \frac{4+2v}{2-v}$

$$X\frac{dv}{dX} = \frac{4+2v}{2-v} - v\cdot\frac{(2-v)}{2-v} = \frac{v^2+4}{2-v}$$

$$\int \frac{2-v}{v^2+4}\,dv = \int \frac{dX}{X} \qquad (v \neq 2,\ X \neq 0)$$ 2

LHS:

- $-\int \frac{v}{v^2+4}\,dv = -\frac{1}{2}\ln(v^2+4)$
- For $I = \int \frac{2}{v^2+4}\,dv$, let $v = 2\tan u$

$$\longrightarrow I = \int \frac{2}{4\tan^2 u + 4}\cdot 2\sec^2 u\,du$$

$$= u = \tan^{-1}\frac{v}{2}$$

So the eqn integrates to:

$$\tan^{-1}\frac{V}{2} - \frac{1}{2}\ln(V^2+4) = \ln X + \frac{1}{2}\ln C$$

& $V = \frac{y-1}{x-2}$, $X = x-2$ $\Rightarrow$

$$2\tan^{-1}\left(\frac{y-1}{2(x-2)}\right) = \frac{1}{2}\ln\left\{C\left[(y-1)^2+4(x-2)^2\right]\right\}$$

2

The method works by changing coords.

$4x + 2y - 10 = 0$ and $2x - y - 3 = 0$

are intersecting lines. The transformation moves the origin to the point of intersection, so that the constants are eliminated and the eqn is then homogeneous.

2

SECTION B

8 ai/

- $v_1 = v_2 = 0 \rightarrow$ span $S = 0$, dim $= 0$
- One of $v_1, v_2 \neq 0 \rightarrow S = \alpha v_i$, $\alpha \in \mathbb{R}$, dim $= 1$
- $v_1, v_2 \neq 0$ but linearly dependent. Same as ↑
- —"— linearly independent : $S = \alpha v_1 + \beta v_2$, dim $= 2$

4

ii/

$$(a \wedge b) \cdot (a \wedge b) = (a \wedge b)_i (a \wedge b)_i$$

$$= \varepsilon_{ijk} a_j b_k \varepsilon_{imn} a_m b_n$$

$$= \varepsilon_{ijk} \varepsilon_{imn} a_j b_k a_m b_n$$

$$= \left(\delta_{jm} \delta_{kn} - \delta_{jn} \delta_{km} \right) a_j b_k a_m b_n$$

$$= a_m a_m b_n b_n - a_n b_n b_m a_m$$

$$= |a|^2 |b|^2 - (a \cdot b)^2$$

3

8b/ For vectors u_1 and u_2 satisfying $u_1 \cdot v = u_2 \cdot v = 0$ we need:

1/ $\alpha u_1 \cdot v = 0$ ✓

2/ $(u_1 + u_2) \cdot v = 0$ ✓

$\Rightarrow$ is a sub-vector space

2

A general plane is given by $\underline{u} \cdot \underline{v} = d$ for a given normal vector $\underline{v}$. For two vectors on the plane we have $\underline{u}_1 \cdot \underline{v} = \underline{u}_2 \cdot \underline{v} = d$

Then 1/ $\alpha \underline{u}_1 \cdot \underline{v} = \alpha d = 0$ only if $d = 0$

2/ $(\underline{u}_1 + \underline{u}_2) \cdot \underline{v} = 2d = 0$ only if $d = 0$

$\Rightarrow$ the plane is a sub vector space $\Leftrightarrow d = 0$

i.e. it's a plane through the origin.

2

8c/ $\dim(U+V) = \dim(U) + \dim(V) - \dim(U \cap V)$ 1

Case 1: $U = V \rightsquigarrow \dim(U \cap V) = 2$, $\dim(U+V) = 2$ 1

Case 2: $U \neq V$. Planes must intersect in a line containing the origin (from (b)).

$\Rightarrow \dim(U \cap V) = 1$, $\dim(U+V) = 3$

So any vector in $\mathbb{R}^3$ can be constructed as a sum of vectors in $U \& V$. 2

(Note there is no situation in which the planes do not intersect.)

8d/ Plane U: $\underline{r} = \lambda(1,1,3) + \mu(1,3,1)$ as it includes O.

$\rightsquigarrow$ normal to U is $\begin{vmatrix} i & j & k \\ 1 & 1 & 3 \\ 1 & 3 & 1 \end{vmatrix} = \begin{pmatrix} -8 \\ 2 \\ 2 \end{pmatrix}$

Similarly, normal to V is $\begin{vmatrix} i & j & k \\ 1 & 1 & -2 \\ 2 & 1 & 2 \end{vmatrix} = \begin{pmatrix} 4 \\ -6 \\ -1 \end{pmatrix}$

So intersection is $\underline{r} = \underline{q}t$ where $\underline{q} = \begin{vmatrix} i & j & k \\ -4 & 1 & 1 \\ 4 & -6 & -1 \end{vmatrix} = \begin{pmatrix} 5 \\ 0 \\ 20 \end{pmatrix}$ 2

$d_{min}^2 = \dfrac{|\underline{d} \wedge \underline{q}|^2}{|\underline{q}|^2}$ where $\underline{d} = \underline{p} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= -\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ as $\underline{p} = 0$

$\underline{d} \wedge \underline{q} = \begin{vmatrix} i & j & k \\ -1 & -1 & -1 \\ 1 & 0 & 4 \end{vmatrix} = \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}$ (using $\underline{q} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$)

$\Rightarrow d_{min}^2 = \dfrac{16+9+1}{1+16} = \dfrac{26}{17}$ 3

9a/ Use $e^x = 1 + x + \frac{x^2}{2!} \dots$

$$\Rightarrow \left(e^{iM}\right)^\dagger = \left(I + iM - \frac{M^2}{2!} - i\frac{M^3}{3!} \dots\right)^\dagger$$

$$= I - iM^\dagger - \frac{M^{\dagger 2}}{2!} + i\frac{M^{\dagger 3}}{3!} \dots$$

$$= e^{-iA^\dagger}$$

2

[have used $(M^n)^\dagger = M^\dagger (M^{n-1})^\dagger = M^{\dagger 2}(M^{n-2})^\dagger \dots$

$= M^{\dagger n}$]

1

Then $\left(e^{iM}\right)\left(e^{iM}\right)^\dagger = e^{iM} e^{-iM^\dagger}$

$= e^{i(M - M^\dagger)}$

$= e^{i0}$ as M hermitian

$= I \Rightarrow e^{iM}$ is unitary 2

Note that here we need M & $M^\dagger$ to commute, in order to re-arrange the series. Since $M = M^\dagger$ they do commute.

1

a/ • S1 : $\langle A, B\rangle = a_1 b_1 + a_2 b_2 + \dots = \langle B, A\rangle$ ✓

• S2 : $\langle A, \alpha B + \beta C\rangle = a_1(\alpha b_1 + \beta c_1) + a_2(\alpha b_2 + \beta c_2) \dots$

$= \alpha(a_1 b_1 + a_2 b_2 + \dots) + \beta(a_1 c_1 + a_2 c_2 + \dots)$

$= \alpha \langle A, B\rangle + \beta \langle A, C\rangle$ ✓

• S3 : $\langle A, A\rangle = \sum_{i=1}^{\infty} a_i^2 \geq 0$

equality $\Leftrightarrow$ $a_i = 0 \ \forall i$ ✓ 3

Using Cauchy - Schwarz :

$\langle A, B\rangle \leq \langle A, A\rangle\langle B, B\rangle = \sum_{i=1}^{\infty} a_i^2 \sum_{i=1}^{\infty} b_i^2 < \infty$

Hence the sum converges for any $A, B \in V$ 2

9c/ Can write as $\begin{pmatrix} G_{k+2} \\ G_{k+1} \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} G_{k+1} \\ G_k \end{pmatrix}$ 2

& diagonalise: $\begin{vmatrix} 1/2 - \lambda & 1/2 \\ 1 & -\lambda \end{vmatrix} = 0 \rightsquigarrow \lambda^2 - \frac{\lambda}{2} - \frac{1}{2} = 0$

$(\lambda - 1)(\lambda + 1/2) = 0$ 1

$\underline{\lambda = 1}$: $\begin{pmatrix} -1/2 & 1/2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \rightsquigarrow V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\underline{\lambda = -1/2}$: $\begin{pmatrix} 1 & 1/2 \\ 1 & 1/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \rightsquigarrow V_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ 2

Write original matrix A as $PA'P^{-1}$ where $P = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$

$\Rightarrow A^n = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1^n & 0 \\ 0 & (-1/2)^n \end{pmatrix} \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$ 2

Let $n \to \infty$: $A^\infty = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$

$= \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} G_\infty \\ G_\infty \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} G_1 = 1 \\ G_0 = 0 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 2/3 \end{pmatrix}$ 2

$\Rightarrow \underline{G_\infty = 2/3}$

10a/ Let $p = \frac{dy}{dx} \rightsquigarrow p^2 x - 2yp - x = 0$

or $y(x,p) = \frac{x(p^2-1)}{2p}$

Then $p\,dx = dy = \left.\frac{\partial y}{\partial x}\right|_p dx + \left.\frac{\partial y}{\partial p}\right|_x dp$ 2

$= \frac{p^2-1}{2p}\,dx + \frac{x}{2}\,\frac{2p^2-p^2+1}{p^2}\,dp$

$= \frac{p^2-1}{2p}\,dx + \frac{x}{2}\,\frac{p^2+1}{p^2}\,dp$

$\left(p - \frac{p^2-1}{2p}\right)dx = \frac{x}{2}\,\frac{p^2+1}{p^2}\,dp$

$\frac{(p^2+1)}{2p}\,dx = \frac{x}{2}\,\frac{p^2+1}{p^2}\,dp \rightsquigarrow \frac{dp}{dx} = \frac{p}{x}$ 3

Integrating gives $p = cx$, so original eqn is

$c^2x^3 - 2ycx - x = 0$ ⊛

$y = \frac{1}{2}\left(cx^2 - \frac{1}{c}\right)$ 3

(Note: should only be one constant...)

$\circledast$: $c^2x^3 - 2xyc - x = 0$ has trivial sol^n $x = 0$ 1

o/wise it's a quadratic for c, for a given $x, y \rightsquigarrow$ 2 curves.

Sol^n is not unique because original eq^n is quadratic in $dy/dx \rightsquigarrow$ two sol^{n}s for the gradient. 2

10a, ALT) Let $p = dy/dx \rightsquigarrow p^2x - 2yp - x = 0$

$\partial/\partial x$: $p^2 + x2pp' - 2(yp' + p^2) - 1 = 0$

$p'(2xp - 2y) - p^2 - 1 = 0$

Now replace y using the original eq^n : $y = \dfrac{p^2x - x}{2p}$ (2)

$\Rightarrow p'\left(2xp - px + \dfrac{x}{p}\right) - p^2 - 1 = 0$

$p'x\left(p + \dfrac{1}{p}\right) - p\left(p + \dfrac{1}{p}\right) = 0$

$\Rightarrow p' = \dfrac{dp}{dx} = \dfrac{p}{x}$ (3)

10b/ Write as $\left(y' - f_1(x,y)\right)\left(y' - f_2(x,y)\right) = 0$

where $f_1 f_2 = -1$ and $f_1 + f_2 = -f$ 2

For a given point (x_1, y_1), the solutions $y' = f_1(x_1, y_1)$ and $y' = f_2(x_1, y_1)$ are the gradients of curves passing through (x_1, y_1). Furthermore, the product of these gradients is $-1 \Rightarrow$ they intersect at right angles. 2

For $f = -\frac{2y}{x}$, eqn is $y'^2 - \frac{2y}{x} y' - 1 = 0$

$\rightsquigarrow$ same as in part (b). Solns are $y = \frac{1}{2}\left(cx^2 - \frac{1}{c}\right)$

$(x,y) = (3,4) \Rightarrow \quad 8c = 9c^2 - 1$

$(9c+1)(c-1) = 0 \rightsquigarrow c = 1, -1/9$ 3

These two values of c give two quadratic curves, with gradients $\frac{dy}{dx} = cx$. At $x = 3$, these gradients are 3 and $-1/3$, so the curves cross at right angles there. 2

11a/ • $A = 0$, $B = 1$, C and D fns of x only

$\rightsquigarrow y' + C(x)y = D(x)$

Then I.F. is $e^{\int C(x)\,dx}$. 3

• Or, if the order of the equation can be reduced

eg if $C = 0$, let $u = \frac{dy}{dx}$

$\rightsquigarrow A\frac{du}{dx} + Bu = D$

Need $A = 1$, B and D fns of x only,

then I.F. is $e^{\int B(x)\,dx}$ 3

11b/ $\frac{d}{dt}$ ① $\rightsquigarrow$ $\frac{dz}{dt} = 2t - \frac{dx}{dt} - \frac{dy}{dt}$

Sub into ②: $\frac{dx}{dt} + 2\left(2t - \frac{dx}{dt} - \frac{dy}{dt}\right) = 3x - y$

$$-\frac{dx}{dt} - 2\frac{dy}{dt} = 3x - y - 4t$$

$+$ ③ $\rightsquigarrow$ $$-2\frac{dx}{dt} = 2x - 4t$$

i.e. $\frac{dx}{dt} + x = 2t$ I.F. $e^{\int dt} = e^t$ 3

$\Rightarrow \frac{d}{dt}(xe^t) = 2te^t$ (u dv)

By parts: $xe^t = 2\left(te^t - \int e^t\right) = 2(t-1)e^t + c$

$$x = 2(t-1) + ce^{-t}$$

$t=0, x=2 \Rightarrow 2 = -2 + c \Rightarrow c = 4 \Rightarrow x = 2(t-1) + 4e^{-t}$ 3

Sub. into ②: $2\frac{dy}{dt} - 2 + 4e^{-t} = y - 2(t-1) - 4e^{-t}$

$$2\frac{dy}{dt} - y = -8e^{-t} + 4 - 2t$$

$\frac{dy}{dt} - \frac{y}{2} = -4e^{-t} + 2 - t$ I.F. $e^{\int -\frac{1}{2}dt} = e^{-t/2}$ 2

$\frac{d}{dt}\left(ye^{-t/2}\right) = \left(-4e^{-t} + 2 - t\right)e^{-t/2}$

$= -4e^{-\frac{3t}{2}} + 2e^{-t/2} - te^{-t/2}$

I_1 I_2 I_3

$$I_1 = -4\int e^{-3t/2}\,dt \qquad I_2 = 2\int e^{-t/2}\,dt$$

$$= \frac{8}{3}e^{-3t/2} \qquad = -4e^{-t/2}$$

$$I_3 = -\int \underset{u}{t}\,\underset{dv}{e^{-t/2}}\,dt$$

$$= -\left(t(-2)e^{-t/2} - \int(-2)e^{-t/2}\,dt\right)$$

$$= 2te^{-t/2} + 4e^{-t/2}$$

$$\Rightarrow ye^{-t/2} = I_1 + I_2 + I_3 = \frac{8}{3}e^{-3t/2} + 2te^{-t/2} + d$$

$$y = \frac{8}{3}e^{-t} + 2t + de^{t/2}$$

$$t=0,\ y=\frac{8}{3} \Rightarrow \quad \frac{8}{3} = \frac{8}{3} + d \quad \Rightarrow \quad d = 0$$

$$\Rightarrow y = \frac{8}{3}e^{-t} + 2t \qquad 3$$

Then $z = t^2 - x - y$

$$= t^2 - \left(2(t-1) + 4e^{-t}\right) - \left(\frac{8}{3}e^{-t} + 2t\right)$$

$$= t^2 - 4t + 2 - \left(4 + \frac{8}{3}\right)e^{-t}$$

$$z = t^2 - 4t + 2 - \frac{20}{3}e^{-t} \qquad 1$$

So $\underline{r} = \begin{pmatrix} 2(t-1) + 4e^{-t} \\ \frac{8}{3}e^{-t} + 2t \\ t^2 - 4t + 2 - \frac{20}{3}e^{-t} \end{pmatrix}$

Accln $= \ddot{\underline{r}} = \begin{pmatrix} 4e^{-t} \\ \frac{8}{3}e^{-t} \\ 2 - \frac{20}{3}e^{-t} \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$ as $t \to \infty$

The force is not constant, but it does asymptote to a constant force for large t.

2