CP1 special relativity: problem sheet 2

Main questions

1. A K^0 meson at rest can decay into a pair of charged π mesons, (also called pions). Given that the rest masses of the K^0 and $\pi^{+,-}$ are 498MeV/c^2 and 140MeV/c^2 respectively, show that the speeds of the pions are $0.83c$ in the rest frame of the K^0.

2. Energy from the Sun (a distance 1.50×10^{11} m from Earth) arrives at the Earth (above the atmosphere) at a rate of about 1400Wm^{-2}. How fast is the sun losing mass due to radiating energy?

3. An electron accelerated by the Stanford Linear Accelerator (SLAC) in California has a total energy of 50 GeV. (a) What fraction of this is kinetic energy? (b) What is the 3-momentum of the electron? (c) What is its speed?

4. (a) Neutral π mesons can be formed by

$$p + p \to p + p + \pi^0 \ ,$$

where p is a proton. What is the threshold proton energy for π^0 production by this process if the target proton is stationary? (The masses of the π^0 and proton are 135 and 938 MeV/c^2 respectively.)

(b) What is the threshold electron kinetic energy for production of an electron pair by the collision of an incident electron with a stationary electron $e^- + e^- \to e^- + e^- + e^+ + e^-$.

5. A particle of mass m, moving at speed $v = 4c/5$, collides inelastically with a particle of the same mass at rest such that the two particles stick together to form a single composite particle. What is the mass m_C and speed v_C of the composite particle?

6. (a) Show that a free electron moving in a vacuum at speed v cannot emit a single photon. (b) A hydrogen atom in an excited state can emit a single photon, why is this different from (a) above?

7. In parts (a), (b) and (c) of this question consider an atom with a ground state mass of M and an excited state with energy Q_0 greater than the ground state energy, i.e. mass $M' = M + Q_0/c^2$.

(a) An atom in the excited state is initially stationary in the lab frame. It decays to its ground state by emitting a photon of energy Q_1 (as measured in the lab frame). Find an expression for Q_1. Give an approximation to Q_1 that is valid in the limit $M \gg Q_0$ accurate to order Q_0/M.

(b) An atom an excited state is moving towards a photon detector with speed v when it then decays emitting a photon of energy Q_2 (as recorded by the detector), coming completely to rest as it does so. Find an expression for Q_2.

(c) What frequency photons could a stationary atom in the ground state absorb?

(d) *(*) optional* There is a similar mismatch between the frequencies of photons emitted and absorbed in nuclear transitions. In experiments it is found that in gasses the rate of reabsorption of emitted photons

is larger at higher temperatures. However, the reabsorption is more efficient at lower temperatures in some solids (the Mössbauer effect). Suggest an explanation for this.

8. Light of frequency ν_0 is emitted from a source moving away from an observer with speed βc. If the light is emitted along the direction of the motion show that the frequency measured by the observer is $\nu = \nu_0\gamma(1-\beta)$. An absorption line in Ca+ has a wavelength of 394 nm as measured for a stationary source. The same line in the spectrum of Hydra occurs at 475 nm. What can be deduced about the motion of Hydra with respect to the Earth?

9. A neutral pion of rest mass $135\text{MeV}/c^2$ is moving with a kinetic energy of 1 GeV. It decays in flight into two photons whose paths are along its direction of motion. Find the energies of the two photons.

Interesting questions to think about

These extra questions vary from a bit harder than the other questions (*) to much harder (***).

Unless told otherwise, you do not need to do these questions for tutorials (it might make sense to try them over the vacation as a challenge). They are beyond the syllabus for the present course, so don't worry if you can't do them.

1. A limit on the graviton mass (*)

There are highly speculative unproven theories in which the graviton (which mediates the gravitational force) has a tiny but non-zero mass.

The LIGO detector has observed a gravitational wave signal from a merger of blackhole a distance 1 Gpc away from Earth. The gravitational wave signal is made up of waves of frequencies roughly in the vicinity of 100 Hz, and lasts about a second. The gravitational wave signal as a function of time is observed to have roughly the shape predicted theoretically. Estimate a bound on the graviton mass from this observation. *Note: in quantum mechanics $E = \hbar\omega$ holds for massive as well as massless particles.*

2. A toy model of a black hole (**)

Consider an observer undergoing acceleration with the acceleration in their instantaneous rest frame constant (in the x-direction say), as in the final example in Section 16 of the lecture notes.

(a) Sketch the observer's trajectory on a spacetime diagram. Show that there is a region of spacetime that they can never receive information from.

If an observer starts accelerating from a position x_0 at t_0, their trajectory is

$$ct = ct_0 + \frac{c^2}{a_{\text{inst}}} \sinh\left(a_{\text{inst}}\tau/c\right) \ , \qquad x = x_0 + \frac{c^2}{a_{\text{inst}}} \left(\cosh\left(a_{\text{inst}}\tau/c\right) - 1\right) \ , \tag{0.1}$$

where τ is its proper time.

Suppose there are two accelerating observers. Both are at rest with respect to the frame S at $t = 0$ and both feel equal constant acceleration a_{inst}. One (the "trailing" observer) begins at $x_0 = c^2/a_{\text{inst}}$ and the other (the "leading" observer) begins at $x_0 = L + c^2/a_{\text{inst}}$.

(b) Sketch the trajectories of these two observers on a space-time diagram.

(c) Suppose that a light signal is travelling in the positive x-direction such that it first passes the trailing observer and then the leading observer. The frequency of the light in the frame in which the two observers are initially at rest is ν. Show that the difference in photon energy measured by the two observers is

$$\Delta E = h\nu \left(\frac{a_{\text{inst}} L}{c^2}\right) \ . \tag{0.2}$$

Hint: The 4-velocity of an observer can be written as $U = c(\cosh\psi, \sinh\psi, 0, 0)^T$ where ψ is the rapidity. Use Eq. (0.1) to relate the observer's rapidities to their proper times, and hence find expressions for $x - ct$ in terms of the two observer's rapidities.