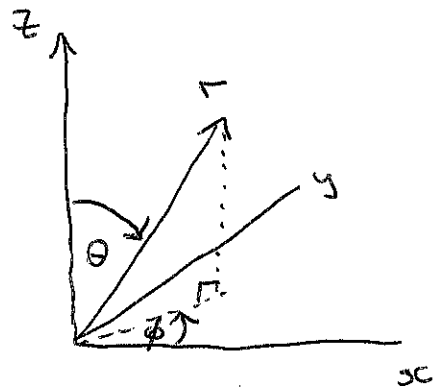
CP1 Jun 2025 A1

(a)



Let density be ρ

Use spherical polars

Volume element $dV = dr\, r\sin\theta\, d\phi\, r\, d\theta$ (1)

$dI_z = \rho\, dV\, (r\sin\theta)^2$ (1)

$dI_z = \rho\, r^4 dr \sin^3\theta\, d\theta\, d\phi$

$I_z = \rho \int_0^R r^4 dr \int_0^\pi \sin^3\theta\, d\theta \int_0^{2\pi} d\phi$ (1)

$= \rho\, 2\pi \frac{R^5}{5} \underbrace{\int_0^\pi (1-\cos^2\theta)\sin\theta\, d\theta}$

$-[\cos\theta]_0^\pi + \left[\frac{\cos^3\theta}{3}\right]_0^\pi$

$2 \qquad -2/3$ (1)

$I_z = \frac{2}{5}R^2 \cdot \underbrace{\frac{4}{3}\pi R^3 \rho}_{M}$

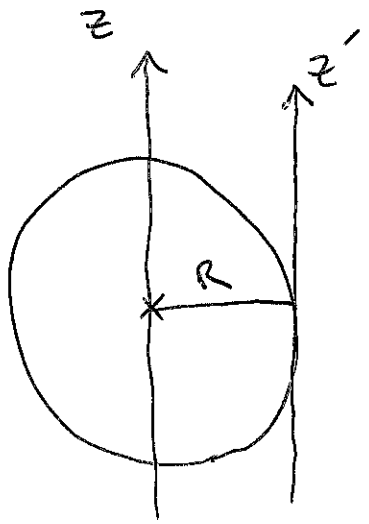
$= \frac{2}{5}MR^2$ (1)

TOTAL (5)

(b) Parallel axis theorem:

The MI of a body of mass M about an axis ∥ to an axis through the c.o.m. is:

$$MI_{CM}^2 + Mr^2 \qquad / r = \perp \text{ distance between axes}$$



(1)

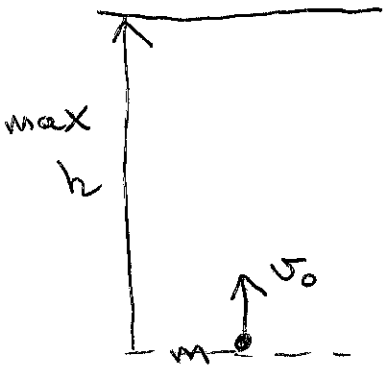
$$I_{z'} = I_z + MR^2$$

$$= \frac{7}{5} MR^2$$

(1)

TOTAL (2)

CP1 Juin 2025 A2



Newton 2 :

$$m\frac{dv}{dt} = -mg - mKv$$

$$\frac{dv}{dt} = -g - Kv \quad (1)$$

$$\int_0^{t_h} dt = -\int_{v_0}^{0} \frac{dv}{g + Kv} \quad (1)$$

$$t_h = -\frac{1}{K}\left[\ln(g + Kv)\right]_{v_0}^{0} \quad (1)$$

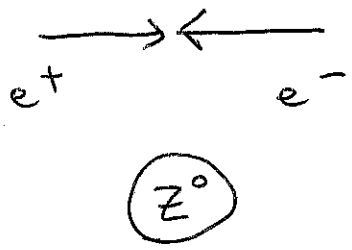
$$= -\frac{1}{K}\left(\ln g - \ln(g + Kv_0)\right)$$

$$= \frac{1}{K}\ln\left(1 + \frac{Kv_0}{g}\right) \quad (1)$$

TOTAL (4)

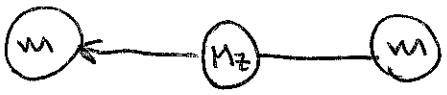
CP1 June 2025 A3

(a)



$$E_e = \frac{M_Z}{2} \quad (1)$$

(b)



Each particle has energy $\frac{M_Z}{2}$ (1)

Using "$E^2 = p^2 + m^2$" for each particle

$$p = \sqrt{E^2 - m^2} = \left(\left(\frac{M_Z}{2}\right)^2 - m^2\right)^{1/2} \quad (1)$$

(c) $\gamma = E/m = 1/\sqrt{1 - v^2/c^2}$ (1)

$$1 - v^2/c^2 = \left(\frac{m}{E}\right)^2 = \left(\frac{2m}{M_Z}\right)^2$$

$$v^2/c^2 = 1 - \left(\frac{2 \times 1.5}{91}\right)^2 = 1 - \left(\frac{3}{91}\right)^2$$

$$v/c = \left(\frac{91^2 - 9}{91^2}\right)^{1/2}$$

$$v \simeq 0.9995\,c \quad (2)$$

TOTAL (6)

CP1 June 2025 A4



Con. of $\underline{p}$: $M\underline{v} = M(\underline{v_1} + \underline{v_2})$ (α)

Con. of KE : $Mv^2 = M(v_1^2 + v_2^2)$ (β) (1)

$\alpha \cdot \alpha \Rightarrow v^2 = v_1^2 + v_2^2 + 2\underline{v_1} \cdot \underline{v_2}$

e.q. β : $\underline{v_1} \cdot \underline{v_2} = 0$ (1)

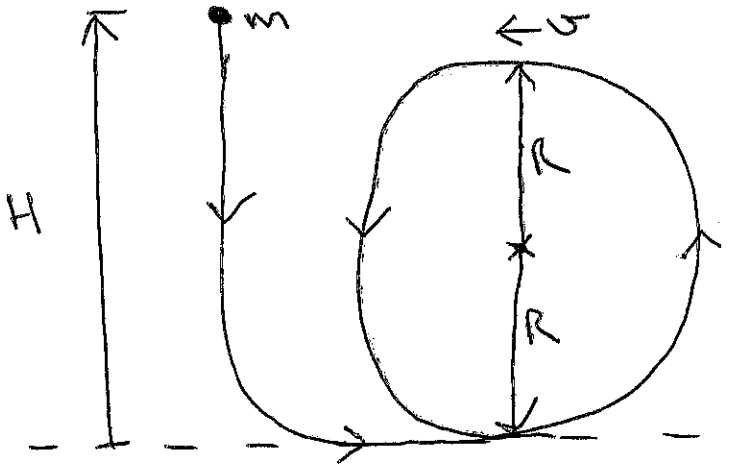
$\Rightarrow$ either v_1 or $v_2 = 0$ (1)

↳ one particle stationary

or $\underline{v_1} \perp \underline{v_2}$ (1)

TOTAL (4)

CP1 June 2025 A5



Let carriage have mass m

Let speed at top of loop = v (1)

Conservation of E:

$$PE \text{ lost} = mg(H - 2R) = KE = \frac{1}{2}mv^2$$

$$v^2 = 2g(H - 2R) \quad (1)$$

To stay on track at top of loop

$$\frac{mv^2}{R} > mg \quad (1)$$

$$v^2 > Rg$$

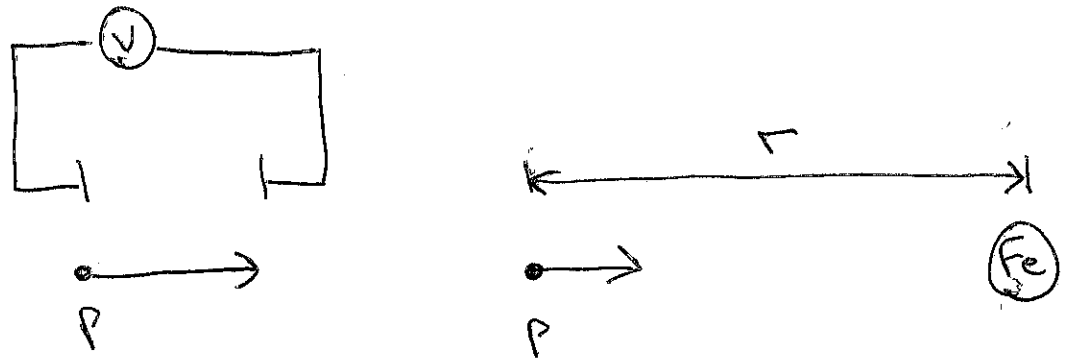
$$2g(H - 2R) > Rg$$

$$H > \frac{R}{2} + 2R$$

$$H > \frac{5R}{2} \quad (1)$$

TOTAL (4)

CP1 June 2024 A6



p KE = eV (1)

PE = $\frac{e \, Ze}{4\pi\varepsilon_0 r}$ (1)

At POCA $eV = \frac{e^2 Z}{4\pi\varepsilon_0 r_{min}}$

$V = \frac{Ze}{4\pi\varepsilon_0 r_{min}}$ (1)

For $r_{min} < 10^{-13}$ m $\quad V > \frac{26 \times 1.6 \times 10^{-19}}{4\pi \, 8.85 \times 10^{-12} \times 10^{-13}}$

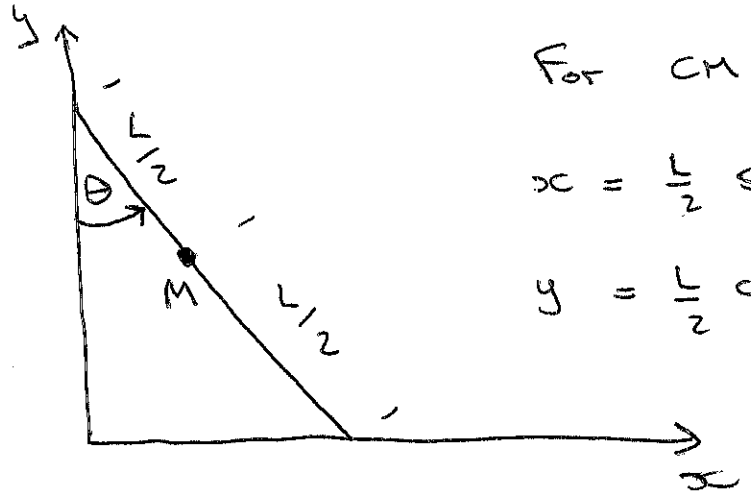
$V > \frac{26 \times 1.6}{4\pi \, 8.85} \times 10^6 = 374 \text{ kV}$ (1)

Assumptions: Any 2 of 4

- p non-relativistic
- ignore nuclear recoil
- ignore e^- in Fe atom
- ignore Strong nuclear force

(2)

TOTAL (6)



For CM of ladder:

$x = \frac{L}{2}\sin\theta, \quad \dot{x} = \frac{L}{2}\cos\theta\,\dot{\theta}$

$y = \frac{L}{2}\cos\theta, \quad \dot{y} = -\frac{L}{2}\sin\theta\,\dot{\theta}$ (1)

(a) $L = T - V$ (1)

KE PE

$T = \frac{1}{2}M(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}I\dot{\theta}^2$ (1)

But $\dot{x}^2 + \dot{y}^2 = \frac{L^2}{4}\dot{\theta}^2$

and $I = \frac{1}{12}ML^2$

$\Rightarrow T = \frac{1}{2}M\frac{L^2}{4}\dot{\theta}^2 + \frac{1}{2}\frac{ML^2}{12}\dot{\theta}^2$

$= \frac{ML^2\dot{\theta}^2}{8}(1 + 1/3) = \frac{ML^2\dot{\theta}^2}{6}$ (1)

$V = Mg\frac{L}{2}\cos\theta$ (1)

$\therefore L = \frac{ML^2\dot{\theta}^2}{6} - \frac{MgL}{2}\cos\theta$ (1)

TOTAL (6)

(b)

Conjugate momentum :

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{ML^2\dot{\theta}}{3} \qquad (1)$$

Euler – Lagrange : $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{\partial L}{\partial \theta}$ (1)

$$\frac{ML^2\ddot{\theta}}{3} = \frac{MgL}{2}\sin\theta$$

$$\ddot{\theta} = \frac{3g}{2L}\sin\theta \qquad (1)$$

TOTAL (3)

CP1 June 2025 B8

Laws of physics are the same in all inertial ref. frames

Speed of light in free space " " " "

(2)

$$p' = \gamma\left(p - \frac{vE}{c^2}\right)$$

$$E' = \gamma\left(E - vp\right) \qquad (1)$$

$$\therefore \quad E'^2 - p'^2 = \gamma^2\left[(E - \beta p)^2 - (p - \beta E)^2\right]$$

$$= \gamma^2\left[E^2 - p^2 + \beta^2\left(p^2 - E^2\right)\right]$$

$$= \gamma^2\left(E^2 - p^2\right)\left(1 - \beta^2\right)$$

But $\gamma^2 = \dfrac{1}{1-\beta^2}$

$$\therefore \quad E'^2 - p'^2 = E^2 - p^2 \qquad \text{QED} \qquad (2)$$

Total (5)

(b)

$$E = \gamma M c^2 \qquad \gamma = \left(1 - v^2/c^2\right)^{-1/2}$$

$$p = \gamma M v$$

For photon: $E = pc = hf = \frac{hc}{\lambda}$

$$p = \frac{h}{\lambda}$$

Total (4)

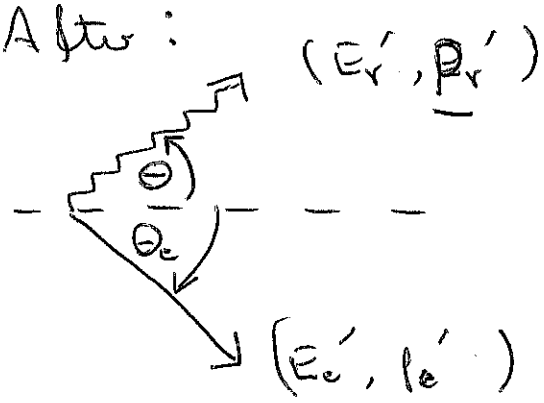
(c)

Before:



(E_γ, p_γ) $(m_e c^2, 0)$

After:



Con E: $E_\gamma + Mc^2 = E_\gamma' + E_e'$ (1)

Con p: $\underline{p}_\gamma + 0 = \underline{p}_\gamma' + \underline{p}_e'$ (2)

$(1)^2$ $(E_\gamma - E_\gamma' + m_e c^2)^2 = E_e'^2$ (3)

$(2)^2$ $(p_\gamma - p_\gamma')^2 = p_e'^2$ (4)

(3) - (4): $E_e'^2 - p_e'^2 c^2 = m_e^2 c^4$

(3) - (4): $(E_\gamma - E_\gamma')^2 + m_e^2 c^4 + 2(E_\gamma - E_\gamma')m_e c^2 - p_\gamma^2 - p_\gamma'^2 + 2\underline{p}_\gamma \cdot \underline{p}_\gamma' c^2$

Equating:

$E_\gamma^2 - p_\gamma^2 + E_\gamma'^2 - p_\gamma'^2 - 2E_\gamma E_\gamma' + 2(E_\gamma - E_\gamma')m_e c^2 + 2\underline{p}_\gamma \cdot \underline{p}_\gamma' c^2 = 0$

$$(E_\gamma - E_\gamma')m_e c^2 - E_\gamma E_\gamma' + E_\gamma E_\gamma' \cos\theta = 0$$

$$(E_\gamma - E_\gamma')m_e c^2 - E_\gamma E_\gamma' (1 - \cos\theta) = 0$$

but "$E = \frac{hc}{\lambda}$"

$$\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)m_e c^2 - \frac{hc}{\lambda\lambda'}(1 - \cos\theta) = 0$$

$$(\lambda' - \lambda)m_e c - h(1 - \cos\theta) = 0$$

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta)$$

Total (7)

$\theta = 0$: $\lambda' = \lambda$

Elastic scattering (1)

$\theta = \pi$: Backward scattering (1)

$$\lambda' = \lambda + \frac{2h}{mec}$$

If $\lambda \ll \frac{h}{mec}$ $\lambda' \approx \frac{2h}{mec}$

$$\therefore E' = \frac{hc}{\lambda'} = \frac{hc}{2h} mec$$

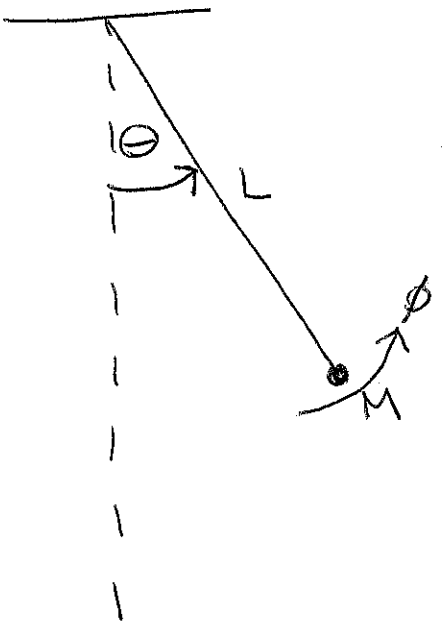
$$= \frac{1}{2} mec^2 \quad (1)$$

$$\approx \frac{1}{4} MeV$$

Total (4)

Constant, independent of λ (1)

CP1 June 2025 B9



(a) $L = T - V$ (1)

$$T = \frac{1}{2} M (L\dot{\theta})^2 + \frac{1}{2} M (L \sin\theta \dot{\phi})^2$$

$$V = -MgL\cos\theta$$

$$L = \frac{1}{2} ML^2 (\dot{\theta}^2 + \sin^2\theta \dot{\phi}^2) + MgL\cos\theta$$ (1)

L does not depend on t => E is conserved (1)

L " " ϕ

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = 0 \Rightarrow \frac{\partial L}{\partial \dot{\phi}} \text{ is conserved}$$ (1)

$$\frac{\partial L}{\partial \dot{\phi}} = \frac{1}{2} M L^2 \sin^2\theta \cdot 2\dot{\phi}$$

$$= M (L\sin\theta)^2 \dot{\phi}$$

$$= J_z = \text{conserved.} \quad (1)$$

Total (5)

(b) $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{\partial L}{\partial \theta}$$

$$\frac{d}{dt}\left(ML^2\dot{\theta}\right) = \frac{1}{2} ML^2 \dot{\phi}^2 \, 2\sin\theta\cos\theta - MgL\sin\theta$$

$$L\ddot{\theta} = L\dot{\phi}^2 \sin\theta\cos\theta - g\sin\theta \quad (2)$$

---

i) ϕ = constant $\Rightarrow \dot{\phi} = 0$

$$\therefore \ddot{\theta} = -\frac{g}{L}\sin\theta \quad (1)$$

If θ <u>small</u>, $\ddot{\theta} \approx -\frac{g}{L}\theta$

SHM with angular frequency $\sqrt{\frac{g}{L}}$ (1)

---

ii) θ = constant $\Rightarrow$ $\ddot{\theta} = 0$

$\therefore \quad 0 = L\dot{\phi}^2 \sin\theta \cos\theta - g\sin\theta$ (1)

Ignore trivial case $\theta = 0$, let $\theta = \theta_0$

$$\dot{\phi}^2 = \frac{g}{L\cos\theta_0}$$

Constant motion with angular freq $\sqrt{\frac{g}{L\cos\theta_0}}$ (1)

Total (6)

(c)

From (a) $J_z = ML^2\sin^2\theta\,\dot{\phi}$

$\theta = \theta_0$ and $\dot{\phi} = \sqrt{\frac{g}{L\cos\theta_0}}$ (1)

$\therefore \quad J_z = ML^2\sin^2\theta_0\sqrt{\frac{g}{L\cos\theta_0}}$

$= ML\sin^2\theta_0\sqrt{\frac{g}{\cos\theta_0}}$ (1)

Total (2)

(d) From (b): $\ddot{\Theta} = \dot{\phi}^2 \sin\Theta\cos\Theta - \frac{g}{L}\sin\Theta$

Now assume $\dot{\phi}^2$ remains constant $= \frac{g}{L\cos\Theta_0}$ (1)

and $\Theta \to \Theta_0 + \beta$

$$\ddot{\beta} = \frac{g}{L\cos\Theta_0}\sin(\Theta_0+\beta)\cos(\Theta_0+\beta) - \frac{g}{L}\sin(\Theta_0+\beta) \quad (1)$$

Expanding using standard trig. relations:

$$\ddot{\beta} = \frac{g}{L\cos\Theta_0}\left\{(\sin\Theta_0\cos\beta + \cos\Theta_0\sin\beta)(\cos\Theta_0\cos\beta - \sin\Theta_0\sin\beta)\right\}$$

$$- \frac{g}{L}(\sin\Theta_0\cos\beta + \cos\Theta_0\sin\beta) \quad (1)$$

Now β is small so $\sin\beta \approx \beta$ and $\cos\beta \approx 1$: (1)

$$\ddot{\beta} = \frac{g}{L\cos\Theta_0}\left\{(\sin\Theta_0 + \beta\cos\Theta_0)(\cos\Theta_0 - \beta\sin\Theta_0)\right\}$$

$$- \frac{g}{L}(\sin\Theta_0 + \beta\cos\Theta_0)$$

$$\ddot{\beta} = \frac{g}{L}\left\{\frac{1}{\cos\Theta_0}(\sin\Theta_0\cos\Theta_0 + \beta\cos^2\Theta_0 - \beta\sin^2\Theta_0 - \beta^2\sin\Theta\cos\Theta_0)\right.$$

$$\left. - \sin\Theta_0 - \beta\cos\Theta_0\right\}$$

↑ IGNORE $O(\beta^2)$ terms (1)

$$\ddot{\beta} = \frac{g}{L}\left\{ \cancel{\sin\theta_0} + \beta\cancel{\cos\theta_0} - \beta\frac{\sin^2\theta_0}{\cos\theta_0} - \cancel{\sin\theta_0} - \beta\cancel{\cos\theta_0} \right\}$$

$$\ddot{\beta} = -\frac{g}{L}\frac{\sin^2\theta_0}{\cos\theta_0}\beta \qquad (1)$$

SHM with period $2\pi\left(\frac{g}{L}\frac{\sin^2\theta_0}{\cos\theta_0}\right)^{-1/2}$ (1)

Total: (7)

CP1 June 2025 B10

(a)

(i) $\bar{J} = \bar{r} \times \bar{p}$

$$\frac{d\bar{J}}{dt} = \dot{\bar{r}} \times \bar{p} + \bar{r} \times \dot{\bar{p}}$$

But $\dot{\bar{r}} \parallel \bar{p}$ and $\dot{\bar{p}} = \bar{F} = F\hat{r}$

So $\frac{d\bar{J}}{dt} = 0 \Rightarrow$ $\bar{J}$ is constant (2)

(ii) $E = \frac{1}{2} M \dot{r}^2 + \frac{J^2}{2Mr^2} + V(r)$ (1)

Since $\bar{J}$ is a constant can define effective potential

$$U(r) = \frac{J^2}{2Mr^2} + V(r) \quad (1)$$

$$\Rightarrow E = \frac{1}{2} M \dot{r}^2 + U(r) \quad (1)$$

simplifies to effectively 1-d motion. (1)

Total (4)

(b) Conservation of energy, using U:

$$\tfrac{1}{2}M v_R^2 - \frac{GMm_E}{R} = \tfrac{1}{2}M v_c^2 - \frac{GMm_E}{r_c} \quad (1)$$

furthest closest

$$v_R^2 - v_c^2 = 2GM_E\left(\frac{1}{R} - \frac{1}{r_c}\right) \quad (\alpha)\ (1)$$

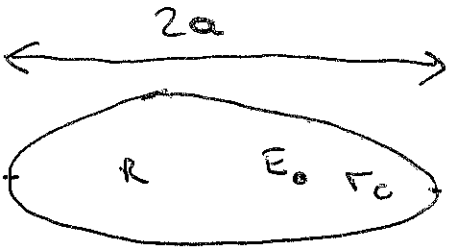
Con. of ang. mom: $MRv_R = Mr_c v_c$

$$v_c = v_R \frac{R}{r_c} \quad (1)$$

So $(\alpha) \Rightarrow v_R^2\left(1 - \frac{R^2}{r_c^2}\right) = 2GM_E\left(\frac{r_c - R}{Rr_c}\right)$

$$\frac{v_R^2}{r_c^2}(r_c + R)(r_c - R) = \frac{2GM_E}{Rr_c}(r_c - R)$$

$$v_R^2 = \frac{2GM_E}{(r_c + R)}\frac{r_c}{R} \quad (1)$$



But $R + r_c = 2a$ (1)

$$v_R^2 = \frac{2GM_E}{2a}\frac{(2a - R)}{R} = GM_E\left(\frac{2}{R} - \frac{1}{a}\right) \quad (1)$$

Total (6)

(c)

Just before retro-rocket fires:

$$v_R^2 = GM_E\left(\frac{2}{R} - \frac{1}{a}\right) \quad (\beta) \qquad (1)$$

and $R = a(1+e)$

$$\Rightarrow v_R^2 = GM_E\left(\frac{2}{a(1+e)} - \frac{1}{a}\right)$$

$$= \frac{GM_E}{a}\left(\frac{2-(1+e)}{1+e}\right)$$

$$= \frac{GM_E}{a}\left(\frac{1-e}{1+e}\right) \quad (\gamma) \qquad (1)$$

Just after rocket fires: $R = a(1+e)$ as before

but major axis reduces to $2a'$

and $v_R \rightarrow v_R(1-\epsilon)$

So $v_R^2(1-\epsilon)^2 = \frac{GM_E}{a}\left(\frac{1-e}{1+e}\right)(1-\epsilon)^2$ (1)

(γ)

But $v_R^2(1-\epsilon)^2 = GM_E\left(\frac{2}{a(1+e)} - \frac{1}{a'}\right)$ (1)

(β)

$$\frac{GM_E}{a}\left(\frac{1-e}{1+e}\right)(1-\epsilon)^2 = GM_E\left(\frac{2}{a(1+e)} - \frac{1}{a'}\right)$$

$$\frac{1}{a'} = \frac{2}{a(1+e)} - \left(\frac{1-e}{1+e}\right)\frac{(1-\epsilon)^2}{a}$$

$$\frac{1}{a'} = \frac{1}{a(1+e)}\left(2 - (1-e)(1-\epsilon)^2\right)$$

$$a' = \frac{a(1+e)}{2-(1-e)(1-\epsilon)^2} \quad (1)$$

So change in major axis is $2a - 2a'$:

$$2a - 2a' = 2a\left(1 - \frac{1+e}{2-(1-e)(1-\epsilon)^2}\right)$$

$$= 2a\left(\frac{2-(1-e)(1-\epsilon)^2 - (1+e)}{2-(1-e)(1-\epsilon)^2}\right) \quad (1)$$

So $(1-\epsilon)^2 = 1 - 2\epsilon + \epsilon^2 \approx 1 - 2\epsilon$ (1)

if $\epsilon \ll 1$

Hence $2a - 2a' \simeq 2a\left(\frac{2-(1-e)(1-2\epsilon)-(1+e)}{2-(1-e)(1-2\epsilon)}\right)$

$$2a - 2a' \simeq 2a\left(\frac{2 - (1 - e - 2\epsilon + 2\epsilon e) - 1 - e}{2 - (1 - e - 2\epsilon + 2\epsilon e)}\right)$$

$$\simeq 2a\left(\frac{1 + 2\epsilon - 2\epsilon e - 1}{1 + e + 2\epsilon + 2\epsilon e}\right)$$

$$\simeq 2a\left(\frac{2\epsilon(1 - e)}{1 + e + O(\epsilon)}\right)$$

$$\simeq 4a\epsilon\left(\frac{1 - e}{1 + e}\right) \qquad (1)$$

Total (8)

CP1 June 2025 B11

(a) Elastic collision conserves KE and momentum. (2)

(b)

(i)

(A) Just Before bounce	(B) Bounce initial	(C) Bounce final
m_1 ↓	m_1 ↓v	m_1 ↑u
m_2 ↓v	m_2 ↑v	m_2 ↑
	lower ball hits ground and rebounds	lower ball hits upper ball, upper ball rebounds (4)

(ii) Let speed of m_2 just before bounce be v (given)

Elastic collision ∴ speed of m_2 just after $= v$ (1)

Speed of m_1 just before collision $= v$ (1)

Let speed of m_1 just after collision be u

Max. energy transfer to upper ball when lower ball at rest after collision (1)

Con of momentum: $m_2 v - m_1 v = m_1 u$

(B) (C)

$$v = \frac{m_1 u}{m_2 - m_1}$$ (1)

Cons. of KE : $\frac{1}{2}m_1u^2 + \frac{1}{2}m_2v^2 = \frac{1}{2}m_1u^2$ (1)

(B) (C)

$$m_1u^2 = (m_1+m_2)v^2$$

$$m_1u^2 = (m_1+m_2)\left(\frac{m_1}{m_2-m_1}\right)^2 u^2 \qquad (u \neq 0)$$

$$(m_2-m_1)^2 = m_1(m_1+m_2)$$

$$m_2^2 + m_1^2 - 2m_1m_2 = m_1^2 + m_1m_2$$

$$m_2^2 = 3m_1m_2$$

$$\frac{m_1}{m_2} = \frac{1}{3}$$ (3)

Total (8)

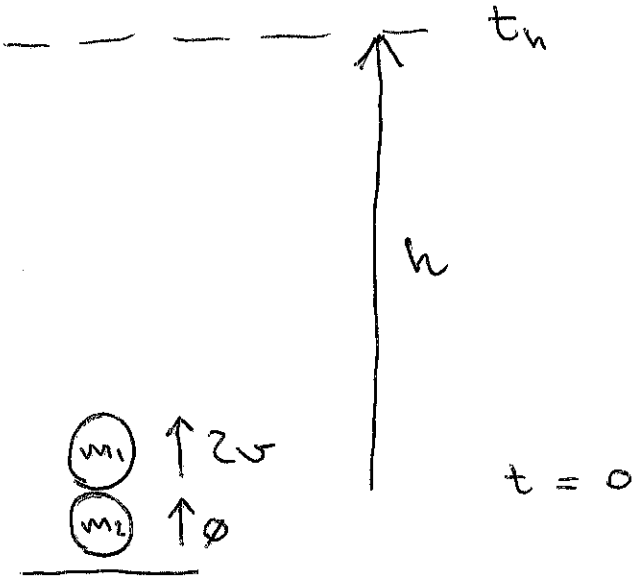
(iii) Max height of top ball corresponds to previous case of max. energy transfer (1)

So initial speed of top ball is given by

$$v = \frac{m_1}{m_2-m_1}u$$

and $m_2 = 3m_1 \Rightarrow u = 2v$ (1)

Case (c) again:



h given by standard formula ("$v^2 = u^2 + 2as$")

$$0 = (2v)^2 - 2gh$$

$$2gh = 4v^2$$

$$h = \frac{2v^2}{g}$$ (2)

t_h given by standard formula ("$v = u + at$")

$$0 = 2v - gt$$

$$t = \frac{2v}{g}$$ (2)

Total (6)