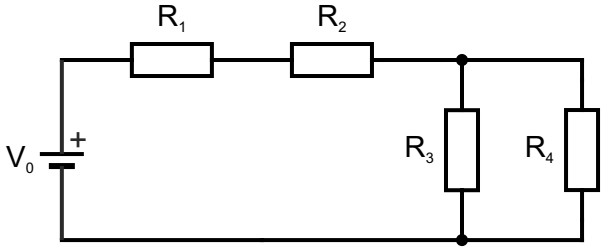
Problem set for 1st-year Circuits

Robert Smith (with thanks to Todd Huffman)

Michaelmas 2021

1 Introduction to simple circuits of resistors

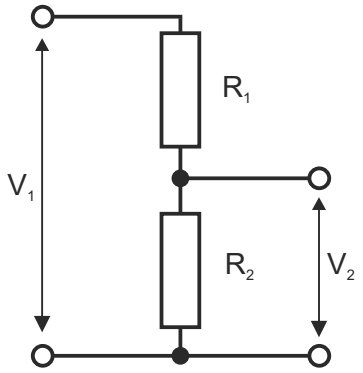
1. For the following circuit calculate the currents through and voltage drops across all resistors. The voltage $V_0 = 12\,\mathrm{V}$ and the resistances are: $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 3\,\mathrm{k\Omega}$, $R_3 = 3\,\mathrm{k\Omega}$ and $R_4 = 6\,\mathrm{k\Omega}$.



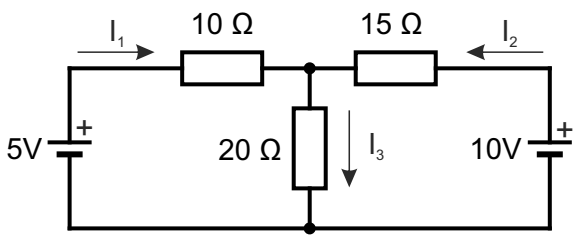
2. The structure of a cube is soldered by using resistors R for its edges. What is the resistance between two most distant corners (diametrically opposite)?

3. Harder: In problem 2 above, what is the resistance between two adjacent corners and between two corners on the same face but not adjacent?

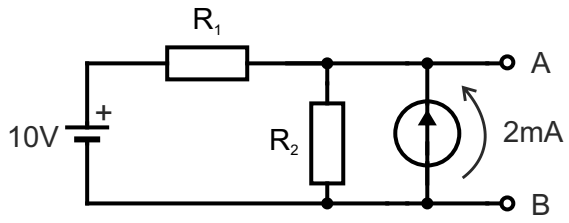
4. For the voltage divider (below), where $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 4\,\mathrm{k\Omega}$ and $V_1 = 5\,\mathrm{V}$ find the voltage drop V_2 across R_2 . When a load resistor R_L is fitted in parallel with R_2, what minimum value must R_L have in order not to change V_2 by more than 5%?



5. Consider the following circuit: find I_1, I_2 and I_3. Hint: you may wish to tackle the problem using mesh currents.

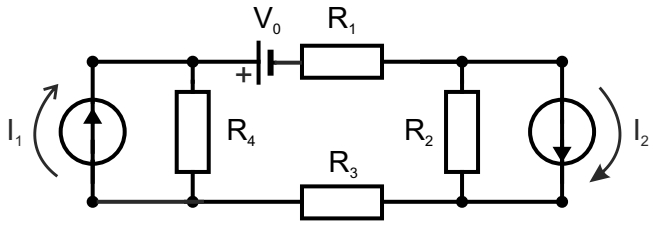


6. Consider the following circuit with resistors $R_1 = R_2 = 1\,k\Omega$:



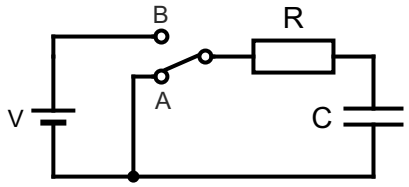
Find the voltage between A and B and the currents I_1 and I_2 through the resistors R_1 and R_2. Considering Thevenin's theorem, what are V_{eq} and R_{eq} in the equivalent circuit? What is the equivalent circuit according to Norton's theorem? Find I_{eq} and R_{eq} in this case.

7. Find the currents through resistors R_1 to R_4. Give magnitudes and directions. The values are: $V_0 = 2\,V$, $I_1 = 1\,mA$, $I_2 = 4\,mA$, $R_1 = 1\,k\Omega$, $R_2 = 1.5\,k\Omega$, $R_3 = 0.5\,k\Omega$ and $R_4 = 2\,k\Omega$.



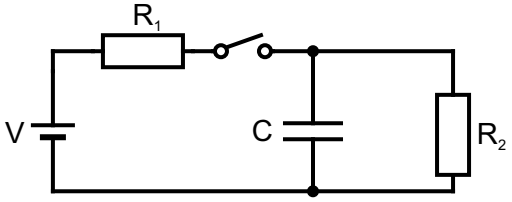
2 Response of linear circuits to transients

8. The capacitor C is initially uncharged. At time $t = 0$ the switch is moved from position A to position B. Derive an expression for the current flowing through R at time t.

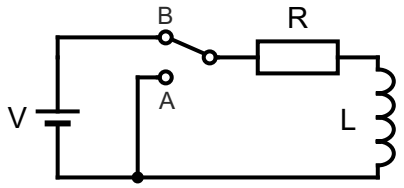


By performing an integration over time, derive an expression for the total energy dissipated in the resistor. What is the final energy stored in the capacitor? Hence show that the total energy supplied by the battery is CV^2 .

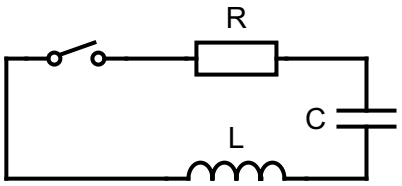
9. Initially the switch in the circuit below is open and the capacitor is uncharged. At time $t = 0$ the switch is closed. Show that the voltage $V_C(t)$ across the capacitor as a function of time goes as $V_C = V_0(1 - \exp(-t/\tau))$. Find V_0 and τ for $V = 6\,V$, $R_1 = 10\,k\Omega$, $R_2 = 22\,k\Omega$ and $C = 20\,\mu F$. When the steady state has been reached, what is i) the power dissipated in each resistor and ii) the energy stored in the capacitor?



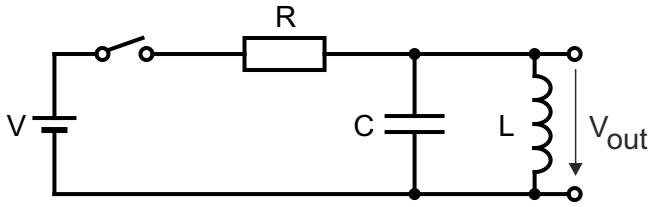
10. At time $t = 0$ the switch is moved from position A to position B. Derive an expression for the current flowing through R at time t.



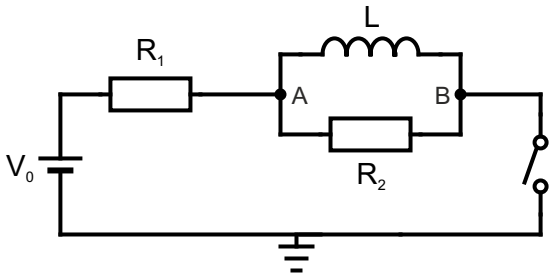
11. At times $t < 0$ the switch is open and the capacitor is charged with 1 V across its terminals. At $t = 0$ the switch is closed. Show that the subsequent time response of the circuit is oscillatory and damped, and sketch (quantitatively) the response of the circuit for $t > 0$, given $R = 150\,\Omega$, $L = 10\,\text{mH}$, and $C = 10\,\text{nF}$.



12. In the RLC circuit shown, at time $t = 0$ the switch is closed. By solving the appropriate differential equation, show that the voltage V_{out} is oscillatory and damped with an exponential decay-time constant given by $t_0 = 2RC$. If the component values are $R = 100\,\text{k}\Omega$, $C = 10\,\text{nF}$ and $L = 4\,\text{mH}$, show that the resonant frequency, $\omega_0 = 1.6 \times 10^5$ rad/s.



13. The switch in the figure has been closed for a long time, ie. a steady state has been reached, it is then suddenly opened (disconnected). Using the values $V_0 = 10V$, $R_1 = 100\,\Omega$, $R_2 = 10\,\text{k}\Omega$ and $L = 100\,\text{mH}$ demonstrate that $V_\text{A} = V_\text{B} = 0\,\text{V}$ before the switch is opened, $V_\text{A} = 10\,\text{V}$, $V_\text{B} = 1010\,\text{V}$ immediately after the switch is opened, then that V_B falls exponentially to 10 V with a time constant of 10 µs.



After many time constants have elapsed, the switch is again closed. Show that the voltage across R_1 rises exponentially with a time constant of 1 ms.

3 Complex impedances and response of linear circuits to AC

14. RMS-values of voltages:

    a) What is the RMS-voltage of a constant voltage $V(t) = V_C$?

    b) What is the RMS-voltage of a square wave between 0 and V volts?

    c) What is the RMS-voltage of a square wave between $-V/2$ and $+V/2$ volts? Why are the answers to b) and c) different?

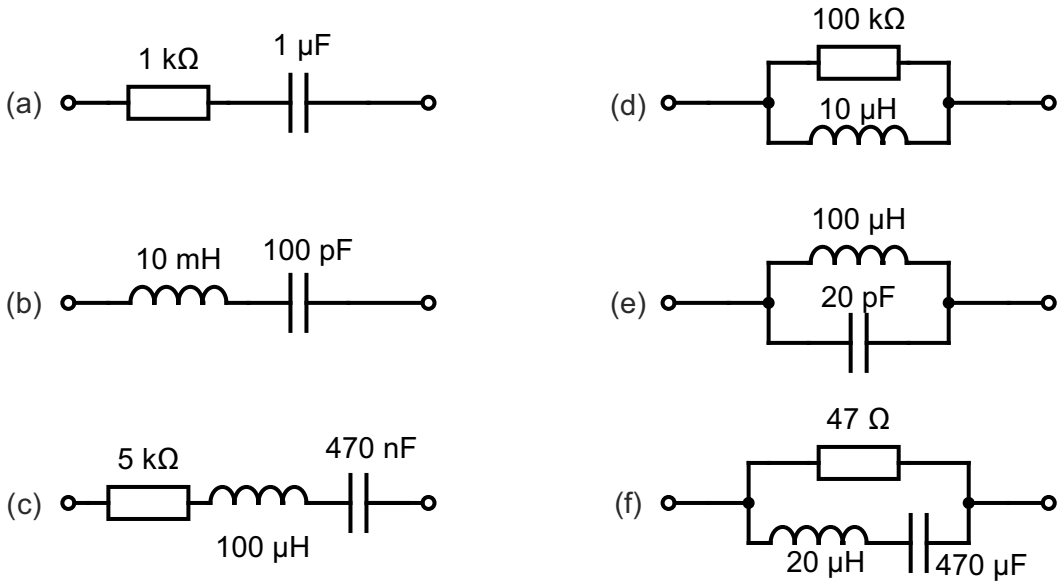
    d) Calculate the RMS-value of a saw-tooth voltage $V(t) = V_0 t/T$ (defined for $-T < t < T$) of period $2T$.

15. For each of the networks below:

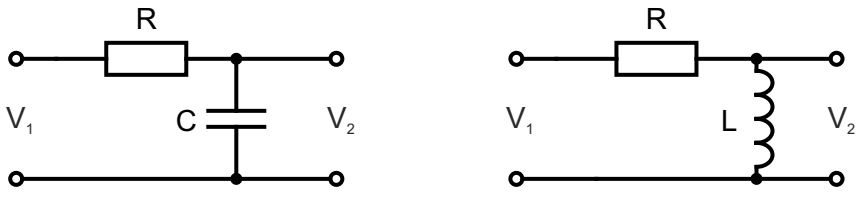
    i) Calculate the (complex) impedances;

    ii) A voltage of the form $V = V_0 \sin(\omega t)$ is applied across the network. Evaluate the phase shift between the voltage and the current flowing through the network. State whether the current leads or lags;

    iii) Calculate the peak voltage drop across the resistors in circuits (a) and (d), the capacitors in circuits (b) and (e), and the inductors in circuits (c) and (f). Take the driving voltage to be of the form $V = V_0 \sin(\omega t)$ with $V_0 = 10\,\mathrm{V}$ and frequency $(\omega/2\pi) = 10\,\mathrm{kHz}$.



16. The two circuits below are driven by sinusoidal input voltages $V_1 = V_0 \sin(\omega t)$.

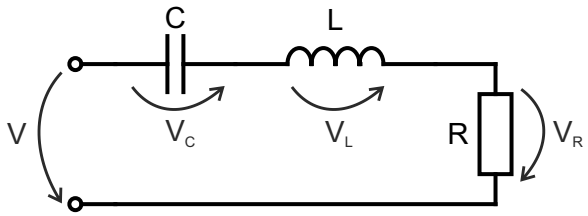


    a) Draw the phasor diagrams of all voltages.

    b) With the help of the phasor diagrams determine the ratio of the amplitudes of voltages V_2 and V_1 as well as the phases of the output voltages V_2 for the two circuits.

    c) Sketch the ratios of amplitudes and the phases as a function of normalised frequency $x = \omega/\omega_0$, with $\omega_0 = 1/RC$ and $\omega_0 = R/L$ respectively for the two circuits.

17. An AC current $I(t) = I_0 \sin(\omega t)$ is flowing through the circuit below made from a series combination of capacitance C, inductance L and resistance R. The numerical values are: $C = 10\,\text{nF}$, $L = 0.2\,\text{mH}$, $R = 100\,\Omega$, $I_0 = 0.1\,\text{A}$ and $\omega = 10^6$ rad/s.



a) Find the amplitudes and phase angles of the voltages across the capacitor, inductor and resistor $V_C(t)$, $V_L(t)$ and $V_R(t)$ and of the total voltage $V(t)$.

b) Find the total power dissipated by the circuit.

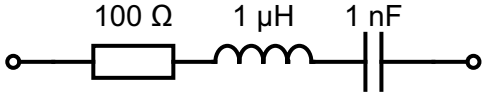
c) Find expressions for the energy contents $W_L(t)$ and $W_C(t)$ of the inductor and capacitor.

d) What are the maximum values for these energies?

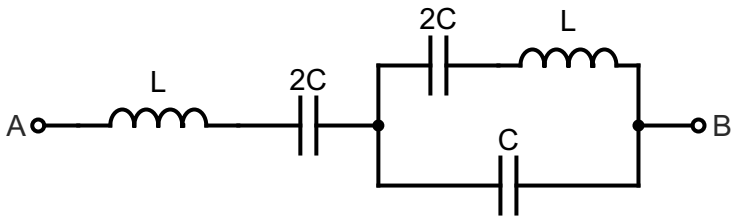
e) Sketch $W_L(t)$ and $W_C(t)$.

f) For which value of L (provided all other values remain constant) would the sum of $W_L(t)$ and $W_C(t)$ be constant?

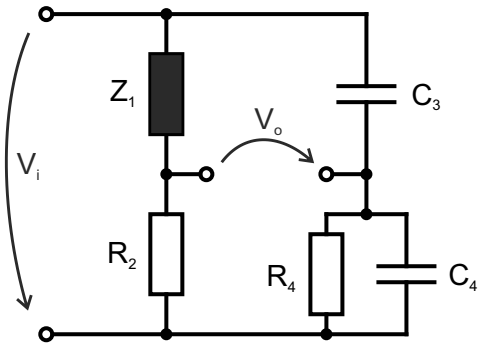
18. At what frequency does the network below have its minimum impedance? If the driving voltage is of the form $V = V_0 \sin(\omega t)$ with $V_0 = 10\,\text{V}$, show that the voltage across the capacitor at the frequency of minimum impedance is 3.16 V.



19. Show that there are two frequencies, $\omega_1 = \sqrt{\frac{5}{2LC}}$ and $\omega_2 = \sqrt{\frac{1}{2LC}}$, for which the impedance between points A and B in the network below is zero.



20. In the following bridge circuit R_2, C_3, R_4 and C_4 are fixed and Z_1 is variable

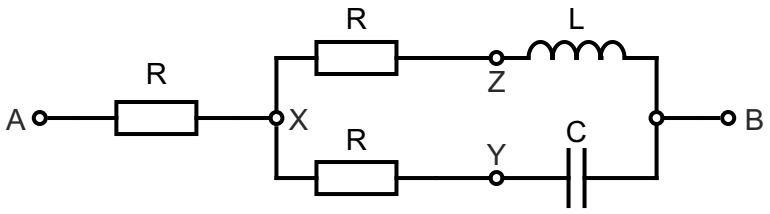


a) Find the complex value of Z_1 for which the bridge is balanced.

b) When Z_1 is a series combination of R_1 and C_1 what values must they have to balance the bridge?

c) Given that Z_1 is the parallel combination of a resistor R_1 and a capacitor C_1, for which values of R_1 and C_1 is the bridge balanced?

21. A voltage $V_{\mathrm{AB}} = V_0 \cos(\omega_0 t)$ is applied between points A and B in the circuit below. $C = (\omega_0 R\sqrt{3})^{-1}$ and $L = (R\sqrt{3})/\omega_0$.



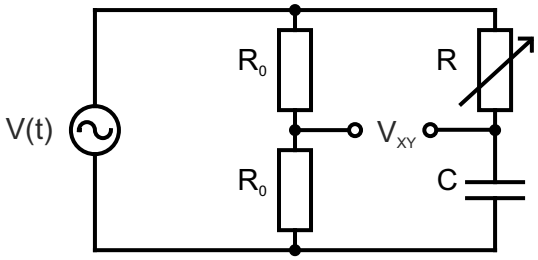
a) Show that the total impedance between A and B is $3R$.

b) Verify that voltages of equal amplitude are developed between the points X-A, the points X-Y and the points X-Z.

c) Show that the phases of these three voltages relative to V_{AB} are 0, $\pi/3$ and $-\pi/3$.

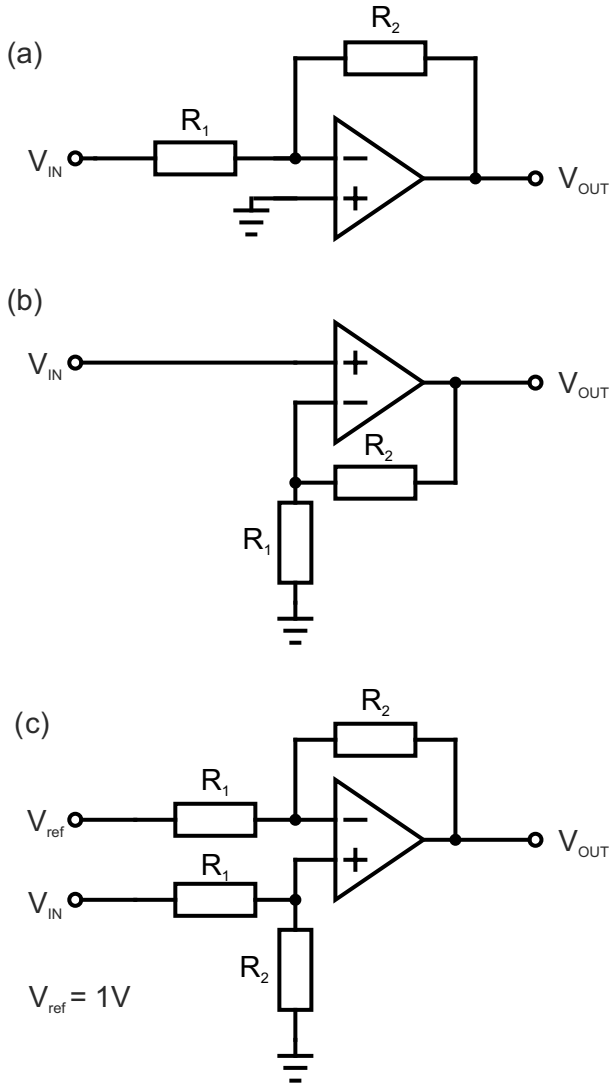
d) A current of source of $I = I_0 \cos(\omega t)$ is now connected across the points A-B, plot the magnitude and phase of the resulting voltage between A and B, as a function of ω.

22. In the circuit below, show that the amplitude of the voltage V_{XY} is independent of R, and show that if $R = 1/\omega C$, the phase of V_{XY} with respect to the applied voltage V is $\pi/2$.

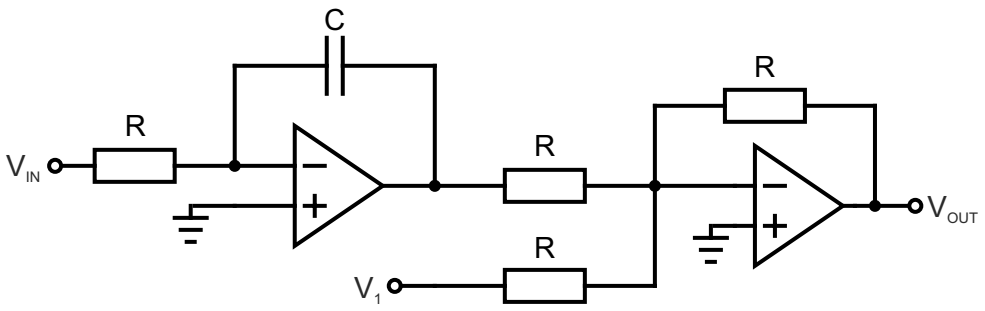


4 Operational amplifier circuits

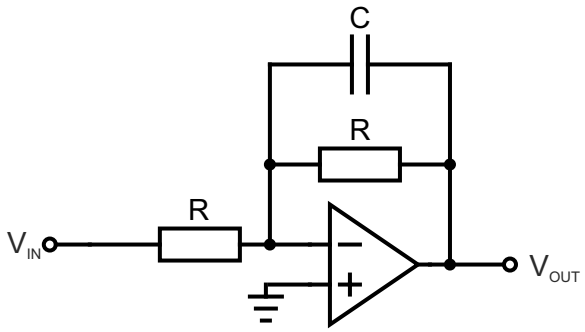
23. For circuits (a) to (c) below, calculate V_{OUT} as a function V_{IN}, R_1 and R_2 assuming the ideal op-amp model. When $R_2 = 4R_1$, give the value of V_{IN} which would produce an output voltage of $V_{\mathrm{OUT}} = 2\,\mathrm{V}$.



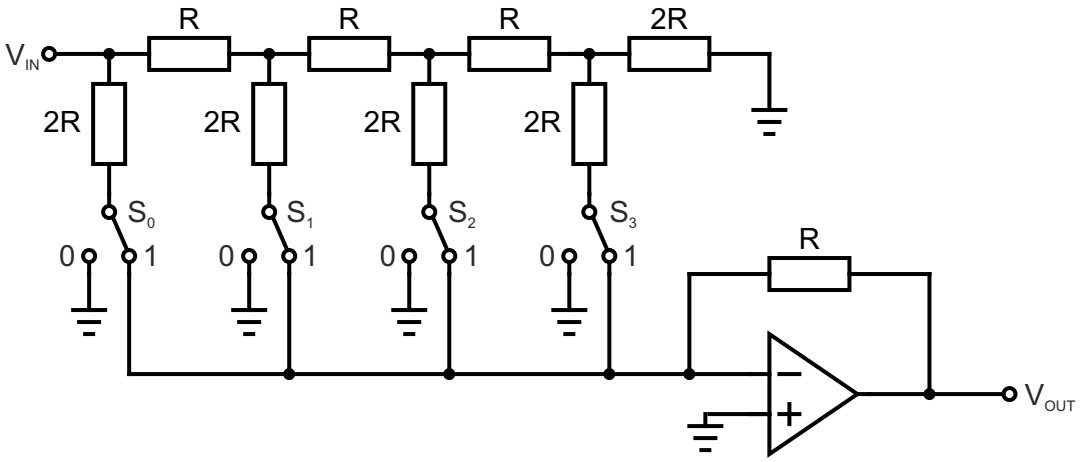
24. For the circuit below, calculate V_{OUT} as a function of time for an input voltage of $V_{\mathrm{IN}} = 0$ for $t < 0$ and $V_{\mathrm{IN}} = 1\,\mathrm{V}$ for $t \geq 0$. When $V_1 = 10\,\mathrm{V}$, $R = 100\,\mathrm{k\Omega}$, $C = 10\,\mu\mathrm{F}$, sketch the value of V_{OUT} against time. What happens as $t \to \infty$?



25. For the circuit below, calculate the ratio $|V_{\mathrm{OUT}}/V_{\mathrm{IN}}|$ as a function of R, C and the angular frequency ω. At what frequency does $|V_{\mathrm{OUT}}/V_{\mathrm{IN}}| = 1/\sqrt{2}$? What are the benefits of an active filter such as this, over a simple RC filter?



26. Show that the circuit below functions as a DAC (digital to analogue converter) by calculating the output voltage V_{OUT} as a function of V_{IN} and the switch settings S_0, S_1, S_2, and S_3 (which can have a value of 0 or 1).



When $V_{\mathrm{IN}} = 10\,\mathrm{V}$, what is the range of V_{OUT}? Design a simple circuit to rescale the output to the range 0 to $+5\,\mathrm{V}$.