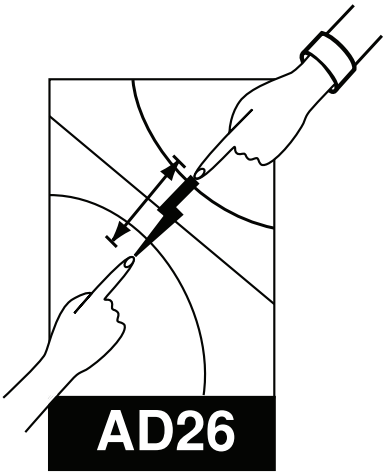
# Error guide in first-year Electronics



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## 1 Introduction

AD02 — *Estimating Experimental Errors*[1] gives a good description of error handling for labs and should be studied. This simplified treatment is for first-year students with little or no experience using errors. It is a "minimum knowledge" on errors and combining errors for first-year electronics labs.

## 2 Summary


- Nomenclature: A true value $A_0$ is measured with result $A$ and error $\pm\sigma_A$
- Error types: "Random" (aka "statistical", "reading"); "systematic" (aka "calibration"); tolerance.
- Combining random errors:

$$C = A \pm B \implies {\sigma_C}^2 = {\sigma_A}^2 + {\sigma_B}^2$$

$$C = A \times or \div B \implies \left(\frac{\sigma_C}{C}\right)^2 = \left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2$$

$$y = F(x) \implies \sigma_y = \left.\frac{\partial y}{\partial x}\right|_{x_0} \sigma_x$$

- Results should take the form $A \pm \sigma_{A1}(stat) \pm \sigma_{A2}(sys)$
- Truncation errors are: $\pm\sigma = \pm\frac{LSB}{\sqrt{12}}$


## 3 Content

A measurement result, given without an error or an implied error, is of little value. The distance to the Sun can be correctly stated as 5 km if the error is ten light minutes. It would not be a correct statement if the error was 1 cm. All measurement results ***must*** be stated with their errors.

An experimental result often requires measuring multiple quantities and computing the answer. The individual measurement errors will in general be correlated with each other, so that a proper treatment will involve a matrix that expresses correlations. For the first-year electronics practical labs a simplified methodology is sufficient. For those doing the labs in Michaelmas, simply applying the results above (and below) will be enough. Those undertaking the labs in Hilary should understand more details.

[1] `www-teaching.physics.ox.ac.uk/practical_course/Admin/AD02.pdf`


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# 4 Types of ”errors”

There are different kinds of inaccuracies accrued in experimental measurements. In this context they will be divided into:

- random errors (”reading errors” or ”statistical errors”);
- systematic errors (”calibration errors”);
- tolerances.

There are other kinds of errors, but those above with the definitions below, are sufficient for the electronics labs. Note that errors in $A$ are usually quoted either as ”absolute”, $\pm\sigma$, or as fractional, $\delta = \pm\frac{\sigma}{A}$. Fractional errors may also be expressed as percentage errors by simply multiplying them by 100.

## 4.1 Random errors

If a measurement is repeated many times and the results plotted in a histogram, the resulting distribution can be fitted to a curve as shown in Figure 1. If the curve is Gaussian,

$$N = N_0 e^{-(x-x_0)^2/2\sigma^2} \tag{1}$$

then the measurement can be said to have a random error of size $\pm\sigma$, where $\sigma$ is a parameter in the fit, $x_0$ is the true value, and $N_0$ is a normalisation constant. This is also called a ”normal distribution” or a ”bell curve”. The importance of this particular distribution is that the ”central limits theorem” shows that in many situations when independent random variables are added the sum tends to this distribution, even if the variables individually come from a range of other distributions.

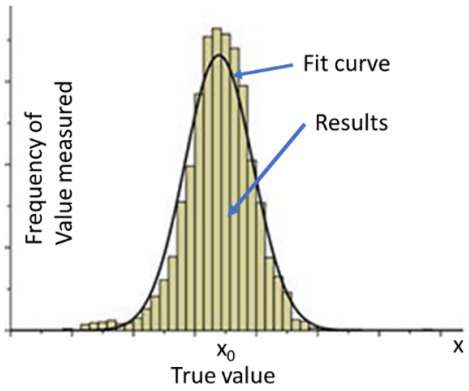



Figure 1: Fitted curve to a histogram of data.

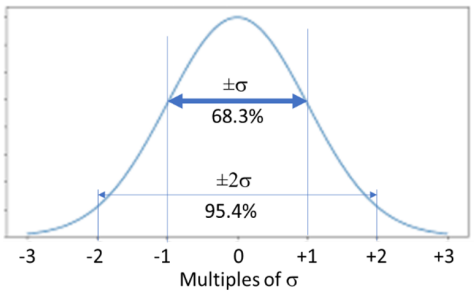



Figure 2: Gaussian distribution showing one and two standard deviations.

Figure 2 shows the meaning of the ‘error’ when it is a random error: it is the standard deviation of the Gaussian distribution. 68% of the time a measurement will result in an answer which is within $\pm 1$ standard deviation of the correct answer. Note that this means that 32% of the time it will not be. If many measurements are made and they always agree within errors, then probably the error has been overestimated. Importantly, $\sigma$ is also the root mean square, RMS, for the distribution.

A random error is the kind that occurs when reading a needle on a dial, or the truncation error from a digital reading. The error associated with truncation for an instrument is taken as the RMS error, which is the least significant bit in the reading (LSB) divided by $\sqrt{12}$.

This is because $\left[\frac{1}{\alpha}\int_{A-\frac{\alpha}{2}}^{A+\frac{\alpha}{2}}(A-x)^2dx\right]^{\frac{1}{2}} = \frac{\alpha}{\sqrt{12}}$, where $\alpha$ is the LSB.

## 4.2 Systematic errors

Measurements are usually made using an instrument, from a simple ruler to much more complex devices. Suppose a 1 m metal ruler is calibrated at 25 °C; if the ruler is naïvely used in the Arctic at -25 °C, it will have contracted by 0.5 mm. Every measurement made with the ruler would be 0.05% too small. This is a systematic error because every measurement is wrong in the same way. In this case the effect is known and can be corrected for, but if the ruler were calibrated at an unknown temperature between -50 °C and +50 °C, an error of 0.1% would have to be ascribed to any set of measurements made. The value of the error is not known, but it is the same for every measurement (at a given temperature).

This kind of error is very different to a random error, and in particular will behave differently when combined or used in a calculation. When reporting the errors on a measurement it is best to keep systematic and random errors separate. So the result of a measurement will be, for example,
$27.3 \pm 0.2(\text{stat}) \pm 0.3(\text{sys})$ . It is usual to refer to the random errors as "(stat)", which is short for "statistical", and to the systematic errors as "(sys)".

A multiplicative systematic error[2] is commonly called a "calibration error". A simple offset, where a measurement has an unknown constant added, is sometimes called a "pedestal" (because the true value "sits on top").

A common practice is to treat systematic errors as if they come from a normal distribution. This allows "easy" manipulations, but is not strictly correct for small numbers of variables.

## 4.3 Tolerances

A tolerance is a statement about the accuracy of a nominal value. For example if a resistor is sold as 10 ohm, it will also have a tolerance quoted. The tolerance is a statement about the maximum possible deviation from the nominal value. So if the tolerance is 5% a particular 10 ohm resistor will never be larger than 10.5 Ω or smaller than 9.5 Ω. This is different to random error where the equivalent statement is only that 66% will be inside ±5%.

Note that the tolerance for a component will have a limited regime of validity. For example, for a resistor there will be a limited temperature range within which the tolerance applies. In practice if many resistors with a 10 Ω±5% value are measured, they will cluster strongly at one value, which will depend on the temperature. At 25 °C they may vary between 9.95 Ω and 10.05 Ω, and at 50 °C between 10.3 Ω and 10.4 Ω. At all allowed temperatures the value will be within the tolerance, but at any one temperature the values will cluster more closely.

The reason tolerances are specified for components is that an engineer will need to know a design will work, not that it will work 66% of the time.

[2] When $A \cong A_0(1 \pm \frac{\delta}{100})$ , where $\delta$ is the percentage error.

# 5 Combining errors

The outcome of an experiment often requires the combining of multiple measurements. It also occurs frequently that measured values need to be used in a function. Therefore, it is necessary to know how to propagate errors through functions, including combining multiple measurements each with its own errors.

## 5.1 Combining random errors

Without explanation[3], for three random, independent, variables $x$, $y$, $z$ with errors of $\sigma_x$, $\sigma_y$, $\sigma_z$, it can be stated that if $C = F(x, y, z)$ then

$$\sigma_C{}^2 = \left(\left.\frac{\partial F}{\partial x}\right|_{x_0} \sigma_x\right)^2 + \left(\left.\frac{\partial F}{\partial y}\right|_{y_0} \sigma_y\right)^2 + \left(\left.\frac{\partial F}{\partial z}\right|_{z_0} \sigma_z\right)^2 \tag{2}$$

where $x_0$, $y_0$, $z_0$ are the true values and are approximated with the measured values for $x$, $y$, and $z$. In Michaelmas use of partial derivatives is not expected, but by Hilary it is. This formula generalises in the obvious way to any number of variables.

From this formula some special cases emerge:

$$C = \quad A \pm B \quad \Longrightarrow \quad \sigma_C{}^2 = \sigma_A{}^2 + \sigma_B{}^2 \text{ (called "adding in quadrature")} \tag{3}$$

$$C = \left\{\begin{matrix} A \times B \\ A \div B \end{matrix}\right\} \Longrightarrow \left(\frac{\sigma_C}{C}\right)^2 = \left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 \tag{4}$$

$$y = F(x) \qquad \Longrightarrow \quad \sigma_y = \left.\frac{\partial y}{\partial x}\right|_{x_0} \sigma_x \tag{5}$$

In practice it is often easier to use the special case formulae above in steps to work out a more complicated case. For example, if $F(x, y) = \sqrt{(x^2 + y^2)}$, then work out the errors in $x^2$ and $y^2$ separately first ($\sigma_{y^2} = 2\sigma_y$), followed by the error in their sum, and then propagate the resulting single error through the root.

## 5.2 Combining systematic errors

In this case the process of combining errors depends on the way the variables are being combined, and the type of systematic error. For calibration errors, as above, when $\delta$ is the percentage error and $A$ and $B$ are measured with the same instrument, $A \cong A_0(1 \pm \frac{\delta}{100})$, then $C = \frac{A}{B} = \frac{(A_0)(1\pm\delta)/100)}{(B_0)(1\pm\delta)/100)} = \frac{A_0}{B_0} = C_0$, so there is no systematic error in the ratio. Similarly the percentage systematic error in the sum or difference is identical to the individual measurements, and in the product it is (for small errors) twice the size.

Basically, systematic errors need to be evaluated on a case by case basis.

## 5.3 Combining tolerances

For the purposes of the electronics labs, if a calculation involves just tolerances then the extreme limits can be computed and a final "tolerance" assigned on that basis (so, for example, adding two 5% resistors in series results in a tolerance on the sum also of 5%). If, however, they are being combined with other kinds of errors, they should be treated as if they were random. This is not actually correct, but it is the best "simple" approach.

[3]See the first-year course on errors, and AD02 — *Estimating Experimental Errors*.