A10288H1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Wednesday 3rd September 2025, 09:30am – 12 noon

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. A thin equiconvex lens (a lens where the two convex surfaces have the same radius of curvature) is made of glass with refractive index n. Its spherical surfaces have radius of curvature R. Show that its focal length f is given by

$$f = \frac{R}{2(n-1)}\,.$$

[5]

An imaging system suffers from chromatic aberration so that an image in blue light is formed at a position 10 mm beyond an image in red light. A student wishes to correct for the aberration and form a corrected image on a screen 1 m from the red light image using a thin equiconvex lens placed between the aberrated images and the screen. Find expressions for the required focal lengths of the lens f_b and f_r at blue and red wavelengths. The lens has refractive indices $n_\mathrm{b} = 1.44$ and $n_\mathrm{r} = 1.40$ at blue and red wavelengths respectively. Calculate the radius of curvature of the lens surfaces and the focal lengths at blue and red wavelengths. [5]

2. A long solenoid of length L and radius R is uniformly wound with N_1 closely spaced turns. It is tightly surrounded about the middle by a short coil closely wound with N_2 closely spaced turns on the same axis. What is the mutual inductance of the coils? [4]

3. A hollow conducting sphere of radius R has an initial charge Q. By consideration of Gauss' law, find an expression for the electric field strength and for the potential inside and outside the sphere as a function of r, the distance from the centre of the sphere.

A second hollow conducting sphere of radius $2R$ and with no initial charge is brought into contact with the first sphere. Find expressions for the final charge on each sphere in this arrangement. [7]

4. Draw simple circuit diagrams in which an operational amplifier is used as (a) an integrator and (b) a differentiator. In each case find an equation for how the output voltage V_out depends on the input voltage V_in and any resistors and/or capacitors used in your circuit. [6]

5. Consider a circuit comprising, in series, a resistor of resistance R, a capacitor of capacitance C, an inductor of inductance L and a switch. At time $t < 0$ the switch is open and the capacitor carries charge Q_0. At time $t = 0$ the switch is closed. Determine the condition for which the subsequent current in the circuit will be oscillatory. [6]

6. A slit of width w is illuminated at normal angle by a collimated beam of coherent light of wavelength λ. Derive the angular intensity distribution in the diffraction pattern and hence determine the Rayleigh (diffraction) limit for a single slit. [7]

Section B

7. (a) Describe, using a clearly labelled diagram, the essential features of a grating spectrograph. [6]

(b) A grating of ruling spacing d is used in a spectrograph at normal incidence. Show that the angular dispersion of the diffracted beam is

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = \frac{\tan\theta}{\lambda}$$

where θ is the angle of diffraction. [6]

(c) An astronomer wishes to use a diffraction grating to disperse collimated light collected from a distant nebula comprising gases including helium, to obtain a spectrum that includes both the helium emission lines at 587.6 nm and at 667.8 nm. The astronomer is in possession of a lens of focal length of 85 mm, and a detector size that is 35 mm along its maximum dimension. Describe a setup whereby these could be used to obtain the desired spectrum, specifying the grating spacing that would be needed to simultaneously detect both of these two emission lines within a single spectrum. [8]

8. (a) A long insulating cylinder of radius R_1 has a cylindrical hole parallel to the axis of the cylinder. The radius of the hole is R_2, and the distance from the centre of the cylinder to the centre of the hole is a. There is a uniform fixed charge per unit volume ρ throughout the cylinder, except in the hole where there is no charge. Using a cylindrical coordinate system with the z axis along the centre of the cylinder, evaluate the components of the electric field inside the cylinder and inside the hole (you may neglect edge effects at the ends of the cylinder). [8]

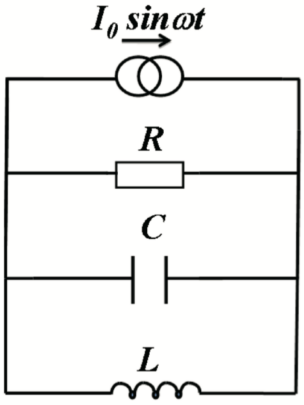
(b) Consider a metallic sphere carrying a charge Q, placed a distance R above an infinite grounded conducting plane. Evaluate the electric field in the region of space outside the sphere and above the conducting plane. [6]

(c) Hence determine the induced charge density on the plane as a function of the distance r from a point on the plane to the point on the plane that is directly below the sphere. [3]

(d) Show that the total induced charge on the conducting plane is $-Q$. [3]

9. (a) Define what is meant by capacitance in terms of charge stored and voltage applied. Hence show that the complex impedance of a capacitor of capacitance C, at an angular frequency ω, is given by $1/(j\omega C)$. Similarly define inductance in terms of voltage V and current I and use this to show that the complex impedance of an inductor of inductance L, at an angular frequency ω, is given by $j\omega L$. [6]

(b) The circuit shown in the figure below is driven by an alternating current source with amplitude I_0 and angular frequency ω. Show that as ω is varied, the maximum amplitude of the current through the resistor I_{R} is I_0, and find an expression for the corresponding angular frequency ω_0. [6]



(c) Sketch a graph of I_{R} versus ω. Let ω_+ and ω_- be the larger and smaller values of ω for which ${I_R}^2 = {I_0}^2$ and let $\delta\omega = \omega_+ - \omega_-$. Find an approximate expression for $\delta\omega$, assuming that $RC/L \gg 1$. Suggest a possible application for this circuit. [8]

10. (a) Show that, for a thin convex lens of focal length f, an object at a distance u from the lens will produce a real image at a distance u from the lens where

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

[6]

(b) If the distance between the object and its real image is held fixed, show that it is possible for the lens to be placed in two positions, provided that $u + v > 4f$. [9]

(c) Derive two expressions for the height of the object H, in terms of $d = u + v$, f, with (i) the height of the image I_1 with the lens in one position and (ii) the height of the image I_2 with the lens in the other position. [10]