A10288H1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday 10 June 2025, 9.30 pm – 12.00 pm

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

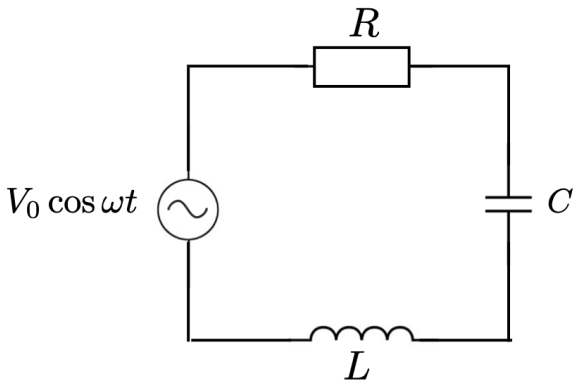
Section A

1. The bottom surface of a thundercloud is 1 km above the Earth's surface and has an area of $1\,\mathrm{km}^2$. Estimate the electrostatic energy stored if it is just on the point of generating lightning. You may assume that the electric field strength required to cause an electrical arc in air is $3\,\mathrm{MV\,m^{-1}}$. If the resulting lightning bolt were to completely discharge the cloud and if it were to persist for $100\,\mu$s, what current would flow through the bolt? State any assumptions you make. [6]

2. A wire forms a closed circular ring of enclosed area A, resistance R and self-inductance L. A uniform magnetic field is applied normal to the plane of the ring. The amplitude of the magnetic field, B, varies with time as $B = B_0 \cos \omega t$ where B_0 and ω are both constants. Find an expression for the voltage and the current induced in the ring due to the applied magnetic field. Find an expression for the power dissipated in the ring and its time-average. [6]

3. Consider a diffraction grating on which light is incident normally. When light of 589 nm wavelength passes through the grating, the transmitted light in the third order is observed at an angle of 30 degrees to the incident beam. How many lines per mm are ruled on the grating? What is the maximum order of light that can be observed? [6]

4. A resistor R, capacitor C, and inductor L are connected in series and a power source supplying a voltage $V = V_0 \cos \omega t$ is connected across them as shown in the diagram below



(i) Find the angular frequency $\omega_\circ$ such that the current through the circuit is a maximum. [5]

(ii) Find the phase difference between the current and the voltage when (a) $\omega = 0$, (b) $\omega = \omega_\circ$ and (c) $\omega = \infty$. [4]

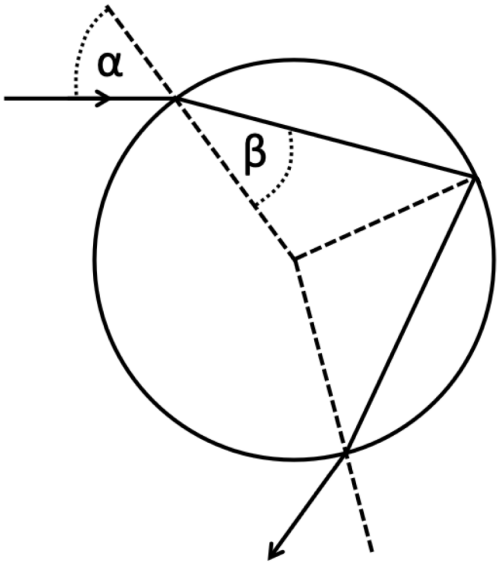
5. What is the relationship between voltage and current in an inductor? [2]

A solenoid of length D and radius a has n turns per unit length. Find an expression for its inductance. Justify any assumptions that you make. [4]

6. A cube is constructed by soldering one resistor of resistance R along each of its edges. Calculate the resistance between two adjacent corners. [7]

Section B

7. Consider a spherical droplet of water. Light is incident on this sphere at an angle α to the surface normal, and is refracted at an angle β, as shown in the diagram below.



Calculate the total angle δ through which the incident light has been deviated, assuming it has experienced a single internal reflection. [6]

Show that the stationary value of δ as a function of α occurs when

$$\frac{\cos\,\beta}{\cos\,\alpha} = \frac{m}{n},$$

where n is the refractive index of water and m is an integer. State the value of m. [7]

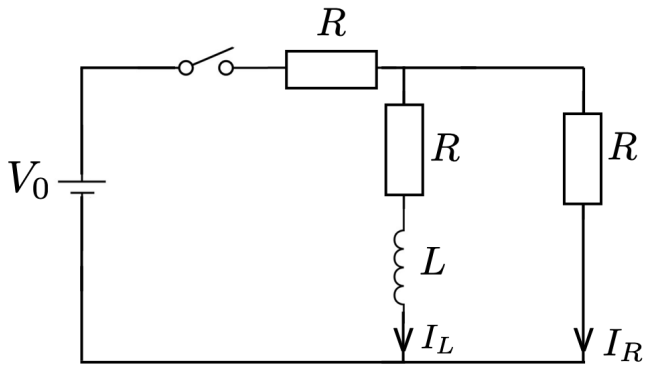
Hence show that when δ is stationary

$$\sin\,\beta = f(n),$$

and determine $f(n)$ which depends only on n, [5]

Hence describe how rainbows can form, and why a "double rainbow" is sometimes observed. [2]

8. Consider the circuit shown in the diagram below. At time $t = 0$, the switch is closed to complete the circuit.



(i) Find a differential equation for the current through the inductor, I_{L}, and show that this leads to a solution of the form

$$I_{\mathrm{L}}(t) = I_{\mathrm{L}}(\infty)\left(1 - \mathrm{e}^{-t/\tau}\right).$$

[9]

(ii) Find $I_{\mathrm{L}}(\infty)$ and τ in terms of V, L and R. [3]

(iii) Draw a labelled sketch of the currents $I_{\mathrm{L}}(t)$ and $I_{\mathrm{R}}(t)$ as a function of t, clearly indicating the values at $t = 0$ and $t \gg \tau$. [5]

(iv) Determine the energy stored in the circuit (excluding the battery) as a function of time. [3]

9. A uniformly charged ring having total charge $+q$ lies in the $z = 0$ plane and its shape is defined by $x^2 + y^2 = a^2$.

(i) Show that the electric field along the axis of the ring is

$$E_z(0, 0, z) = \frac{q}{4\pi\epsilon_0} \frac{z}{(a^2 + z^2)^{3/2}}.$$

[4]

(ii) Sketch $E_z(0, 0, z)$ as a function of z, labelling the positions of any turning points. [4]

A second uniformly charged ring, also defined by $x^2 + y^2 = a^2$ and with total charge $+q$, is placed in the $z = b$ plane. You may assume that $b \gg a$.

(iii) Sketch $E_z(0, 0, z)$ for this case. [3]

(iv) Suppose that the motion of particles is confined to the z axis. It is then possible to stabilise either a proton or an electron at the positions $(0, 0, 0)$, $(0, 0, b/2)$, and $(0, 0, b)$. Determine which particle is stable at each position. [3]

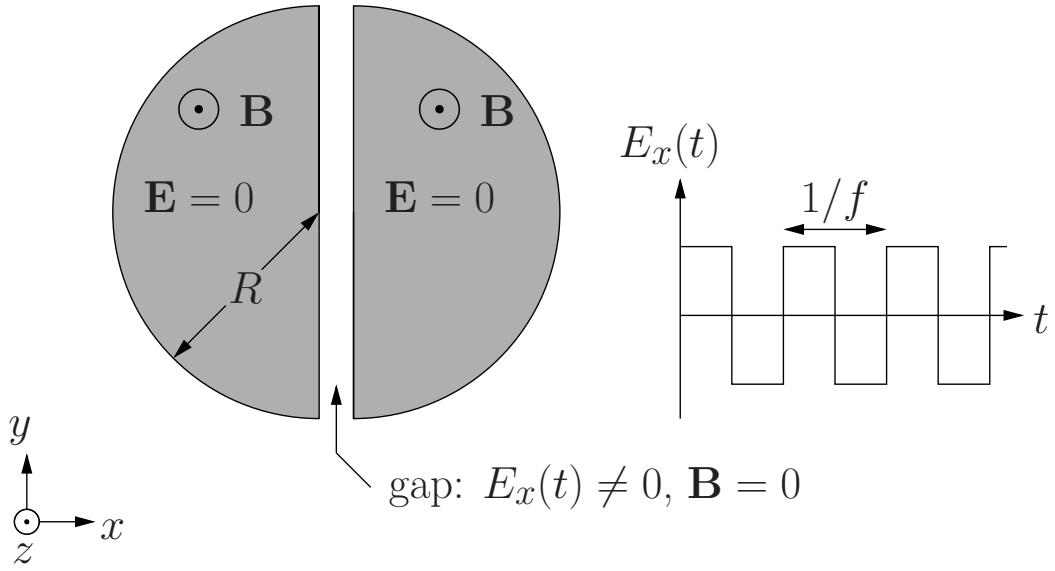
(v) If a particle is stabilised at one of these positions, but then very slightly displaced in the z-direction (by an amount $\delta \ll a$), show that it will execute simple harmonic motion, and determine the oscillation frequency at each of the three positions. [6]

10. A particle of charge q and mass m moves with velocity $\mathbf{v}$ in a region of space with electric field $\mathbf{E}$ and magnetic field strength $\mathbf{B}$.

(a) Write down an expression for the force $\mathbf{F}$ on the particle. If $\mathbf{E} = 0$ and $\mathbf{B}$ is time independent, explain whether it is possible to increase the kinetic energy of the charged particle. [4]

(b) Show that this particle, placed in a region of uniform magnetic field strength $\mathbf{B}$ and $\mathbf{E} = 0$ can undergo circular motion and determine the angular frequency ω and the radius of the circle r. [4]

The diagram below shows the main features of a *cyclotron*, a machine for accelerating charged particles. The particles can be considered to move within a thin disc of radius R which lies in the xy plane. Whenever the particles are in the shaded region of the diagram, they experience a uniform magnetic field $\mathbf{B}$, aligned along the z-direction, perpendicular to the disc, but zero electric field. Whenever a particle passes through the narrow gap between the two shaded halves of the cyclotron, it experiences no magnetic field but an electric field $\mathbf{E}$ aligned in the x-direction and which is driven by a square wave oscillator, so that it changes direction back and forth. The square-wave oscillator has frequency f.



(c) A proton is placed at rest, right in the centre of the cyclotron and inside the narrow gap.

(i) Describe its subsequent motion.

(ii) Calculate the maximum value of the proton velocity $v_{\max}$ and proton kinetic energy $T_{\max}$ that could be achieved in the cyclotron. [You may assume that the motion can be treated in the non-relativistic limit].

(iii) Evaluate $v_{\max}$ (as a fraction of the speed of light, so that you can check how reliable the non-relativistic assumption is) and $T_{\max}$ (in eV) for the case of protons in a cyclotron with $R = 1\,\mathrm{m}$ and $B = 0.3\,\mathrm{T}$. [10]

(d) In order to accelerate protons, what frequency f (in MHz) should be chosen for the square-wave oscillation of $E_x(t)$? [2]