Section A

① Treat as parallel plate capacitor:

$$V = 3\,MVm^{-1} \times 1\,km = 3 \times 10^9\ V$$

$$C = \frac{\varepsilon_0 A}{d} \quad \text{and energy stored} = \frac{1}{2}CV^2$$

$$= \frac{\varepsilon_0 \cdot 10^6}{10^3} \qquad \sim 40\ GJ$$

$$= 8.85 \times 10^{-9}\ F \qquad (4 \times 10^{10}\ J)$$

$$Q = CV \sim 27 \text{ Coulombs}$$

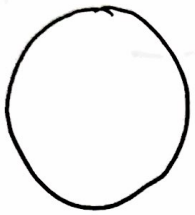
$$I = \frac{Q}{t} \sim \underline{\underline{0.3\ MA}}$$

$t \leftarrow 100\,\mu s$

(This is an estimate question and answers should not be given to high precision!)

[6]

(2)



$$\Phi = B_0 \cos \omega t \; A$$

$$V = -\dot{\Phi} = A B_0 \omega \sin \omega t$$

$$I = \frac{V}{R + i\omega L} = \frac{A B_0 \omega \sin \omega t}{R + i\omega L}$$

$$\text{Power} = |I|^2 R$$

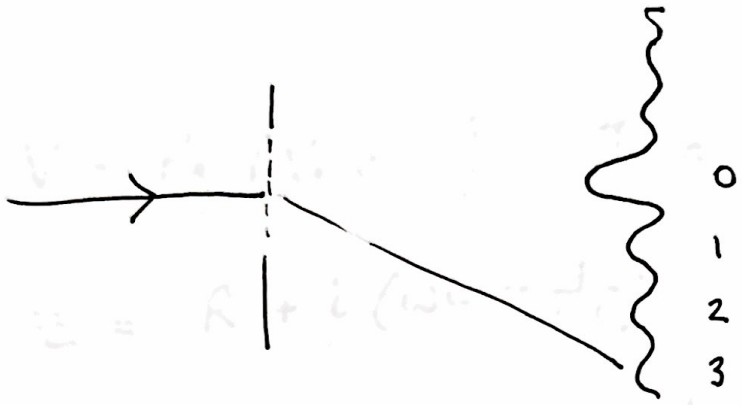
$$= \left(\frac{A^2 B_0^2 \omega^2 \sin^2 \omega t}{R^2 + \omega^2 L^2} \right) R$$

$\langle \text{Power} \rangle$ time-averaged

$$= \frac{R A^2 B_0^2 \omega^2 \langle \sin^2 \omega t \rangle}{R^2 + \omega^2 L^2}$$

$$= \frac{A^2 B_0^2 \omega^2 R}{2(R^2 + \omega^2 L^2)} \qquad \because \langle \sin^2 \omega t \rangle = \frac{1}{2}$$

③



①

$\lambda = 589\,nm$

$\theta = 30°$

$\frac{\pi}{180} \times 30$ rad

$n\lambda = d\sin\theta$

order number = 3

589nm

spacing of lines in nm

$$d = \frac{3 \times 589}{\sin(\pi/6)} \quad nm$$

$$= 3534\,nm$$

④

$\therefore$ Lpmm = 283

$$n_{max} = \frac{d \times \sin 90}{\lambda} = \frac{3534}{589} = 6$$ ①

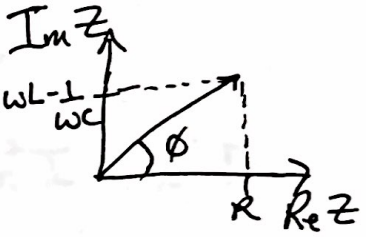
④ (i) $V = Re\left(V_0 e^{i\omega t}\right)$ $\quad I = Re\left(I_0 e^{i(\omega t+\phi)}\right)$

$$Z = R + i\left(\omega L - \frac{1}{\omega C}\right)$$

$$|I| = \frac{V_0}{|Z|} = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}}$$

maximised when $\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$ [5]

(ii) $\tan\phi = \frac{\frac{1}{\omega C} - \omega L}{R}$ [1]



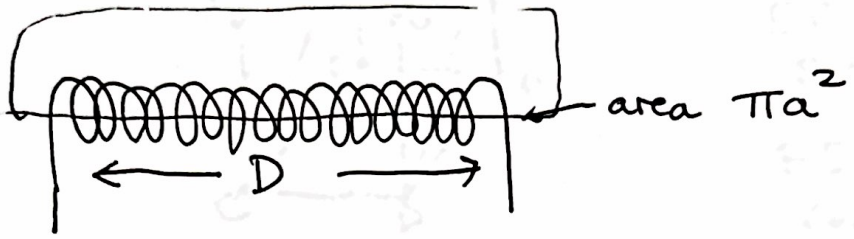
a) when $\omega = 0$, $\phi \to \frac{\pi}{2}$

b) $\omega = \omega_0$, $\phi = 0$

c) when $\omega = \infty$, $\phi = \frac{-\pi}{2}$

5.

$$V = L \frac{dI}{dt}$$ (2)



Ampère's theorem: $\oint \underline{B} \cdot d\underline{l} = \underbrace{nD}_{\text{number of turns}} I \mu_0$

(*) $B \cdot \cancel{D} = n \cancel{D} I \mu_0$

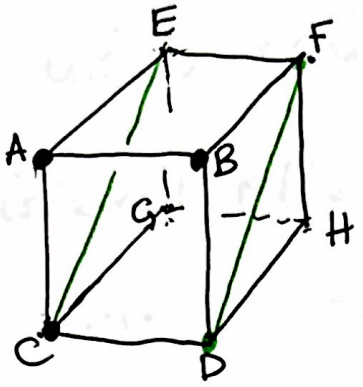
$$\Phi = B \underbrace{\pi a^2}_{\text{area}} nD = \mu_0 n^2 I \pi a^2 D$$

$\Rightarrow$ Inductance $L = \frac{\Phi}{I} = \mu_0 n^2 \pi a^2 D$ (3)

(*) Have <u>assumed</u> that $B = 0$ outside solenoid, i.e. ignored edge effects. (1)

[6]

(6)

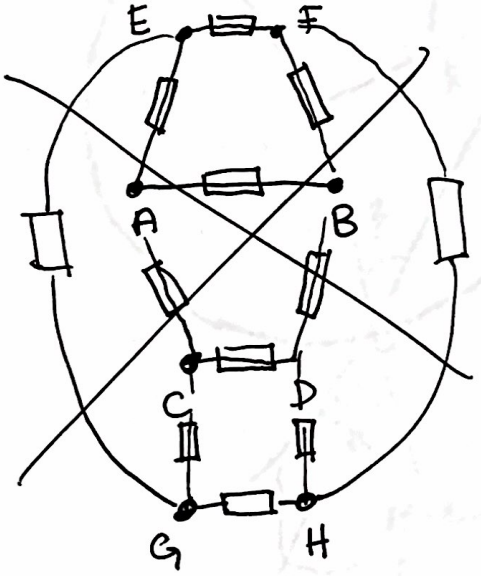


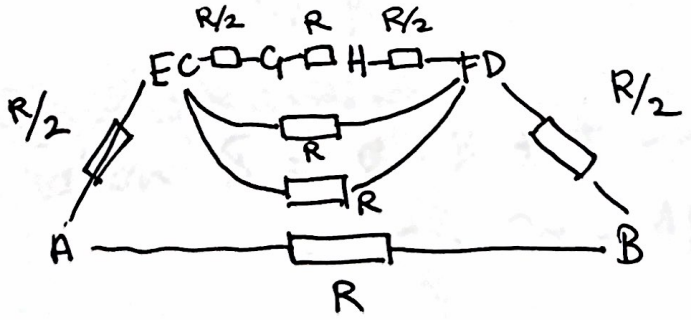
by symmetry,
EC are at same potential

FD are at same potential

$\Rightarrow$ Connect them.

$\Downarrow\Downarrow$





$$\Rightarrow R_{AB} = \left(\frac{1}{R} + \frac{1}{\frac{R}{2}+\frac{R}{2}+\left(\frac{1}{R}+\frac{1}{R}+\frac{1}{2R}\right)^{-1}}\right)^{-1}$$

$$= \left(\frac{1}{R} + \frac{1}{R+\frac{2R}{5}}\right)^{-1} = \frac{7}{12}R$$

Section B

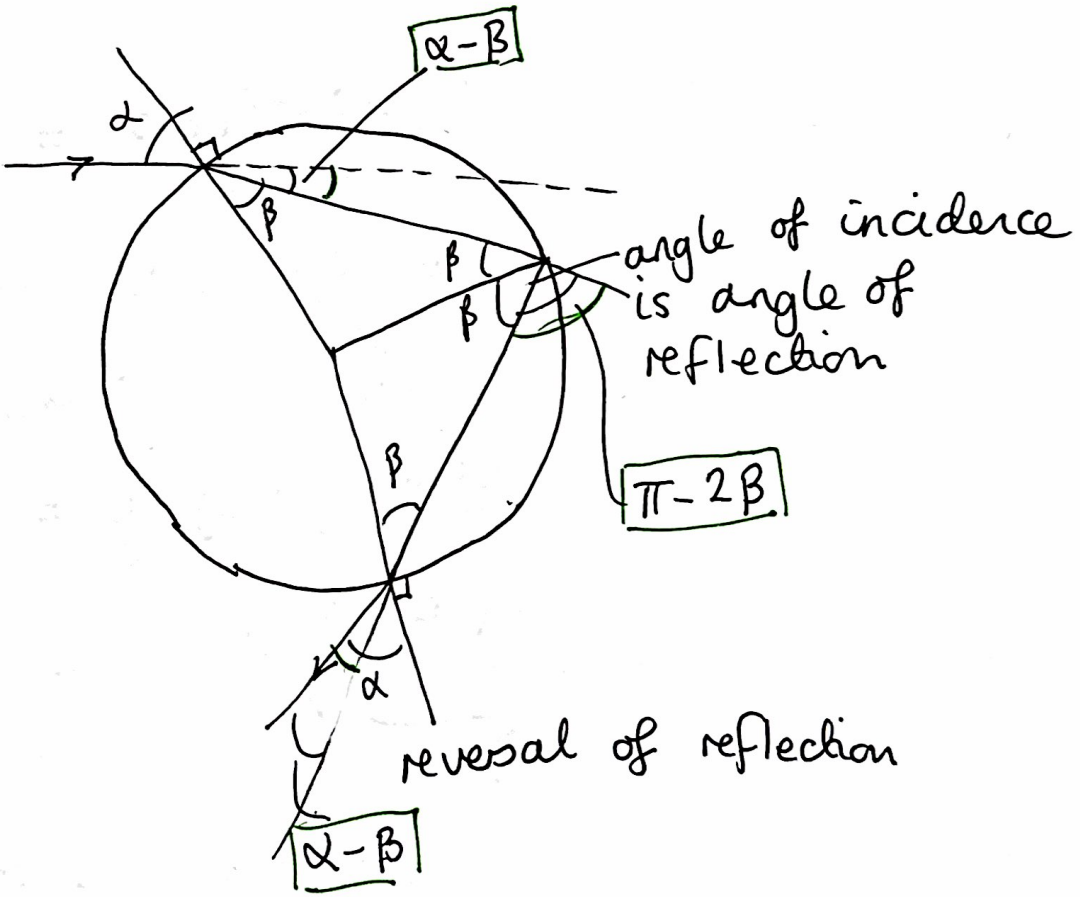
(7)

Traigles inside sphere are isoceles

single internal reflection

Snell's law: $\sin\alpha = n \sin\beta$

n ↑ refractive index



total deviation $\delta = \alpha - \beta + \pi - 2\beta + \alpha - \beta$

$= \pi + 2\alpha - 4\beta$ [6]

(1)

(7₂)

what is the stationary value of δ as a function of α?

$$\delta = \pi + 2\alpha - 4\beta$$

$$\delta = \pi + 2\alpha - 4\sin^{-1}\left(\frac{\sin\alpha}{n}\right)$$

$$\sin\frac{\alpha}{n} = \sin\left(\frac{\pi + 2\alpha - \delta}{4}\right)$$

$\frac{d}{d\alpha}$ ↴

$$\frac{\cos\alpha}{n} = \cos\left(\frac{\pi + 2\alpha - \delta}{4}\right) \times \left(\frac{1}{2} - \frac{1}{4}\frac{d\delta}{d\alpha}\right) \quad \text{——②}$$

stationary when $\frac{d\delta}{d\alpha} = 0$

insert δ from ① into ②

$$\frac{\cos\alpha}{n} = \frac{1}{2}\left(\cos\left(\frac{\pi + 2\alpha - \pi - 2\alpha + 4\beta}{4}\right)\right.$$

$$= \frac{1}{2}\cos\left(\underbrace{\sin^{-1}\left(\frac{\sin\alpha}{n}\right)}_{\beta}\right)$$

$$\Rightarrow \cos\alpha = \frac{1}{2}n\cos\beta$$

so $\frac{\cos\beta}{\cos\alpha} = \frac{2}{n}$ $\quad \therefore m = 2$

[7]

(7.3)

$$\sin\alpha = n\sin\beta \quad \text{—③}$$

$$\cos\alpha = \frac{1}{2}n\cos\beta \quad \text{—④}$$

Eliminate α:

$$\sin^2\alpha + \cos^2\alpha = 1 = n^2\sin^2\beta + \frac{1}{4}n^2\cos^2\beta$$

and use: $\sin^2\beta + \cos^2\beta = 1$

$$n^2\sin^2\beta + \frac{1}{4}n^2(1-\sin^2\beta) = 1$$

$$n^2\sin^2\beta - \frac{1}{4}n^2\sin^2\beta = 1 - \frac{1}{4}n^2$$

$$\Rightarrow \sin\beta = \left(\frac{1-\frac{1}{4}n^2}{n^2-\frac{1}{4}n^2}\right)^{1/2} = \left(\frac{4-n^2}{3n^2}\right)^{1/2} \quad [5]$$

The refractive index n depends on the wavelength of the light that is (reflected and) refracted by the raindrop therefore the minimum deflection angle is also dependent on wavelength.

Light will be dispersed from the sun by raindrops; double internal reflection → double rainbow

[2]

(8.)

$$V_0 = (I_L + I_R)R + I_R R \quad \text{——①}$$

$$I_L R + \dot{I}_L L = I_R R \quad \text{——②}$$

$$① \Rightarrow V_0 = I_L R + \underbrace{2I_R R}_{②}$$

$$= I_L R + 2I_L R + 2\dot{I}_L L$$

$$\Rightarrow V_0 = 3RI_L + 2L\frac{dI_L}{dt} \qquad [5]$$

$$\frac{dI_L}{dt} = \frac{V_0 - 3RI_L}{2L} = -\frac{(I_L - V_0/3R)}{2L/3R}$$

$$\frac{d\left(I_L - \frac{V_0}{3R}\right)}{\left(I_L - \frac{V_0}{3R}\right)} = -\frac{3R}{2L}\,dt$$

$$\Rightarrow \left[\ln\left(I_L - \frac{V_0}{3R}\right)\right]_{I_L=0}^{I_L(t)} = \left[-\frac{3R}{2L}t\right]_0^t$$

$$\Rightarrow \frac{\left(I_L - \frac{V_0}{3R}\right)}{-V_0/3R} = e^{-3R/2L\,t}$$

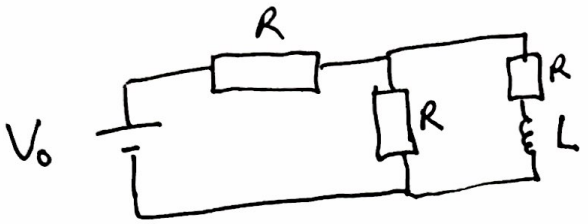
$$\Rightarrow I_L(t) = \underbrace{\frac{V_0}{3R}}_{I_L(\infty)}\left[1 - e^{-t/\tau}\right] \qquad [4]$$

$$\tau = \frac{2L}{3R} \qquad [3]$$

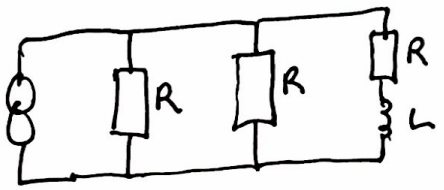
(82)

Alternative method:

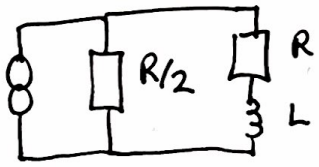
(Therevin equivalents)



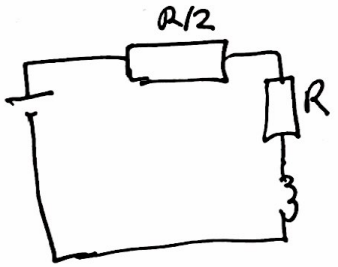
$\equiv \frac{V_0}{R}$



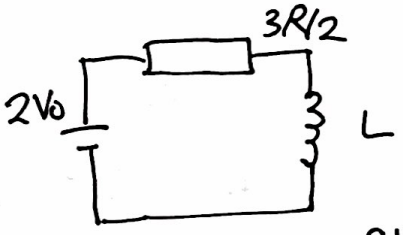
$\equiv \frac{V_0}{R}$



$\equiv 2V_0$



$\equiv$



gives the same time constant

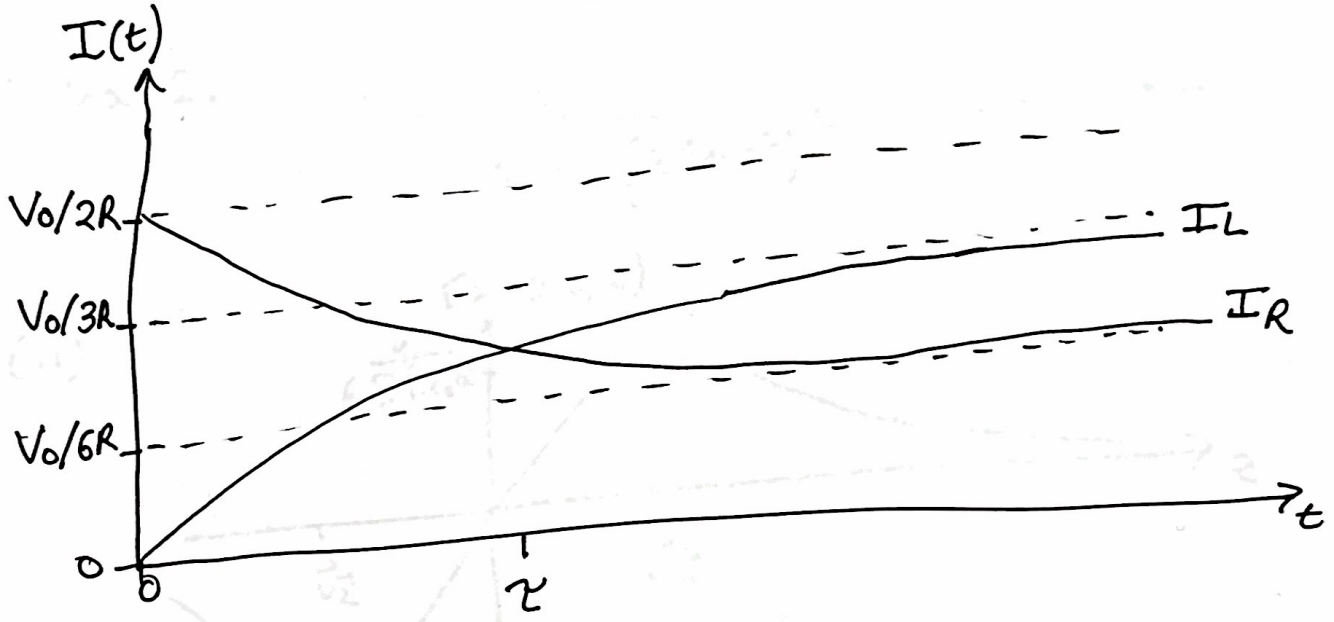
(8₃)

(iii) $V_0 = I_L R + 2I_R R \Rightarrow I_R = \frac{1}{2R}(V_0 - I_L R)$

$$I_R = \frac{V_0}{2R} - I_L$$

$$\Rightarrow I_R(t) = \frac{V_0}{6R} + \frac{V_0}{3R} e^{-t/\tau}$$

$$= \frac{V_0}{6R}\left(1 + 2e^{-t/\tau}\right)$$



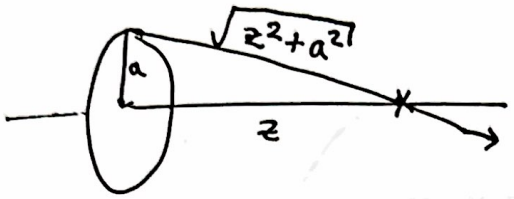
[5]

(iv) Energy stored is

$$\frac{1}{2} L I_L^2(t) = \frac{V_0^2 L}{18R^2}\left(1 - e^{-t/\tau}\right)^2$$

[3]

(9)



Method 1:

$$E_z = \frac{q}{4\pi\varepsilon_0 (z^2+a^2)} \underbrace{\frac{z}{\sqrt{z^2+a^2}}}_{\cos\theta}$$

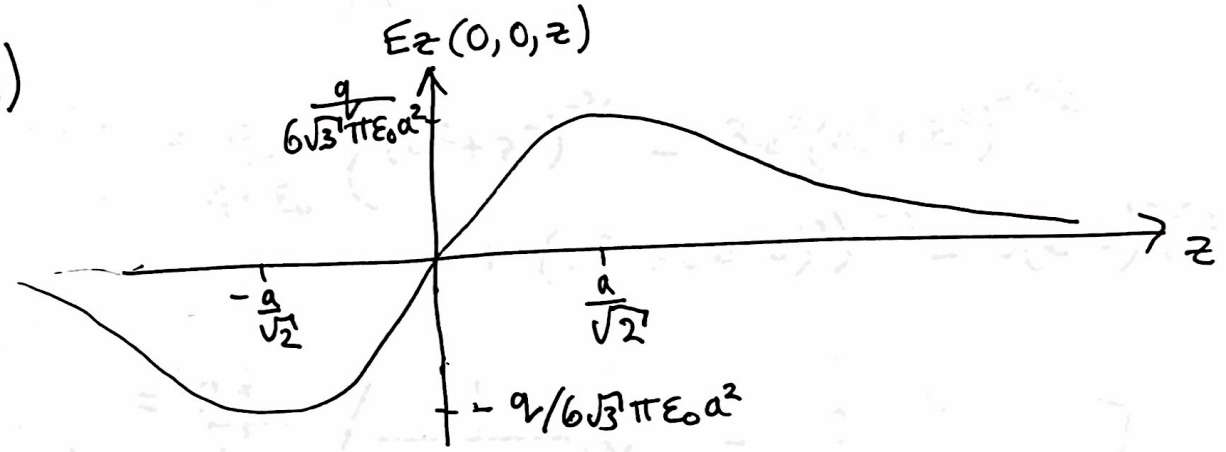
by symmetry, perpendicular components cancel.

Method 2:

$$\phi = \frac{q}{4\pi\varepsilon_0 (z^2+a^2)^{1/2}} \quad \text{and} \quad E_z = -\frac{\partial \phi}{\partial z}$$

[4]

(ii)



Stationary points when $\frac{d}{dz}\left(\frac{z}{(z^2+a^2)^{3/2}}\right) = 0$

$\frac{v\,du - u\,dv}{v^2}$

$$\frac{(z^2+a^2)^{3/2} \cdot 1 - z\,\frac{3}{2}(z^2+a^2)^{1/2} \cdot 2z}{(z^2+a^2)^3} = 0$$

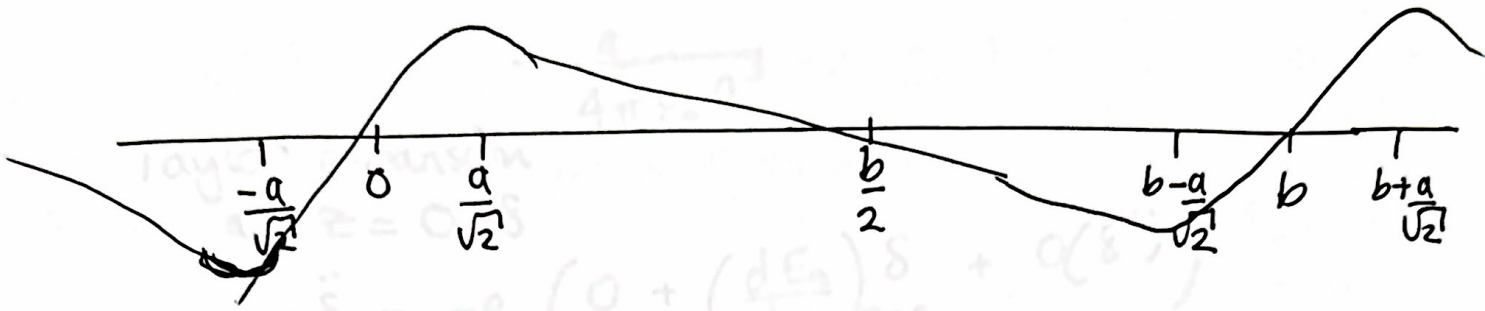
$$(z^2+a^2)^{1/2}\left[z^2 + a^2 - 3z^2\right] = 0$$

$$a^2 - 2z^2 = 0$$

$$\Rightarrow z = \pm\frac{a}{\sqrt{2}}$$

q_2

(iii)



(iv) $(0,0,0)$ electron is stable

$(0,0,\frac{b}{2})$ proton is stable

$(0,0,b)$ electron is stable

$$\frac{dE_z}{dz} = \frac{q}{4\pi\varepsilon_0}\left((a^2+z^2)^{-3/2} - 3z^2(a^2+z^2)^{-5/2} + (a^2+(z-b)^2)^{-3/2} - 3(z-b)^2(a^2+(z-b)^2)^{-5/2}\right)$$

$$\left(\frac{dE_z}{dz}\right)_{z=\frac{b}{2}} = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{(a^2+(\frac{b}{2})^2)^{3/2}} - \frac{3(\frac{b}{2})^2}{(a^2+(\frac{b}{2})^2)^{5/2}}\right]$$

$$= \frac{2q}{4\pi\varepsilon_0}\left[\frac{a^2+(\frac{b}{2})^2 - 3(b/2)^2}{(a^2+(\frac{b}{2})^2)^{5/2}}\right]$$

$$= \frac{2q}{4\pi\varepsilon_0}\left[\frac{a^2 - 2(b/2)^2}{(a^2+(\frac{b}{2})^2)^{5/2}}\right]$$

if $b \gg a$ then

$$\approx -\frac{4q}{4\pi\varepsilon_0}\frac{b^2/4}{b^5/4\sqrt{2}} = -\frac{\sqrt{2}q}{\pi\varepsilon_0 b^3}$$

$$\Rightarrow m\ddot{\delta} = -\frac{e\sqrt{2}q}{\pi\varepsilon_0 b^3}\delta \quad \Rightarrow (2\pi\nu)^2 = \frac{e\sqrt{2}q}{\pi\varepsilon_0 b^3 m}$$

$$\Rightarrow \nu^2 = \sqrt{2}eq/4\pi^3\varepsilon_0 b^3 m$$

(9₃)

$$\left(\frac{dE_z}{dz}\right)_{z=0} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{a^3} - 0 + \text{small terms}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 a^3}$$

Taylor expansion at $z = 0+\delta$

$$m\ddot{\delta} = -e\left(0 + \left(\frac{dE_z}{dz}\right)_{z=0}\delta + O(\delta^2)\right)$$

$$= -\frac{eq\delta}{4\pi\varepsilon_0 a^3} + O(\delta^2)$$

$$\Rightarrow \text{SHM with } \omega^2 = (2\pi\nu)^2 = \frac{eq}{4\pi\varepsilon_0 a^3}$$

$$\Rightarrow \nu = \left(\frac{eq}{16\pi^3\varepsilon_0 a^3}\right)^{1/2}$$

by analogy, same at $z = b$.

(10) $\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$ Lorentz force law (2)

$\underline{E} = 0 \Rightarrow \underline{F} = q\underline{v} \times \underline{B} \Rightarrow \underline{F} \perp \underline{v}$

$\Rightarrow$ no work done $\Rightarrow$ no increase in KE (2)

[4]

$\underline{E} = 0$, $\underline{B}$ uniform

either method 1 or method 2 acceptable

method 1: without loss of generality $\underline{B} \parallel z$

$$\Rightarrow \quad m\dot{v}_z = 0$$
$$m\dot{v}_x = qv_yB$$
$$m\dot{v}_y = -qv_xB$$

$$\Rightarrow \quad \ddot{v}_x + \omega^2 v_x = 0 \quad \text{with} \quad \omega = \frac{qB}{m}$$

solutions $v_x(t) = v_0 \cos(\omega t + \phi)$

$v_y(y) = v_0 \sin(\omega t + \phi)$

$\Rightarrow$ circular motion $r = \frac{v}{\omega} = \frac{mv}{qB}$

method 2: centripetal force $\frac{mv^2}{r} = qvB$

always $\perp \underline{v}$

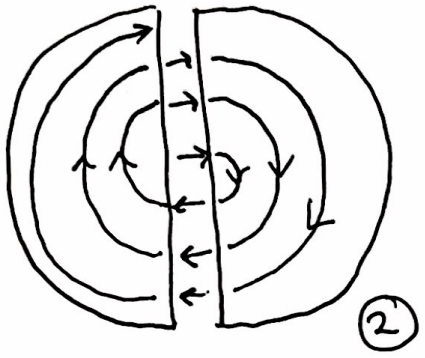
$\Rightarrow$ circular motion $r = \frac{mv}{qB}$

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

[4]

2 marks method
1 for $\omega = qB/m$
1 for $r = mv/qB$

Particles get a kick from E_x in gap, every time they cross gap $\Rightarrow$ spiral out



(2)

ω constant, so

$v = r\omega$ increased as

r goes from 0 to R

→ (2) marks for method and showing understanding

$$V_{max} = \frac{qBR}{m_p} = 2.87 \times 10^7 \text{ ms}^{-1} = 0.096\,c \quad (2)$$

$$T_{max} = \frac{1}{2} m_p V_{max}^2 = \frac{1}{2} \frac{R^2 q^2 B^2}{m_p} = 6.90 \times 10^{-13} \text{ J}$$

$$= 4.31 \text{ MeV} \quad (3)$$

[$V_{max} \ll c$ so non-relativistic OK] (1)

Choose f to match $\frac{\omega}{2\pi}$

$$\left(\omega = \frac{qB}{m} = 2.87 \times 10^7 \text{ rad s}^{-1}\right)$$

$$\Rightarrow \quad f = 4.57 \text{ MHz} \quad (2)$$

[10]