A10287H1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Thursday 4th September 2025, 9.30 am – 12.00 noon

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. (a) State the parallel axis and perpendicular axis theorems for moments of inertia. [2]

(b) A weightlifter's dumbbell comprises a thin rod of mass m and length l with a thin disc of radius b and mass M attached at each end perpendicular to the axis of the rod. Calculate the moment of inertia of the dumbbell about an axis perpendicular to the rod and through its centre. Assume that the moment of inertia of a disc of radius b and mass M about an axis through its centre and perpendicular to its surface is $Mb^2/2$. [5]

2. An athlete throws a ball with speed v towards a vertical wall located at a distance L away. At what angle must she throw the ball in order for it to hit the wall at the highest possible point? What is this angle for the case $L = v^2/g$? Describe briefly the trajectory if $L < v^2/g$. [5]

3. A point-like particle of rest mass M and relativistic energy E collides head-on with a stationary point-like particle of rest mass m; the collision is elastic. Defining the speed of light $c = 1$, show that E', the final energy of M, is given by:

$$E' = \frac{2mM^2 + E(m^2 + M^2)}{2Em + m^2 + M^2}.$$

[8]

4. A simple harmonic oscillator with mass M and spring constant K is constrained to move in the x direction. Evaluate both the Lagrangian and the Hamiltonian for the oscillator and show that the total energy is conserved. [4]

5. A spaceship is moving in a frame S' with speed v'. Relative to an observer in frame S, S' is moving with speed v in the same direction as v'. Write down the Lorentz transformations between S' and S, and hence derive an expression for the speed of the spaceship in frame S. [4]

6. An alpha particle (which contains two protons) is accelerated from rest through a potential difference of 100 kV and is directed towards a stationary Beryllium nucleus fixed in a target. Stating any assumptions made, calculate the smallest possible distance of closest approach and comment on this value in relation to the typical size of an atom. (A Beryllium nucleus contains 4 protons.) [6]

7. A high-energy particle is travelling in space with speed v towards an observer. The particle emits a photon with energy E in its rest frame. What is the frequency of the photon as detected by the observer? [6]

Section B

8. (a) Prove the parallel axis theorem for the moment of inertia of a solid body. [5]

(b) A solid body of mass m is free to oscillate about a fixed frictionless axis. Show that if the body is displaced by a small amount from its equilibrium position and then released, it will undergo simple harmonic motion with the period of oscillations, T, given by

$$T = 2\pi\sqrt{\frac{I}{mgh}},$$

where I is the moment of inertia of the body about the axis and h is the distance from the axis to the body's centre of mass. [6]

(c) A uniform rod of length l swings as a pendulum about a fixed position x from its centre of mass. For what value of l does the rod have the same period of oscillation as a rod of the same density, but with length $2l$, which is pivoted at its top end? (The moment of inertia of a uniform rod of mass m and length a is $ma^2/12$.) [9]

9. Define the angular momentum $\mathbf{L}$ of a body and state the circumstances in which $\mathbf{L}$ is conserved. [3]

A planet of mass M and radius R has no atmosphere and is not rotating.

(a) A body is launched vertically upwards from the planet's surface with speed v_0 which is less than the escape velocity. Derive an expression for the maximum radial position r_0 reached by the body. [4]

(b) A second identical body is launched horizontally from the surface of the planet with speed v. Assuming $r_0 \gg R$ show that the speed required for the body to just reach the same radial position r_0 (from part (a)) is given by:

$$v = v_0\left(1 + \frac{R^2}{2r_0^2}\right).$$

[9]

(c) Explain whether the magnitude of the escape velocity of the body is dependent on the launch angle, and explain qualitatively any difference if the planet were rotating. [4]

10. A small particle of mass m is fixed to the bottom end of a thin, massless rod of length l. The rod is fixed to a pivot at its top end and is free to swing in any direction. At a given moment, the angle between the rod and the downward vertical axis is θ and the azimuthal angle is ϕ.

(a) Write down the Lagrangian of the system. [2]

(b) Show that the energy, and the angular momentum J about the principal axis, are constants of the motion. [3]

(c) Derive the equation of motion for the θ coordinate. Show how this equation simplifies in the two special cases of constant ϕ and constant θ. In each case derive the angular frequency of the motion, stating any assumptions made. [6]

(d) In the case of constant $\theta \simeq \theta_0$, find an expression for J (from part (b)) in terms of θ_0. [2]

(e) Now consider small oscillations in θ such that $\theta = \theta_0 + \alpha(t)$. Stating any assumptions made, derive the period of the oscillations. [7]

11. (a) State Einstein's postulates of special relativity. Write down the Lorentz transformations for momentum p and energy E and show that the quantity $E^2 - p^2$ is invariant under the Lorentz transformation. [6]

(b) A particle of rest mass $4m$ is at rest in the laboratory frame and decays into two particles with rest masses m and $2m$. What are the speeds of the two particles (in units of c) in the laboratory frame? [9]

(c) The particle with rest mass m has a proper mean lifetime of 1μs. What is the probability that it will travel further than 2000 m in the laboratory frame before it decays? [5]