A10287H1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 9th June 2025, 9.30 am – 12.00 noon

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. (a) Derive the moment of inertia of a solid sphere, of uniform density, about an axis through its centre of mass (the 'principal axis'); assume that the sphere has mass M and radius R.

(b) State the parallel axis theorem. Hence determine the moment of inertia of the sphere about an axis parallel to its principal axis and tangential to its surface. [7]

2. A soldier fires a bullet vertically upwards. The bullet has mass m and initial speed v_0. If the bullet experiences a force due to air resistance of mKv, where v is its instantaneous speed and K is a constant, derive an expression for the time taken for it to reach its maximum height. Assume that the acceleration due to gravity is constant. [4]

3. A particle accelerator at CERN collides electrons and positrons together head-on. In these collisions Z^0 bosons are produced via the reaction $e^- + e^+ \rightarrow Z^0$.

(a) What is the energy of the electron if the Z^0 boson (of rest mass M_Z) is produced at rest in the centre-of-mass frame?

(b) The Z^0 boson decays into two particles of equal rest mass m that travel with relativistic speeds. What are the energies and momenta of the particles in the Z^0 boson rest frame?

(c) If the Z^0 boson rest mass is 91 GeV/c^2 and the two particles are c-quarks of rest mass 1.5 GeV/c^2, what is the speed of each c-quark in the rest frame of the Z^0 boson? [6]

4. A particle of mass M travelling at speed v ($\ll c$) collides elastically with an identical stationary particle. Show that, after the collision, either one particle is stationary, or the particles move at right angles to each other. [4]

5. Consider a roller coaster carriage that rolls in a vertical plane on a frictionless track at an amusement park. The track comprises a vertical straight section joined smoothly at its lower end to a circular loop of radius R. The carriage starts from rest at a height H above the base of the loop. What is the minimum value of H required for the carriage to pass around the loop without losing contact with the rails? [4]

6. A proton is accelerated from rest through a potential difference V. It is then directed head-on towards a stationary iron nucleus which is fixed in a target. Stating clearly any assumptions made, calculate the minimum value of V required for the proton to get closer than 10^{-13}m to the nucleus. (The atomic number of iron is 26.) [6]

7. A rod of mass M and length l leans against a frictionless vertical wall at an angle θ with respect to the wall. The bottom of the rod stands on a frictionless horizontal floor. The rod slides in a vertical plane and remains in contact with the wall and the floor at all times.

(a) Define the Lagrangian, and derive an expression for the Lagrangian of the system. (The moment of inertia of a uniform rod of length l and mass M about an axis through its centre of mass and transverse to its length is $Ml^2/12$.)

(b) Hence derive the conjugate momentum and the equation of motion. [9]

Section B

8. (a) State Einstein's postulates of special relativity. Write down the Lorentz transformation for momentum and energy and show that the quantity $E^2 - p^2c^2$ is invariant under the transformation. [5]

(b) A relativistic particle of rest mass M is moving with speed v. Write down expressions for its linear momentum p and total energy E. Write down the corresponding expressions for a photon of wavelength λ. [4]

(c) In the process known as Compton scattering, a photon scatters by an angle θ from a free electron. Show that the scattered photon wavelength λ' is given by:

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta)$$

where m_e is the electron rest mass, c is the speed of light in vacuum, and h is Planck's constant. [7]

Interpret physically the cases where $\theta = 0$ and π. If an incoming photon has wavelength $\lambda \ll h/m_e c$ what is its energy in MeV when it is scattered through an angle of π radians? Comment on the result. [4]

9. A particle of mass M is fixed at the bottom end of a massless rod of length l. The rod is held at its upper end so that it is free to pivot in any direction. Let the polar angle between the rod and the downward vertical axis be θ, and the azimuthal angle be ϕ.

(a) Write down the Lagrangian of this system and hence show that the energy, and angular momentum about the vertical axis, are constants of the motion. [5]

(b) Derive the equation of motion in terms of the polar angle θ, defining any other terms. Derive the angular frequency in the two special cases where (i) ϕ is constant and (ii) θ is constant, stating any assumptions made. [6]

(c) In the case of constant $\theta = \theta_0$, find an expression for the angular momentum J_z about the vertical axis in terms of θ_0. [2]

(d) If now θ undergoes small oscillations such that $\theta = \theta_0 + \beta(t)$, find the period of the oscillations, stating any assumptions made. [7]

10. (a) A point-like particle of mass M moves within a central potential $V(r)$.

(i) If $\mathbf{r}$ is the spherical radial coordinate of the particle, define its angular momentum $\mathbf{J}$ about an axis through the origin and show that J is constant. [2]

(ii) Explain the concept of an *effective potential* and derive an expression for it in terms of M, r, J and $V(r)$. [4]

(b) A satellite moves in an elliptical orbit around the Earth with major axis $2a$. If the satellite's distance of furthest approach to Earth is R, and its speed at that point is v_R, show that

$$v_R^2 = Gm_E\left(\frac{2}{R} - \frac{1}{a}\right)$$

where m_E is the mass of the Earth and G is the gravitational constant. [6]

(c) When at the point of furthest approach, a retro-rocket on the satellite fires so as to reduce its speed from v_R to $v_R(1-\epsilon)$, where $\epsilon \ll 1$. Neglect any change in the satellite's mass when the retro-rocket is fired; hence show, stating any assumptions made, that the major axis of the new orbit is reduced by an amount

$$4\epsilon a\frac{(1-e)}{(1+e)}$$

where e is the eccentricity of the original orbit, defined by $R = a(1+e)$. [8]

11. (a) Explain what is meant by an elastic collision, stating clearly which quantities are conserved. [2]

(b) A ball of mass m_1 is held just above a second ball of the same size with mass m_2, such that there is a very small vertical gap between them. The balls are then released simultaneously, fall towards the ground, and bounce. The speed of m_1 just before its bounce is v.

(i) Assuming all collisions are elastic, draw simple diagrams showing the collisions which occur as the balls bounce. [4]

(ii) What is the mass ratio m_1/m_2 if the top ball receives the largest possible fraction of the total energy available after the bounce? (Ignore any effects of the sizes of the balls and the gap between them.) [8]

(iii) What is the maximum height the top ball can reach, and what is the time taken for it to reach this height starting from the time of its bounce? [6]