A10288W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Wednesday 4th September 2024, 9:30 am – 12 noon

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. An astronomer suspects that a star at a distance of 4×10^{17} m from the Earth is actually a binary system with an orbital separation of 1×10^{11} m. What diameter telescope is required to confirm this by imaging it at wavelength $\lambda = 410$ nm? [3]

2. Write down Maxwell's equations in a vacuum in differential form. Show that the equations allow wave solutions for the electric field **E** and the magnetic flux density **B**, and derive the speed of these waves. [6]

3. A thin conducting disc has radius a and mass m. It is rotating freely with angular frequency ω about an axis through its centre of mass, normal to its plane. It is immersed in a uniform magnetic field B parallel to this axis. At time $t = 0$, a resistor R is connected between the rim and the centre of the disc. Derive an expression for the initial current $I(0)$ through the resistor. Hence derive an expression for the current $I(t)$ at time $t > 0$. [You may assume that the moment of inertia of the disc about the axis of rotation is $\frac{1}{2}ma^2$.] [6]

4. Sketch and clearly label an experimental arrangement for using a diffraction grating as a spectrometer. Comment on the conditions required for proper illumination of the diffraction grating. [4]

If a diffraction grating of 2000 lines per cm is used to analyse the spectrum of mercury, find the angular separation in the second-order spectrum of the two lines of wavelengths 579.0 nm and 577.0 nm. [3]

5. Consider a square, each side of length $2a$, with corners ABCD. A circular loop of wire is passed through the same four points ABCD, and a current I flows around this loop. Find the magnetic flux density at the centre of the loop. [6]

6. Draw and label a circuit using one operational amplifier and resistors which gives an output

$$V_{\text{out}} = -2(V_{\text{a}} + 2V_{\text{b}})$$

for two voltage inputs V_{a} and V_{b}. Describe the behaviour of the circuit if the feedback resistor is replaced by a capacitor. [6]

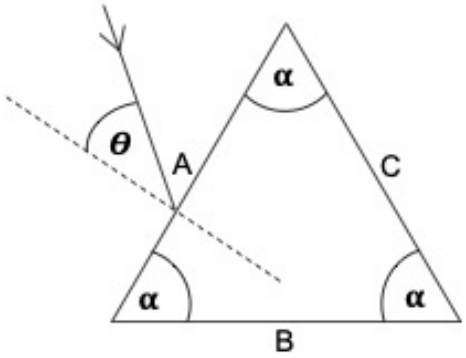
7. A thin equiconvex lens (that is, a lens where the two convex surfaces have the same radius of curvature) is made of glass with refractive index n. Both its spherical surfaces have radii of curvature R. Show that its focal length f is given by

$$f = \frac{R}{2(n-1)}.$$

[6]

Section B

8. State Huygen's Principle, and use it to derive Snell's Law of refraction. [5]



The figure above shows a prism of refractive index $n > 1$ which has all angles $\alpha = 60°$, and has faces labelled A, B and C.

Consider a ray of green monochromatic light which enters the prism at face A at an angle θ to the face normal, as shown in the figure above.

(i) Sketch and comment on the possible light paths that the light ray may take through the prism. You should consider only positive angles of θ as measured clockwise from the normal to face A. [4]

(ii) If the refractive index of the prism is $n = 1.5$, for what range of angles θ does the ray undergo total internal reflection at face B of the prism? [4]

(iii) For a ray that does undergo total internal reflection at face B of the prism, calculate the angle of deviation, δ, of the ray that subsequently emerges from face C of the prism with respect to the initial direction of the ray incident on face A. Express your answer in terms of θ and α. [4]

(iv) The incident angle θ is fixed such that total internal reflection is just occurring at face B for green monochromatic light. Describe what would be observed if the monochromatic light source is replaced with a white light source incident at the same angle θ. [3]

9. Current I flows through a wire coil, in the y–z plane, of radius a with N turns. Find an expression for the magnetic flux density at distance x, measured from the coil's centre, along an axis perpendicular to its plane (its principal axis). [5]

A second, identical coil shares the same principal axis; these coils are separated by distance a. Identical currents I flow through the coils in the same direction. What is the magnetic flux density B at the mid-point on the line joining the centres of the two coils? What is B at the centre of each coil? Sketch the total magnetic flux density and the contribution from each coil as a function of position on the axis between the coils. Describe briefly the properties of this coil arrangement. [9]

A stream of electrons is accelerated by a potential V and introduced into the region between the coils, perpendicular to the principal axis of the coils, and close to the mid-point. An experimenter wants to deflect the electron stream so that it emerges from the region between the coils travelling parallel to, but in the opposite direction to the input stream, and separated by 2 cm (you may assume that $2\,\text{cm} \ll a$). Find the approximate value of the magnetic flux density between the coils required to achieve this when the accelerating potential $V = 10\,\text{kV}$, stating any assumptions you make. [6]

10. An infinitely long hollow metal cylinder of radius r_1 is placed inside another infinitely long metal cylinder of radius r_2 so they are coaxial. There is a vacuum between the two cylinders. The outer cylinder is maintained at a constant potential difference V with respect to the inner one. Find expressions for the electric field $\mathbf{E}$ between the two cylinders ($r_1 < r < r_2$), and the capacitance per unit length. [5]

Derive an expression for the energy per unit length stored in the cylinders described above. [4]

Between the cylinders there is a uniform magnetic flux density $\mathbf{B}$ parallel to the axis of the cylinders. Show that protons (mass m and charge $+e$) can have a trajectory of fixed radius r between the cylinders if they have a speed v whose minimum value satisfies a quadratic equation of the form

$$av^2 + bv + c = 0.$$

Find expressions for a, b and c in terms of the parameters given, stating your assumptions. [8]

Calculate this minimum speed v for protons orbiting between the cylinders at a radius of 5 cm in a B-field of 0.01 T if the cylinders have radii $r_1 = 4\,\text{cm}$ and $r_2 = 6\,\text{cm}$ and the potential difference between them is 20 V. [3]

11. A voltage source produces a voltage $V_0 \cos(\omega t)$. For each of the components listed below, state its complex impedance and sketch the voltage across, and the current through each component, for several cycles of the sinusoidal waveform when they are connected across the terminals of the voltage source.

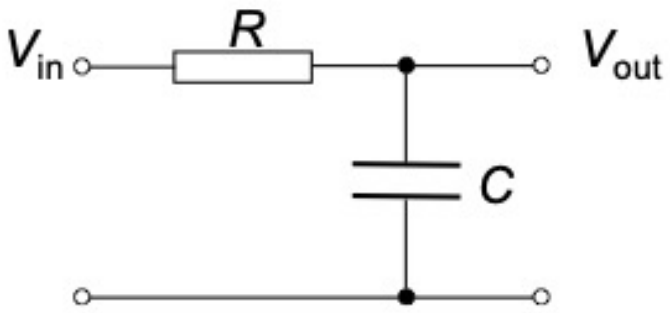
(i) A resistor of resistance R.

(ii) A capacitor of capacitance C.

(iii) An inductor of inductance L.

Calculate the total impedance of the system if a resistor, capacitor and inductor are connected together in series. [6]

The circuit diagram below shows a simple low-pass filter, consisting of a resistor of resistance R and a capacitor of capacitance C.



(a) If a sinusoidal voltage of $V_0 \cos(\omega t)$ is applied to the filter, find the amplitude and phase of V_{out} relative to V_{in}. Sketch the resulting amplitude and phase as a function of the angular frequency ω, and comment on the cases when ω tends to 0 and ∞. [4]

(b) Calculate the angular frequency for which

$$|V_{\text{in}}| = \frac{|V_{\text{in}}|}{\sqrt{2}}.$$

[2]

(c) Now a constant voltage V is applied to the input of the filter at time $t = 0$. If the capacitor is initially uncharged, determine an expression for the output voltage V_{out} as a function of time t. [3]

(d) If instead, a square-wave voltage of fixed period $T = 2\pi/\omega$ is applied to the input, sketch V_{out} for the cases when:
(i) $\omega \ll 1/RC$, (ii) $\omega \gg 1/RC$ and (iii) $\omega \sim 1/RC$. [5]