A10288W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday 4 June 2024, 2.30 pm – 5.00 pm

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. A tetrahedron has a single $1\,\Omega$ resistor between each of its four corners. Pick any two corners and find the resistance between them. [4]

2. Calculate the potential energy of a spherical, conducting shell of charge Q and radius a. [4]

3. A parallel plate capacitor, for which each plate has an area $A = 1\,\text{cm}^2$ and separation distance $d = 0.5\,\text{mm}$ between the plates, is connected in parallel with a $1.0\,\text{mH}$ inductor. The permittivity of the material between the parallel plates is ϵ_0. What is the resonant frequency f of this circuit? [3]

If a change of $1\,\text{Hz}$ in f can be measured, what is the fractional change in d that could be detected if this is the only parameter that changes in the circuit? Give your answer in parts per million (ppm). [3]

4. What is the Rayleigh criterion?

The pupil of a human eye has a diameter of about $5\,\text{mm}$. Estimate the angular resolution, in degrees, of the human eye.

A radio telescope has a circular dish having a diameter of $10\,\text{m}$ and it detects radio waves at a frequency of $10\,\text{GHz}$. Estimate its angular resolution in degrees. [6]

5. Four straight sections of wire, each of length $2a$, are connected in a square loop with corners ABCD. A current I flows around this loop. Find the magnetic flux density at the centre of the square. [6]

6. A particle of mass m and charge q moves in the z-direction with velocity $\mathbf{v}$ through a region with uniform electric field $\mathbf{E} = (E_x, 0, 0)$ and uniform magnetic flux density $\mathbf{B} = (0, B_y, 0)$. What $\mathbf{v}$ would lead to the particle continuing in a straight line? [4]

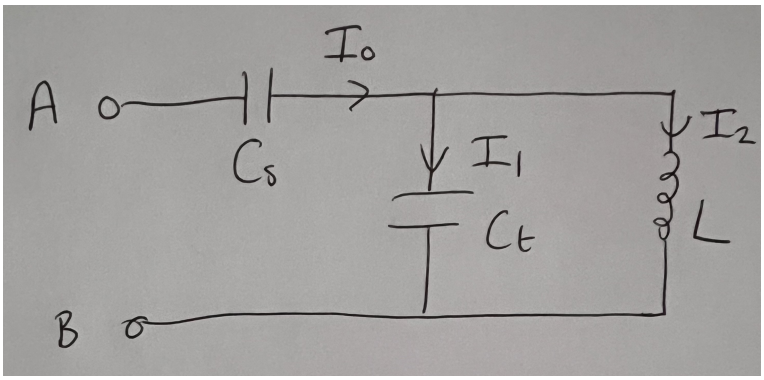
The power supply producing the electric field is suddenly switched off. Derive an expression for the subsequent trajectory of the particle, and hence describe its motion. [5]

7. Two coaxial solenoids with radii a and $3a$ are fully overlapping. Each solenoid has n turns per unit length and length l. Neglecting end effects, calculate the self-inductance of each coil and their mutual inductance. [5]

Section B

8. Explain the usefulness of complex impedances in circuits driven by a sinusoidal voltage of angular frequency ω. Derive the complex impedance of (a) a resistor, (b) a capacitor and (c) an inductor. [5]

The circuit below is to be driven by a sinusoidal voltage $V_o \sin \omega t$ connected across A and B.



For the case that $C_\mathrm{s} = 0$, calculate the impedance across A and B when $\omega = 1/\sqrt{C_\mathrm{t} L}$ and calculate the currents I_1, I_2 and I_0 (shown in the diagram) at this frequency. [8]

For the case when $C_\mathrm{s} \neq 0$, show that the magnitude of the current through the inductor is given by

$$|I_2| = \frac{a\,\omega}{1 - b\,\omega^2},$$

and find the value of the constants a and b. Hence find the frequency at which I_2 diverges when $L = 1\,\mu\mathrm{H}$, $C_\mathrm{t} = 80\,\mathrm{nF}$ and $C_\mathrm{s} = 20\,\mathrm{nF}$. [7]

9. Describe, with the aid of a sketch, how a diffraction grating (a transmission grating) is used to disperse incident light into its constituent wavelengths. Discuss the conditions for proper illumination, and how this may be achieved. Where, in principle, would the detector be located to receive the light dispersed by the diffraction grating? Describe the optical properties of a simple element that is often used in practice between the grating and the detector. [6]

Write down the diffraction grating equation and define the terms used. [2]

An astronomer wishes to use a diffraction grating to disperse collimated light collected from a distant nebula composed predominantly of hydrogen, to obtain a spectrum that includes both the Balmer hydrogen-alpha emission line at 656.3 nm and the Balmer hydrogen-beta emission line at 486.1 nm. The astronomer is in possession of a grating with 1000 lines per mm, and a lens of focal length of 85 mm. Describe a setup whereby these could be used to obtain the desired spectrum, and calculate how large the detector has to be to simultaneously detect both of these two emission lines. [10]

Explain what is meant by the orders produced by a grating, and calculate whether for the setup above there would be any contamination of the desired spectrum from a different order. [2]

10. Two point charges $\pm q$ are placed at points $(0, 0, \pm a/2)$, defining an electric dipole $\mathbf{p} = qa(0, 0, 1)$. Show that the potential V at large distance $r \gg a$ from the origin is given by

$$V = \frac{\alpha \mathbf{p} \cdot \mathbf{r}}{r^3},$$

where α is a pre-factor that you should determine. [6]

Derive $\mathbf{E} = (E_{\rm r}, E_\theta, E_\phi)$ in spherical polar coordinates, and hence $|\mathbf{E}|$. [6]

An infinite conducting sheet is placed far away from the dipole in the plane $x = b$ where $b \gg a$. Calculate (i) the force on the dipole and (ii) the torque on the dipole. [4]

How would your answers change if the conducting sheet were brought much closer to the dipole, so that $b = a$? [4]

11. State Maxwell's equations in their differential form assuming the presence of charge density ρ and current density $\mathbf{J}$, defining all terms that you use. [4]

Use one of these equations to derive Ampère's law and explain why a displacement current needs to be included. [3]

Show from Maxwell's equations that plane electromagnetic waves may propagate in a vacuum and derive their speed. [6]

A plane electromagnetic wave in a vacuum is described by an electric field given by

$$\mathbf{E} = (E_x, E_y, E_z) = (3E_0 \cos(kz - \omega t), 4E_0 \cos(kz - \omega t), 0).$$

Determine the Cartesian components of $\mathbf{B}$ and calculate the numerical value of the ratio $\mu_0|\mathbf{E}|/|\mathbf{B}|$. [7]