A10287W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Thursday 5th September 2024, 9.30 am – 12.00 noon

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. (a) State the parallel axis and perpendicular axis theorems for moments of inertia. [2]

(b) Derive the moment of inertia of a solid circular cylinder (of uniform density) about an axis through its centre of mass and perpendicular to its principal axis; assume the cylinder's mass is M and its radius is R. [4]

2. Two particles with masses m and M travel without friction in the horizontal plane. The two particles collide obliquely and elastically. Show that, in the centre of mass frame, the magnitude of the velocity of each particle does not change as a result of the collision. [4]

3. A circular disc of negligible thickness has mass M and radius R and is free to rotate in the vertical plane about a frictionless axle through its centre. A small mass m is attached to the disc at a radius r vertically below the axle. If m is displaced by an angle θ and released, find its equation of motion and, for small angles, the period of its oscillation. (The moment of inertia of a thin disc of mass M and radius R is $\frac{1}{2}\,MR^2$.) Evaluate the period for the special case $m = M$ and $r = R$. [5]

4. An athlete throws a ball with speed v from the top of a tower of height H. Ignoring air resistance, what is the maximum horizontal distance from the base of the tower that the ball can reach? [8]

5. An α particle (which contains 2 protons) is accelerated from rest through a potential difference of 50 kV. It is directed head-on towards a stationary gold nucleus (the atomic number of gold is 79) which is fixed in a target. Calculate the distance of closest approach stating any assumptions made. [4]

6. Explain what is meant in Lagrangian mechanics by a cyclic coordinate and how the existence of a cyclic coordinate implies a conservation law. For an isolated system with a Lagrangian which has no time dependence show that the total energy is conserved. [5]

7. (a) A particle accelerator at CERN fires protons onto a stationary target and produces muons. If each proton produces one muon, the proton momentum = 1 TeV, and the muon is produced with the same speed as the proton, what is the muon momentum? Assume that the proton mass = 1 GeV and the muon mass = 100 MeV. [3]

(b) If N muons are produced at time $t = 0$, and the muon mean proper lifetime is τ, write down the number of muons remaining at a later time t. The muons travel along a beampipe before hitting a detector 10 km downstream of the target. What fraction of the initially-produced muons is recorded in the detector? Assume that the muon mean proper lifetime is 2.2 μs. [5]

Section B

8. A satellite of mass m orbits a planet of mass M ($M \gg m$).

(a) Derive an expression in plane polar coordinates for the total energy of the satellite, explaining each of the terms. [6]

(b) Initially the satellite is moving in a circular orbit of radius R. It then receives an impulse which does not change its angular momentum, but which causes an increase in its kinetic energy by a factor β ($\beta > 1$). Calculate the distances of furthest and closest approach of the satellite to the centre of the planet. [8]

(c) A space station launches a satellite intended to be on a circular orbit of radius b around the planet. The satellite was launched with the correct speed, but by mistake the direction of launch in the plane of the intended orbit was in error by an angle θ. Describe the new orbit and find the maximum and minimum distances of the satellite from the centre of the planet. [6]

9. State Einstein's postulates of special relativity. Write down the Lorentz transformations for momentum and energy and show that the quantity $E^2 - p^2$ is invariant under the Lorentz transformation. [7]

In the laboratory frame, a proton of total energy E undergoes a collision with a stationary proton. After the collision the two protons travel along trajectories each inclined at an angle ϕ with respect to the direction of the initial incoming proton. Derive an expression for ϕ in terms of E and the proton rest mass, m. [8]

Through what potential difference must the incoming proton be accelerated in order to produce an angular deviation $\phi = 30°$? [5]

10. A mass M is attached to the bottom end of a massless spring of natural length L and spring constant K. The top end of the spring is attached to a frictionless pivot.

(a) Initially the spring hangs vertically such that the mass can only move vertically up and down. Derive the Lagrangian for this system and hence find the equation of motion. Verify that this is consistent with the result obtained using Newton's 2nd Law. [5]

(b) Now the spring is free to swing from side to side in a vertical plane such that the spring remains in a straight line. Assume that at time t the extension of the spring beyond its natural length is $x(t)$ and that the spring makes an angle $\phi(t)$ with the vertical axis. Write down the Lagrangian of the system and hence find the equations of motion for M in terms of x and ϕ. [7]

Find the two pairs of values of (x, ϕ) for which the system is in equilibrium and identify which one is a stable equilibrium. For the stable equilibrium show that small oscillations can be decoupled into two independent modes and give a simple physical interpretation of each mode. [8]

11. A raindrop falls vertically in the Earth's gravitational field. As the drop falls through a cloud it accretes water vapour that is initially at rest so that its mass $m(t)$ increases. Ignore air resistance and derive from first principles a differential equation that relates the downward velocity of the raindrop, $v(t)$, to the rate of increase of its mass. [6]

(a) The raindrop accumulates mass at a constant rate αm_0. If at time $t = 0$ its mass is m_0 and its speed is 0, show that

$$\frac{\mathrm{d}}{\mathrm{d}t}[v(1+\alpha t)] = g(1+\alpha t).$$

Hence, or otherwise, find its velocity and acceleration. [8]

(b) If instead the mass of the raindrop increases in proportion to the mass already accumulated, $\dfrac{\mathrm{d}m(t)}{\mathrm{d}t} = \beta m(t)$, where β is a constant, find $v(t)$ assuming $v(0) = 0$. [3]

(c) Compare the results from (a) and (b) by drawing in each case a sketch of $v(t)$ as t varies from zero to large values. [3]