A10287W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 3rd June 2024, 2.30 pm – 5.00 pm

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. (a) Derive the moment of inertia of a thin solid disc, of uniform density, about an axis through its centre of mass and perpendicular to its face (the 'principal axis'); assume the disc mass is M and its radius is R. [3]

(b) State the parallel axis and perpendicular axis theorems. Hence determine the moment of inertia of the disc about an axis perpendicular to its principal axis and tangential to the rim of the disc (assume the disc is negligibly thin). [4]

2. A thin uniform rod of mass M and length L lies on a frictionless horizontal table. A particle, also of mass M, moves along the table top at a constant (non-relativistic) speed and strikes the rod perpendicular to its axis at a certain distance from its centre. Calculate this distance such that the particle and the centre of mass of the rod move with the same velocity after the collision. The collision can be assumed to be perfectly elastic; the moment of inertia of a thin rod about an axis through its centre of mass and perpendicular to its length is $\frac{1}{12}\,ML^2$. [6]

3. (a) An athlete throws a ball of mass M vertically upwards. The ball leaves her hand with speed u and experiences a force due to air resistance proportional to its speed, with proportionality constant K. Derive an expression for the time taken for the ball to reach its maximum height in terms of K. Assume the acceleration due to gravity is constant with height. [5]

(b) Now consider if the athlete were standing on another planet identical to Earth but with no atmosphere such that $K = 0$. By using your result from (a) and considering the limit $K \to 0$, or otherwise, calculate the time taken for the ball to reach its maximum height. [2]

4. (a) A system comprises a positively-charged particle of charge $+qe$, where e is the proton charge, and a nucleus of atomic number Z separated by a distance d. Write down the electrical potential energy of this system. [1]

(b) Now consider the particle to be accelerated from rest, starting at infinity, through a potential difference V and directed towards the nucleus, which is fixed in a target. Calculate the minimum distance of closest approach, stating any assumptions made. [4]

(c) Protons with kinetic energy 4.2 MeV are fired at a fixed gold target. Evaluate the minimum distance of closest approach of a proton to a gold nucleus; the atomic number of gold is 79. Compare this value with the typical size of an atom. [2]

(d) If α particles (which contain 2 protons) with the same kinetic energy, 4.2 MeV, were used instead of protons (as in Rutherford's famous scattering experiment), what would be the distance of closest approach? [1]

5. (a) Write down the Lorentz transformations for x and t between a reference frame S' moving with speed v along the x-axis with respect to a reference frame S. [2]

(b) In the moving frame S', two spatially-separated events occur simultaneously. What is the relationship between the relative times and relative locations of the two events in frame S? [2]

What is the minimum possible spatial separation that these two events can have in frame S if their time separation in that frame is 1 nanosecond, and for what value of v is this the case? [2]

6. (a) A Higgs boson (mass M_H) decays into two particles of equal mass m_q. Assuming that the particles travel with relativistic speeds, what are their energies and momenta in the Higgs boson rest frame? [3]

(b) Assume the Higgs boson mass = 125 GeV/c^2 and the two particles are b-quarks of mass = 5 GeV/c^2, what is the speed of each b-quark in the rest frame of the Higgs boson? (Hint: consider the Lorentz factor γ.) [3]

Section B

7. (a) A satellite of mass m orbits a star of mass M ($M \gg m$). Considering angular momentum, derive an expression for the total energy of the satellite, explaining each of the terms. [6]

(b) A small planet is in a circular orbit of radius R about a massive star. The star suddenly undergoes an explosion in which a small fraction ϵ of its mass is ejected spherically symmetrically to a large distance from the star (assume that none of the ejected mass hits the planet). Calculate the closest and furthest distances of the planet's orbit around the star after the explosion. [14]

8. Define the invariant interval in space-time. Starting with the Lorentz transformations, verify that the invariant interval is indeed invariant with change of inertial reference frame. Hence, or otherwise, calculate the factor by which time is dilated for observers in two inertial reference frames moving at a constant relative speed. [9]

An astronaut sitting at the back of a rocket of proper length l moving at speed $0.5c$ away from Earth throws a ball to the front of the rocket at speed $0.8c$ relative to the rocket. What distance does the ball travel, and how long does it take, according to:

(a) the astronaut

(b) a stationary observer at Mission Control on Earth

(c) a massless flea resting on the ball. [10]

According to Mission Control, one year has passed since the rocket left Earth with speed $0.5c$; how much time has elapsed in the frame of the astronaut? [1]

9. Write down the Lagrangian for a system described by generalised coordinates q_i and $\dot{q}_i$ and write down the Euler-Lagrange equation. Explain briefly what is meant by a cyclic coordinate and how it is related to a conservation law. [3]

A uniform thin rod of length L and negligible mass is supported by a frictionless pivot at one end, and has a mass M glued to the other end, such that it is free to swing in a fixed vertical plane. Write down the Lagrangian of the system. Hence, or otherwise, find the period of small oscillations of the rod in this vertical plane. [8]

Suppose the support is changed to allow the rod to swing in *any* direction. Using suitable generalised coordinates, write down the Lagrangian of the system and hence identify the cyclic coordinate and the respective conserved quantity. [5]

Write down the Hamiltonian of the system and hence state whether the energy is a constant of motion. [4]

10. Draw a simple illustrative diagram and state three advantages of using the centre-of-mass frame when considering the collision of two particles moving with non-relativistic speeds. [4]

Two particles A (of mass m) and B (of mass $M > m$) undergo a head-on elastic collision. Before the collision, particle A has speed u ($u \ll c$) and particle B is at rest. Calculate the speeds of A and B after the collision in:

a) the centre-of-mass frame

b) the laboratory frame in which B was initially at rest. [6]

If instead the collision is **not** head-on, and particle A is deflected through an angle of 30°, calculate the final speeds of A and B in the laboratory frame for the case $M = 2m$. [10]