A10287W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 4 September 2023, 9.30 am – 12.00 noon

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

1. State the parallel axis and the perpendicular axis theorems.

Obtain an expression for the moment of inertia of a solid cylinder of radius r and length h about an axis through its centre of mass and perpendicular to its principal axis.

(Hint: The moment of inertia of a disk of mass M_{disk} and radius R_{disk} is $\frac{1}{2}M_{disk}$ R^2_{disk}) [5]

2. Define the Lagrangian and the Hamiltonian as used in classical mechanics. Prove that the Hamiltonian is conserved in a system for which the Lagrangian has no explicit time dependence.

State Noether's theorem. [6]

3. A spaceship hovers at a constant height just above the surface of a planet where the acceleration due to gravity is g. The exhaust expels fuel at a constant speed u. The initial mass of the spaceship including fuel is M_0. While there is fuel remaining in the spaceship, what should be the rate of ejected mass $(t) = \dfrac{dM(t)}{dt}$ to keep the spaceship hovering at rest at a time t? Write the answer in terms of M_0, u, g and t. [5]

4. A point mass m moves at a speed v_0 perpendicular to a rod of length l and mass m. The rod is initially at rest. Calculate the location where the point mass should collide with the rod, so that the point mass and the centre of mass of the rod move with equal speeds after the collision. Consider that the collision is elastic.

(Hint: the moment of inertia of the rod is $\frac{1}{12}ml^2$) [4]

5. Two events separated by a distance of $1km$ are measured to occur $1\,\mu$s apart in an inertial frame A. In a different inertial frame B, the events are observed to occur simultaneously. What is the velocity of the frame B, as observed from frame A?

Find the maximum time interval possible between the events in the frame A for them to occur simultaneously in frame B. [6]

6. A potential energy function is given by:

$$U(x) = \frac{kx}{(a^2 + x^2)},$$

where k and a are constants and x is the position coordinate.

Sketch the force and potential as a function of x. Find the values of the force and the potential at the positions of stable equilibrium, and define the regions of attraction and repulsion. [8]

7. An alpha particle is accelerated from rest through a potential difference of 20 kV. It travels directly towards a stationary Beryllium nucleus (4 protons, 5 neutrons). Calculate the distance of closest approach. [6]

Section B

8. a) Write down expressions for the momentum p and the total energy E of a relativistic particle, and use them to verify that $E^2 - p^2c^2$ is an invariant. [3]

A particle of mass m travelling at speed v_1 collides head-on with a particle of mass $2m$ travelling at speed $v^2 = c/3$. They merge to form a single particle of mass M at rest in the same frame. Find expressions for v_1 and M. Discuss why M is not equal to $3m$. [6]

(b) Write down the Lorentz transforms for energy and momentum. Consider a photon of energy E' travelling in a frame S' which is moving at speed $v = \beta c$ with respect to a frame S. Show that the relativistic Doppler factor, D, is given by

$$D = (\frac{1+\beta}{1-\beta})^2,$$

where $D = E/E'$ is the ratio of the photon energies. [3]

In the case where the relative speed is close to the speed of light, c, i.e. $\beta = (1-\epsilon)$, where ϵ is small, show that the Doppler factor can be written as

$$D = (4\gamma^2 - 1)^{1/2},$$

where γ is the Lorentz factor. [2]

(c) The propagation of extremely high energy photons through the universe is limited by a reaction with the cosmic microwave background radiation in which electron-positron pairs are formed: $\gamma_1 + \gamma_2 \rightarrow e^+ + e^-$. Assuming that the background photon γ_2 has an energy of $10^3 eV$, calculate the minimum energy that the other photon γ_1 must have in order for the above reaction to take place. [4]

What is the velocity of the resulting electron-positron pair in the laboratory frame? [2]

9. Write the expression for the energy of a satellite in a non-circular orbit around a massive body, explaining the energy contributions. [2]

A spacecraft bound for Mars is moved from a circular orbit of radius 1 Astronomical Unit (AU) about the Sun into an elliptical orbit with maximum and minimum distances of 1.52 and 1.0 AU, corresponding to the orbital radii of Mars and Earth. Find the speed of the spacecraft at the minimum and maximum distances of the elliptical orbit. What is the difference in the speed of the spacecraft at the minimum distance of the elliptical orbit compared to its circular orbit around the Sun? [10]

When the spacecraft reaches the furthest point in its orbit, a rocket fires to move it into a circular orbit at the distance of Mars. What is the change in orbital speed required and in what direction should the rocket thruster be fired? In order to achieve a successful rendezvous with Mars, what are the relative orbital positions of Mars and the spacecraft when it moves into the elliptical orbit?

Assume that the orbits of the planets around the Sun are circular and that all orbits are in the same plane.

[8]

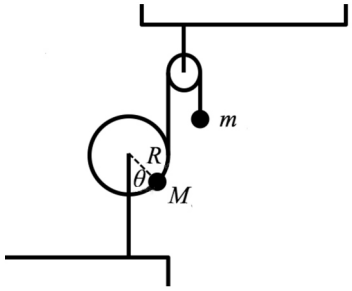
10. (a) A uniform disk of mass $2M$ and radius R, is mounted on a frictionless horizontal pivot through its principal axis. The disk has an additional point-mass, $m = M$, fixed to a point on its circumference. (Hint: The moment of inertia of a disk or mass M_{disk} and radius R_{disk} is $\frac{1}{2}M_{disk}\ \mathrm{R}^2_{disk}$)

(i) Write the Lagrangian for this system. [2]

(ii) Find the frequency of small oscillations of the weight about its lowest point. [3]

(iii) The disk is now placed on its rim on a flat surface and set in motion such that it can roll without slipping. Show that the period of small oscillations has increased by a factor of $\sqrt{3/2}$ relative to the result obtained in part (b). [8]

(b)A weight with mass M, is attached to the rim of a massless, frictionless pulley, mounted through its axis and linked by a massless string to a second weight of mass m ($m < M$) suspended from a second massless, frictionless pulley, as shown in the figure.



(i) Find the equation of motion of the weight M and the condition for equilibrium of the system in terms of the two masses and the angle θ. [3]

(ii) Find the frequency of small oscillations of this system about the point of equilibrium. [4]

11. A rocket of initial total mass m_i leaves a space station in deep space, far from all sources of gravitation. The rocket ejects burnt fuel at a constant speed $u \ll c$ relative to the rocket. Derive an expression for the velocity of the rocket relative to the space station when the total mass of the rocket plus remaining fuel is m_f. [6]

Some time after the rocket has left the space station, a small quantity of the ejected fuel remains stationary in the frame of the space station. What limit does this fact place upon the fraction of the original total mass that was burnt as fuel? [3]

The rocket ejects fuel at a rate proportional to its remaining total mass m, so that $\frac{dm}{dt} = -\alpha m$. Derive an expression for the distance x between the space station and the stationary burnt fuel. Obtain a value for x given that $u = 1000\,\mathrm{ms}^{-1}$, and that m is reduced to half its starting value after 10 minutes. [11]