A10287W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 5 June 2023, 2.30 pm – 5.00 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Use dimensional analysis to calculate (to within a dimensionless constant):

(a) The speed of sound in a gas, given that it only depends on its pressure P and density ρ;

(b) the velocity of a transverse wave on a stretched string, given that it only depends on the length l, the tension T, and the mass per unit length μ of string. [4]

2. A particle moves with constant speed v_0 on a spiral trajectory given by $r(t) = Ae^{k\theta(t)}$, where A and $k > 0$ are constants; r and θ are polar coordinates.

(a) Determine the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ as functions of r and θ.

(b) Show that $\mathbf{v} \cdot \mathbf{a} = 0$ at all times. [6]

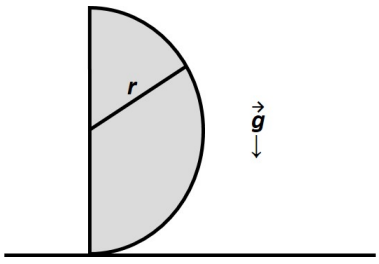
3. A ball is projected from the ground with speed v_0 at 45 degrees to the vertical. It strikes a wall 4 m away and rebounds elastically. The acceleration due to gravity is g.

(a) Calculate the height h of the point where the ball hits the wall.

(b) Calculate the distance from the wall at which the ball hits the ground. [6]

4. (a) Calculate the distance of the centre of mass of a half-cylinder of radius r from the centre of curvature (i.e., from the centre of the cylinder from which the half-cylinder is formed); also calculate its moment of inertia about an axis passing through the centre of mass and parallel to the cylinder axis. Hint: the moment of inertia of a homogeneous disk of radius R and mass M about an axis perpendicular to its centre is $\frac{1}{2}MR^2$. [3]

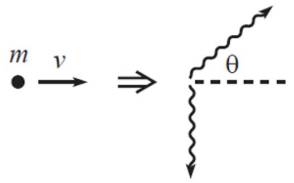
(b) A half-cylinder of radius r is initially held at rest with its flat face vertical and then released. Calculate the angular velocity of the half-cylinder, about the cylinder axis, when the kinetic energy reaches its maximum value. Assume that the half-cylinder rolls without slipping. The acceleration due to gravity is g.



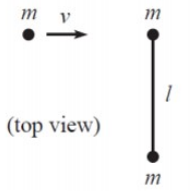
[6]

5. Three relativistic rockets A, B and C are travelling along the same straight trajectory. Rocket A, at the back, is moving at speed $v_A = 4c/5$; rocket C, at the front, is moving with speed $v_C = 3c/5$. At what speed would rocket B, in the middle, have to move for both rockets A and C to appear to be approaching at the same speed? What is this speed? [5]

6. A mass m moving at speed v decays into two photons. One photon is emitted perpendicular to the original direction, and the other photon at an angle θ as shown in the figure. Show that if $\theta = \arctan 1/2$, then $v/c = (\sqrt{5} - 1)/2$. [5]



7. A dumbbell consisting of two equal masses m joined by a massless rod of length l is initially at rest on a frictionless, horizontal surface. An object, also of mass m, moving at speed v perpendicular to the dumbbell axis, collides with the mass at one end of the dumbbell and sticks to it. Assume that all masses have negligible size compared to l.



(a) Calculate the resulting angular velocity ω of the system about a vertical axis through the new centre of mass.

(b) Calculate the speed of the end of the dumbbell that has the two masses on it, after the rod has made one half of a revolution after the collision. [5]

Section B

8. A particle of mass m, angular momentum L and energy $E > 0$, moves in a central potential $V(r) = \dfrac{\beta}{r^2}$; where β is a constant.

(a) Write down the effective potential in which the particle moves. [2]

(b) In the case $\beta > -L^2/2m$, solve the equations of motion, and obtain the expression of the trajectory of the particle r in terms of the polar angle θ. Hint: use the substitution $y = \dfrac{1}{r}$.

Plot the effective potential and show that the trajectory has the form

$\dfrac{1}{r} = \dfrac{1}{a}\sqrt{\dfrac{2mE}{L^2}}sin(a\theta)$, where $a^2 = 1 + \dfrac{2m\beta}{L^2}$.

Calculate the minimum value of r.

Discuss the trajectory when $\beta = 0$

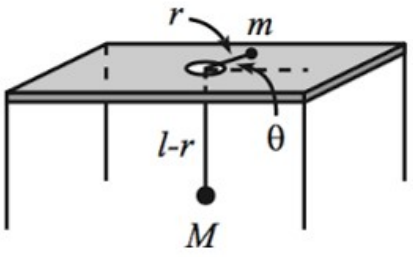
[10]

(c) In the case $\beta = -L^2/2m$, plot the effective potential and show that

$r = \dfrac{1}{\theta}\sqrt{\dfrac{L^2}{2mE}}$.

Does the particle moving from a finite radius on this trajectory towards the origin, reach the origin in finite time? Why? [8]

9. (a) A point mass m is free to slide on the surface of a frictionless table; m is connected to a point mass M that hangs below the table by a string that passes through a small hole in the table as shown in the figure. Assume that M only moves in a vertical line, and that the string is always taut. The gravitational acceleration is g.



(i) Write down the Lagrangian and find the equations of motion for the variables r and θ as shown in the figure. [4]

(ii) Find the conditions under which m undergoes a circular motion.

Calculate the frequency of small oscillations in the variable r about this circular motion. Discuss the values of the frequency for $m \gg M$, $m \ll M$ and $M = 2m$. [6]

(b) A pendulum consists of a mass m connected by a massless rod of length l to a support. The support oscillates horizontally: $x(t) = A\cos\omega t$, where A and ω are constants.

(i) Write down the Lagrangian and the equations of motion of the system. [4]

(ii) Find the general solution for the angle θ of the pendulum as a function of time for small values of θ.

[6]

10. (a) A hot air balloon of constant mass M supports a bag of sand with initial mass m_0. The balloon is initially stably at rest. At a time t_0 the sand starts to leak from the bag at a constant rate, so that the bag becomes empty at t_f. Find the height of the balloon and its velocity at t_f.

[10]

(b) An object of negligible mass starts to slide straight down an inclined plane at time $t = 0$. The object collects dust as it goes, such that its mass is σx, where x is its position measured along the plane and σ is a constant. Friction between the object and the plane can be neglected and the acceleration due to gravity is g. Show that the acceleration of the object is $a = \dfrac{g \sin\theta}{3}$.

Hint: Use the trial solution $x = k\dfrac{t^2}{2}$, where k is a constant.

[10]

11. (a) Twins A and B leave from the same point with their clocks both reading zero. They travel in opposite directions with relative speed v as viewed from either of their rest frames. When B's clock reads T, B sends out a light signal. Considering relativistic effects, calculate the time shown on A's clock, when A receives the signal. Answer this question by doing the calculation entirely in (i) A's frame, and then (ii) B's frame. Why is the time on A's clock the same when observed from either frame? [10]

(b) Twin A stays on the Earth, and twin B flies at speed v to a distant star and back. The star is at a distance L from the Earth. A sends out flashes at intervals of t seconds (as measured in A's frame).

Show that the numbers of red-shifted flashes n_b, and blue-shifted flashes n_r that B receives can be calculated using the expressions: $n_r = \dfrac{1}{t}(\dfrac{L}{v} - \dfrac{L}{c})$ and $n_b = \dfrac{1}{t}(\dfrac{L}{v} + \dfrac{L}{c})$.

By considering the numbers of red-shifted and blue-shifted flashes that B receives, show that B is younger by a factor γ when B returns, i.e. show that if the elapsed time measured by A and B is T_A and T_B respectively, then $T_B = T_A/\gamma$. [10]