A10288W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday 6 September 2022, 9.30 am – 12.00 noon

Time allowed: 2$\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

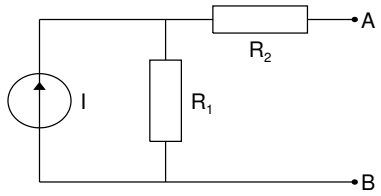
A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

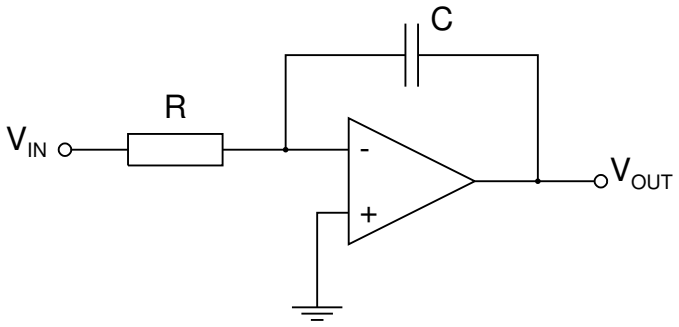
Section A

1. The circuit below consists of an ideal current source, generating current I, and two resistances: R_1 and R_2.



Derive expressions for the voltage V_{eq} and resistance R_{eq} in the Thévenin equivalent circuit between points A and B. [3]

2. (a) Derive an expression for the output voltage $V_{\text{OUT}}(t)$ in terms of the input voltage $V_{\text{IN}}(t)$ in the following circuit containing an ideal op amp



[4]

(b) What would the relationship between V_{OUT} and V_{IN} be if the resistor and capacitor were exchanged? [3]

3. (a) A cube centred at the origin has sides of length $2a$ parallel to the x, y, z axes. Point charges $+q$ are located at the four vertices defined by $z = +a$, and $-q$ at the four vertices $z = -a$. Show that a charge $-q$ located at the origin experiences a force in the $+z$-direction given by

$$F_z = \left(\frac{2}{3\sqrt{3}}\right)\frac{q^2}{\pi\epsilon_0 a^2}.$$

[3]

(b) Suppose that the positive and negative charges at two opposite corners of the cube are exchanged. What is the vertical component F_z of the force now acting on the $-q$ charge at the origin? [2]

4. (a) Show how Gauss' Law leads to the following expression for the capacitance of a pair of coaxial, conducting cylinders of length l

$$C = \frac{2\pi\epsilon_0 l}{\ln(b/a)}$$

where a, b are the radii of the inner and outer cylinders respectively, and $b \ll l$. [3]

(b) Show that, as $a \to b$, this approaches the expression for the capacitance of a parallel plate capacitor. [2]

5. Starting from the Biot-Savart Law, show that the magnitude of the magnetic flux intensity at distance a from an infinite straight wire carrying current I is given by

$$B = \frac{\mu_0 I}{2\pi a}$$

and show that this is consistent with Ampère's Circuital Law.

You may assume the following indefinite integral:

$$\int \frac{\mathrm{d}x}{(a^2+x^2)^{3/2}} = \frac{x}{a^2(a^2+x^2)^{1/2}}.$$ [4]

6. The total electromagnetic energy U in volume $\mathcal{V}$ is given by

$$U = \int_{\mathcal{V}} \frac{1}{2}\left(\epsilon_0 \mathbf{E}.\mathbf{E} + \frac{1}{\mu_0}\mathbf{B}.\mathbf{B}\right) \mathrm{d}\mathcal{V}.$$

Using Maxwell's equations in free space, show that for a surface area element $\mathrm{d}\mathbf{a}$,

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\oint_S \mathbf{N}.\mathrm{d}\mathbf{a} \quad \text{where} \quad \mathbf{N} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}$$

and give a physical interpretation of the vector $\mathbf{N}$. [6]

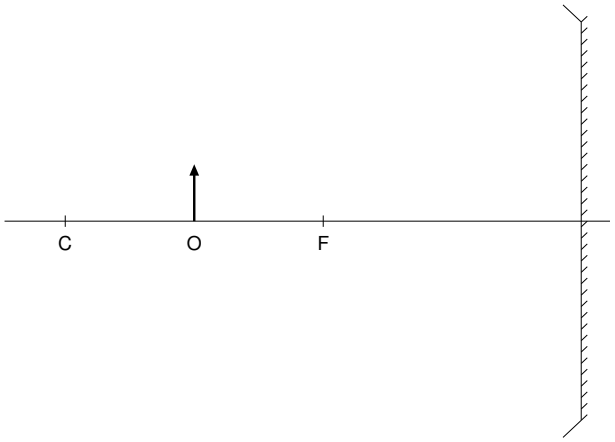
7. (a) Assume that a parabolic mirror focuses all rays parallel to the optical axis to a single point. By considering the reflection of a suitably chosen ray, or otherwise, show that a parabolic mirror surface of the form $y = ax^2$ has its focus at $f = a/4$. [2]

(b) Now assume, instead, that there is a spherical mirror surface following

$$(y-r)^2 + x^2 = r^2$$

where r is the radius of curvature. Show that, near the origin, this behaves similarly to a parabolic mirror and determine the focal distance in terms of r. [3]

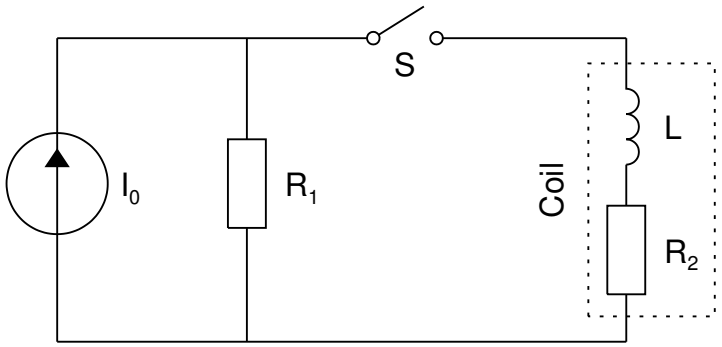
8. The sketch shows an object (O) placed somewhere between the centre of curvature (C) and the focal point (F) of a spherical concave mirror. Reproduce the sketch and, assuming the paraxial approximation, add construction rays to show the location and orientation of the image, and whether it is larger or smaller than the object.



[5]

Section B

9. A constant current source I_0 is connected initially across a resistance R_1 as shown in the figure below. At a time $t = 0$ the switch S is closed, connecting, in parallel with R_1, a coil of self-inductance L and a series resistance R_2.



Find

(i) the differential equation for the current I_c through the coil, [3]

(ii) the value of I_c at any subsequent time, [7]

(iii) the energy stored in the inductor at time $t_c = L/(R_1 + R_2)$, and [4]

(iv) the energy dissipated as heat within the coil in the time interval between $t = 0$ and $t = t_c$. [6]

10. A circular loop of radius a and carrying a current I lies in the y, z-plane.

(a) Show that the magnetic flux intensity at a distance x along the axis of the loop is given by

$$\mathbf{B} = \frac{\mu_0 I}{2}\left(\frac{a^2}{(x^2+a^2)^{3/2}}\right)\hat{\mathbf{x}}.$$ [5]

(b) The axial component of the electric field from an electric dipole of moment p is given by

$$\mathbf{E} = \frac{p}{2\pi\epsilon_0 x^3}\hat{\mathbf{x}}.$$

Explain why the circular current loop may be considered to act as a magnetic dipole and suggest a suitable expression for the magnetic dipole moment which can be applied to any shape of loop. [3]

(c) Defining B as the x-component of $\mathbf{B}$, derive an expression for $\mathrm{d}B/\mathrm{d}x$ and sketch the variation of $\mathrm{d}B/\mathrm{d}x$ from $x \ll -a$ to $x \gg +a$. [6]

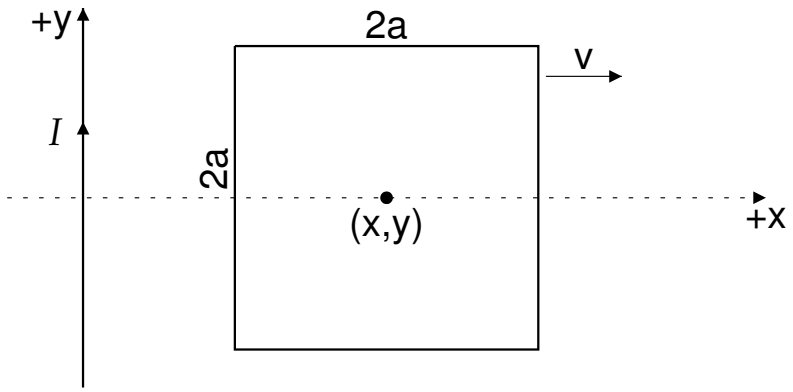
(d) Determine the x values at which $\mathrm{d}^2B/\mathrm{d}x^2 = 0$, hence suggest a location where a second, identical coil might be placed so as to ensure a uniform B field in the x-direction. [6]

11. (a) Explain how the Lorentz force leads to the concept of electromotive force (EMF) in a wire moving in a magnetic field. [4]

(b) A steady current I flows along the y-axis creating an axi-symmetric magnetic field

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

where r is the radial distance from the wire.



As shown in the above figure, a square loop of wire, with sides of length $2a$, lying in the x-y plane, moves at speed v in the $+x$ direction within the magnetic field created by the steady current.

Calculate the EMF induced in each of the four sides of the loop as a function of the x coordinate of the centre of the loop, and show that these sum (using the anticlockwise path integration convention) to give a net EMF

$$\mathcal{E} = -\frac{v\mu_0 I}{\pi}\left(\frac{a^2}{x^2 - a^2}\right).$$ [6]

Give a physical explanation in terms of the Lorentz force for the direction in which the induced current will flow around the loop. [2]

(c) Show that $-\mathcal{E}$ is also the rate of change of magnetic flux through the loop, and that this is consistent with one of Maxwell's Equations. [8]

12. A Young's double-slit experiment is set up with a beam of monochromatic light of wavelength λ parallel to the x-axis normally incident on two long, thin, parallel slits placed at $(x, y) = (0, \pm s/2)$. The light emerging from the slits illuminates a screen at $x = D$. Assume that $s \ll D$ and that the apparatus is in a vacuum.

(a) Show that the relative path difference δ from the two slits to a point situated at position y on the screen (y is not necessarily small) is given by

$$\delta \simeq \frac{sy}{(D^2 + y^2)^{1/2}}.$$ [5]

(b) Assuming that the wave amplitude incident on the slits is given by

$$\Psi(x, t) = A \cos(kx - \omega t)$$

show that the intensity distribution on the screen has the form

$$I(y) = I_0 (1 + \cos(ay))$$

and determine expressions for I_0 and a in the limit $y \ll D$. [8]

If $D = 1$ m, $s = 1$ mm and $\lambda = 532$ nm, calculate the distance from the central maximum to the first minimum. [3]

(c) Suppose now a glass slide of refractive index $n = 1.5$ and thickness 1 mm is used to cover one slit, shifting the diffraction pattern on the screen. The direction of the incident beam is then rotated away from normal incidence ($\theta = 0$) until the diffraction pattern is restored to its original position. At what angle θ does this occur? [4]