A10287W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 5th September 2022, 9.30 am – 12.00 noon

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

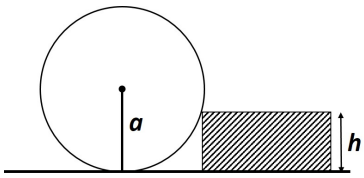
1. In a nuclear explosion there is an approximately instantaneous release of energy E in a small region of space. This produces a spherical shock wave. Assuming that it only depends on E, the time t, and the initial air density ρ_0, use dimensional analysis to calculate how the radius R of this shock wave grows with t. [4]

2. State the parallel and perpendicular axis theorems. [2]

Determine the moment of inertia of a solid cylinder of radius R and length h about an axis through its centre of mass and perpendicular to its principal axis. Hint: the moment of inertia of a uniform disk of mass M and radius R is equal to $\frac{1}{2}MR^2$. [3]

3. Consider the orbit of a point mass in an elliptical orbit about a massive object. Starting from conservation of angular momentum show that the radius vector joining the point mass and the massive object sweeps out equal areas of the orbit in equal times. [6]

4. A uniform solid ball of radius a rolling with velocity v on a level surface collides inelastically with a step of height $h < a$, as shown in the figure. Find the minimum velocity for which the ball will reach the top of the step, in terms of h and a. You may assume that no slipping occurs at the impact point; the moment of inertia of a solid sphere with respect to an axis through its centre is $\frac{2}{5}Ma^2$, where M is the mass of the sphere.



[6]

5. Consider a rocket starting at rest in free space (in the absence of gravitational fields), find the rocket mass (in terms of the initial mass) at which the momentum of the rocket is maximum, including the remaining fuel. You may assume that the exhaust velocity is a constant v_o. [4]

6. A projectile is launched with an initial speed V. At what angle should the projectile be fired so that it travels the maximum distance by the time it returns to the ground? The acceleration of gravity is g. [3]

What is the optimal launch angle if the ground slopes upwards by an angle β? [4]

7. Write down the one-dimensional Lorentz transformation formulas for x and t between two reference frames S and S' moving with relative velocity v. [2]

A train with proper length L moves at speed $5c/13$ with respect to the ground. A ball is thrown from the back of the train towards the front. The speed of the ball with respect to the train is $c/3$. For someone on the ground, how long does the ball spend in the air? How far does it travel? [6]

Section B

8. A raindrop nucleates in uniform fog. It then falls, sweeping up the fog droplets which lie in its path, increasing its mass M. The raindrop retains all the fog which it collects. Let ρ_1 and ρ_2 be the densities of the raindrop and fog respectively, and $R(t)$ the radius of the drop. The rate of accretion ($\dot{M}$) is equal to the cross-sectional area of the raindrop, times the velocity of the rain drop $V(t)$, times ρ_2. The raindrop remains spherical and experiences no viscous drag. The collisions between the raindrop and the droplets are completely inelastic and the buoyancy of the air is neglected.

(a) Show that the equation describing the evolution of R in a gravitational field with constant acceleration g, is $g\frac{\rho_2}{\rho_1}R = 12\dot{R}^2 + 4R\ddot{R}$. [7]

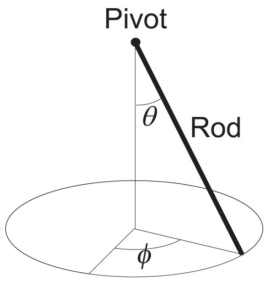
(b) Use dimensional analysis to obtain a solution of the equation obtained in (a). (Hint: R only depends on $\tilde{g}$ and t, where $\tilde{g} = g\frac{\rho_2}{\rho_1}$) [7]

(c) Demonstrate that the acceleration of the raindrop is $g/7$. [6]

9. a) A uniform thin rod of length l, mass m and moment of inertia I hangs from a frictionless pivot at one end and is free to swing in a vertical plane. Write down the Lagrangian of this system. [4]

Use the Euler-Lagrange equation to obtain the equation of motion of the rod and hence find the period of oscillation in the small angle approximation, in terms of l, I, and the acceleration due to gravity g. [7]

b) A rod similar to that in part (a) also hangs from a frictionless pivot at one end, but is free to swing in *any direction*. Write down the Lagrangian of this system in terms of g, m, l and the generalized co-ordinates θ and ϕ, where θ is the angle between the rod and the vertical, and ϕ is the angle of rotation of the rod about the vertical axis, as shown in the figure. [6]



Identify the cyclic co-ordinate and the corresponding conserved quantity, and write down an expression that relates the two.

Explain whether or not the Hamiltonian of the system is a constant of motion. [3]

10. Write down the Lorentz transformations for energy and momentum. Show that $E^2 - c^2p^2$ is invariant under the Lorentz transformation. [6]

An electron is accelerated through a potential difference until it has a total energy E, and collides with a second electron which is initially at rest in the laboratory frame. After the collision, the two particles travel with paths inclined to the original track of the first particle by $\pm\theta$. By considering the motion of the particles in the lab frame, find an expression for θ in terms of E and E_0, the rest energy of the electron. [10]

The angle of deviation of each electron is measured to be 30 degrees. What was the initial accelerating potential? [4]

11. (a) Write the equations of motion for a particle of mass m, subject to a central force $F(r)$ in the polar coordinates of the plane of motion r and θ. [3]

Find the central force which results in an orbit for the particle described by $r = a(1 + \cos\theta)$, where a is a constant. [8]

(b) Now consider a different particle of mass m that is acted on by an attractive force whose potential is given by $U \propto r^{-4}$. Write the equation of the effective potential U_{eff} and calculate the distance r_0 where U_{eff} is maximum. Calculate the maximum U_{eff} and make a plot of U_{eff} vs. r. [4]

Find the impact parameter b for capture of the particle of mass m coming from infinity with an initial velocity v_∞, where $b = \dfrac{L}{mv_\infty}$; L is the angular momentum. Hint: Consider that only particles with total energy larger than the maximum U_{eff} will be captured by the force field. [5]