A10287W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 6 June 2022, 2.30 pm – 5.00 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

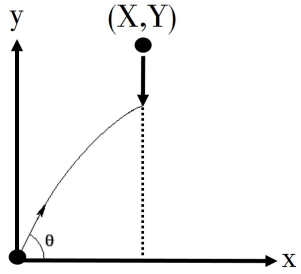
The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Use dimensional analysis to calculate (to within a dimensionless constant) the orbital period T of a planet moving in a circular orbit of radius R around a star of mass M. Consider that the orbital period depends on M, R and G (the universal constant of gravitation); the planet has a much smaller mass than the star. [3]

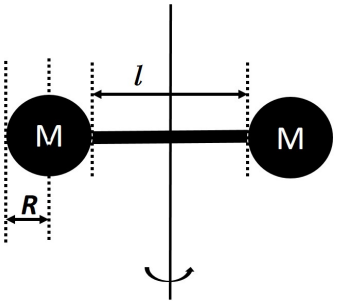
2. A particle is projected from the ground with a velocity u_0 at time $t = 0$; θ is the angle of projection. A target particle is released from rest at the point (X, Y) at $t = t_0$, and it falls under the effect of gravity from a point above the highest point reached by the projected particle, as shown in the image. Neglecting air resistance, demonstrate that the condition for the particles to collide is $\sin\theta = A\cos\theta + B$, where $A(t_0) = \dfrac{Y}{X} - \dfrac{gt_0^2}{2X}$, and $B(t_0) = \dfrac{gt_0}{u_0}$. [6]



3. (a) Show that the moment of inertia of a thin uniform solid rod of mass m and length l about an axis that is perpendicular to the rod and passes through its centre of mass is $\frac{1}{12}ml^2$. [3]

(b) Show that the moment of inertia of a solid, uniform sphere of mass M and radius R about an axis passing through its centre of mass is $\frac{2}{5}MR^2$. [3]

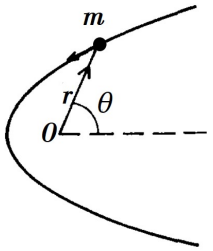
(c) Find the moment of inertia of a dumbbell made up of two identical solid spheres of mass M and radius R connected by a rod of mass m and length l, about an axis that is perpendicular to the rod and passes through its centre (see figure below). [3]



4. A particle of mass m with energy E, and angular momentum L about the origin, experiences an attractive central potential $V = -G(r)$ centred on point 0, as shown in the figure below. The position of the particle is given by the polar coordinates (r, θ).

(a) Find an expression for $dr/d\theta$ in terms of E, L, $G(r)$, r, and the particle mass m.

(b) In the special case $G(r) = \beta r^{-1}$, find an equation for the distance of closest approach, r_{min}, in terms of E, L and m. [7]

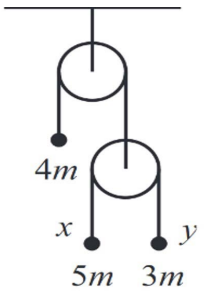


5. Consider the Atwood's machine shown in the figure below with masses $4m$, $5m$, and $3m$; x and y are the heights of the right two masses, relative to their initial positions.

(a) Demonstrate that the Lagrangian of the system is equal to $m(3\dot{x}^2 + \dot{x}\dot{y} + 2\dot{y}^2) - mg(3x + y)$. [4]

(b) Noether's theorem states that for each symmetry of the Lagrangian, there is a conserved "momentum" $\mathbf{P}(\mathbf{q}, \dot{\mathbf{q}})$, where q_i are the spatial coordinates. It can be shown that if the Lagrangian is invariant to continuous transformations of the coordinates $q_i \to q_i + \epsilon K_i$ (where ϵ is an arbitrarily small number), then the conserved momentum $\mathbf{P}(\mathbf{q}, \dot{\mathbf{q}}) \equiv \sum_i \dfrac{\partial L}{\partial \dot{q}_i} K_i$.

Use Noether's theorem to find the conserved momentum of the system (Hint: the Lagrangian is invariant under the transformation $x \to x + \epsilon K_x$, with $K_x = 1$ and $y \to y + \epsilon K_y$ with $K_y = -3$; K_i has length units). [4]



6. A beam of protons having momentum $400\,\mathrm{GeV/c}$ strikes a target, and pions are produced.

(a) Calculate the momentum of the pions, which have the same speed as the protons. The pion rest mass is $0.14\,\mathrm{GeV/c^2}$. The proton rest mass is $0.94\,\mathrm{GeV/c^2}$. [4]

(b) These pions then travel down a decay pipe of 400 m in length where some of them decay. What fraction of the pions emerge from the decay pipe? (Hint: The number of pions in the beam n decays as $n = n_0 \exp(-t/\tau)$, where n_0 is the number of pions at $t = 0$ and τ is the lifetime of the pions in the laboratory frame. The pions' proper mean lifetime is $2.6 \times 10^{-8}\,\mathrm{s}$). [3]

Section B

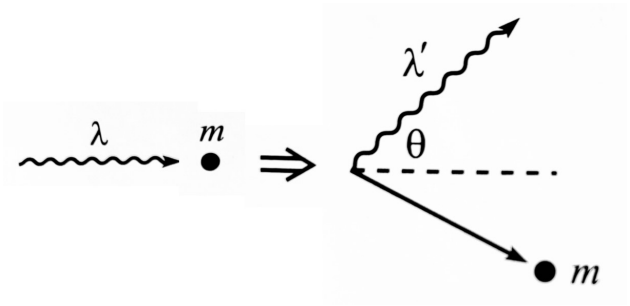
7. (a) Write down the Lorentz transformations which relate the space and time coordinates in two inertial frames, S and S', where S' is travelling with velocity $\mathbf{v}$ relative to S along the positive x direction. [2]

(b) Use the transformations to derive an expression for the phenomenon of length contraction, i.e. the shortening of the measured length of an object moving relative to the observers frame. [4]

Two identical rockets of rest length L_0 are seen from the Earth approaching from opposite directions at velocities $\pm c/2$. How long does one of them appear to the other? [4]

(c) A photon collides with a stationary electron. The photon scatters at an angle θ; demonstrate that the resulting wavelength λ' is given in terms of the incident wavelength λ, by $\lambda' = \lambda + \frac{h}{mc}(1 - cos\theta)$, where m is the mass of the electron. [7]

Discuss the results for $\theta = 0$ and $\theta = \pi$. [3]



8. A particle of mass m moves under the influence of an attractive central force $f(r)$ where r is the distance of the particle from the force centre (which is taken to be the origin).

(a) Write down the effective potential V_{eff} (in terms of the angular momentum J) and the energy of the particle. [2]

(b) Calculate the initial conditions that are consistent with a circular orbit. [3]

(c) Consider the case where the particle moves in a potential $V(r) = \beta r^k$, with angular momentum J, and find the radius r_0 of the circular orbit. [4]

(d) Suppose that a small radial perturbation that produces small oscillations of the orbit can be modelled by the harmonic potential, namely

$$V_{eff} = \frac{1}{2} V''_{eff}(r_0)(r - r_0)^2,$$

where the frequency ω_r of the perturbed orbit can be expressed as

$$\omega_r = \sqrt{\frac{V''_{eff}(r_0)}{m}}.$$

If the circular orbit is subjected to this small radial perturbation so that the radius oscillates around r_0, find the frequency ω_r of the resulting small oscillations in r in terms of J, k, m, and r_0. [7]

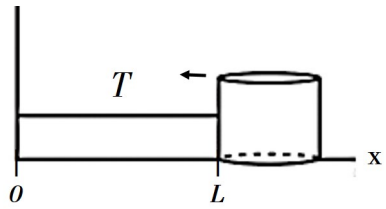
(e) Calculate the ratio of the radial frequency ω_r to the frequency of the (nearly) circular motion $\omega_\theta = \dot{\theta}$. How many radial oscillations occur for each revolution when $k = -1$ and $k = 2$? Sketch the particle trajectory in each case. [4]

9. (a) A massless bucket contains a mass M of sand at $t = 0$. A string with constant tension T (independent of length) pulls the bucket towards the wall, as shown in the image below; the friction with the ground can be neglected. The bucket is initially at rest at a distance L from the wall. When the bucket is released, it moves towards the wall leaking sand at a rate $\frac{dm}{dx} = M/L$, where $m(t)$ is the mass of the sand in the bucket. The bucket empties when it hits the wall.

Calculate the kinetic energy of the sand in the bucket as a function of x (Hint: write the acceleration $\ddot{x}$ as $v\frac{dv}{dx}$).

Identify the position of the maximum value of kinetic energy (Hint: calculate the maximum with respect to $z \equiv \frac{x}{L}$). [5]

Calculate the magnitude of the momentum of the bucket as a function of x, can calculate the maximum value of the momentum. Discuss why the maximum of kinetic energy occurs closer to the wall than the maximum of the momentum. [5]



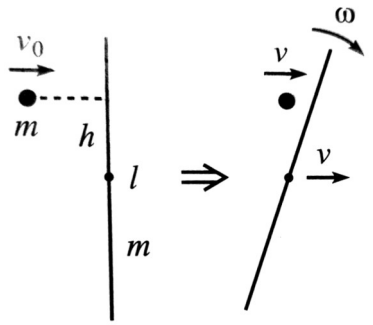
(b) A rocket with initial mass m_0 (including fuel) is launched vertically upwards in a uniform gravitational field with constant acceleration g ($g = 9.8m/s^2$). It ejects fuel with a constant velocity w with respect to the rocket. The fuel is completely ejected during a time T (i.e. T is the "burn-out time"). Find the equation of motion of the rocket ("the Tsiolkovsky rocket equation") in terms of $\frac{dm}{dt}$, w, g, and m, where m is the mass of the rocket at time t, $m(t)$. [3]

Give an expression of the velocity of the rocket $v(t)$, when $dm/dt = -\alpha = constant$. [4]

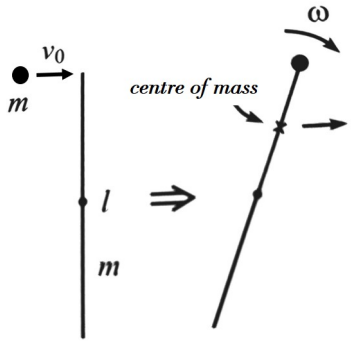
According to the rocket equation, what is the minimum w for the rocket to leave the launch pad? [1]

Calculate the final speed of a rocket that transports a payload of $100kg$, when $\alpha = 500kg/s$, $w = 2500m/s$, and the rocket has an initial mass $m_0 = 10,000kg$. The fuel fraction $f = m_{fuel}/m_0$ is 0.75 (m_{fuel} is the initial mass of the fuel) . [2]

10. (a) A ball with mass m and negligible radius travels at a speed v_0 perpendicular to a rod of mass m and length l, which is initially at rest. At what location should the ball collide elastically with the rod, so that the ball and the centre of mass of the rod move with equal speeds after the collision? (the moment of inertia of the rod is $\frac{1}{12}ml^2$) [7]



(b) A ball with mass m and negligible radius travels at a speed v_0 perpendicular to the rod, which is initially at rest. The ball collides completely inelastically with the rod at one of its ends, and sticks to it. Calculate the resultant angular velocity of the system. [8]



(c) A uniform coin of mass m and radius r rolls down a plane inclined at an angle θ with the horizontal, without slipping. What is its linear acceleration? If the coefficient of static friction between the coin and the plane is μ, what is the largest angle θ for which the coin does not slip? (The moment of inertia of the coin is $\frac{1}{2}mr^2$). [5]