A10288W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

7 September 2021

Opening Time 9.30 am

You have $2\frac{1}{2}$ hours to complete the paper and up to 30 mins technical time to upload your answer file.

Answer **all** *of Section A and* **three** *questions from Section B.*

Write your candidate number, the paper name, paper code and a list of the questions that you have answered on the first page, and submit all of your answers as a single PDF file.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors is available within Inspera.

The numbers in the margin indicate the weight that the examiners expect to assign to each part of the question.

Section A

1. State Gauss's Law and use it to calculate the capacitance of:

(a) two large, flat, conducting sheets of area A, separated by a small distance d,

(b) two concentric conducting spherical shells with radii a and b, where $b > a$,

(c) two concentric conducting cylinders of length L, large compared to their radii a and b, where $b > a$. [6]

2. Many camera viewfinders operate through the lens to provide the photographer with an accurate image for composing a picture. Explain whether the viewfinder image appears upside down to the photographer for:

(a) an old-fashioned camera with the photographer looking from behind the camera directly at a matt screen that is replacing the photographic plate during picture composition. [2]

(b) a single-lens reflex (SLR) camera with a flip mirror inside to produce an image on a matt screen mounted on top of the camera housing, with the photographer bending over the camera from behind and looking down at the screen. [2]

Certainly it is far more convenient for the photographer to stand behind the camera and look forward into the same direction the camera is pointing. For doing so, further mirrors mounted on top of the camera are required to re-direct the image towards the eye. Draw a diagram to illustrate a mirror arrangement for doing so such that the image appears the right-way up, using the minimum number of mirrors. [2]

In any case, the viewfinder image will appear very small at the near point of the eye. To bring the eye closer to a distance of 50 mm, an eyepiece is required. How would a single lens be used to achieve this, and what will be the highest possible magnification? [2]

3. Particles of mass m and positive charge q get accelerated from standstill whilst moving through the electrical potential difference U in a capacitor. After leaving the capacitor through a slit in the cathode, they get forced into a circular trajectory by another uniform field.

What type of field and field direction are required to achieve the latter? Derive the radius R of the particle trajectory as a function of the charge to mass ratio, q/m, and of the other defining parameters. [5]

In extension to the described setup, an additional homogeneous field is applied such that particles of a given q/m ratio continue to move in a straight line after passing through the slit, whilst other particles get deflected. What type of field is required to achieve this, and how does the field strength relate to the charge/mass ratio and all other parameters? [3]

4. Explain what is meant by a complex impedance $Z = X + jY$. State the complex impedances of a resistor R, an inductor L, and a capacitor C. Derive the magnitude and phase of the combined impedance for (a) R and C in series and (b) C and L in parallel. For the latter case, what happens if the angular frequency ω approaches $1/\sqrt{LC}$? [5]

5. Write down Maxwell's equations and show that in a region of zero charge and current density, there exist travelling wave solutions for the electric field. Consider the superposition of two independent travelling waves of different frequencies. Elaborate in detail under which condition one is allowed to simply sum up their average intensities. [3]

How does the maximum electric field strength of a plane travelling wave relate to the intensity of the light, and what is the resulting flux of photons (provided each carries energy hf, where h is Planck's constant and f the frequency of the light)? What is the light pressure on a black absorbing surface as a function of maximum field strength? [3]

6. A single-lens microscope is used to inspect a small translucent object that is illuminated from behind with a strong collimated beam of light. To obtain a 15-fold magnified image at a distance of 200 mm from the object, what is the focal length required for the microscope lens and where should it be placed? [4]

The strong illumination can saturate the eye of the observer. Discuss whether there is a simple way of masking out any direct illuminating light from reaching the image plane, without significantly attenuating the light of interest that is scattered by the object. Where would you place the mask, and what form should it take? [3]

Section B

7. TV satellites in geostationary orbit operate at a carrier frequency of 12 GHz, and receiving satellite dishes on Earth typically have a diameter of 850 mm.

(a) Make use of the Rayleigh criterion to calculate the angular resolution of a satellite dish, and deduce the maximum number of satellites in geostationary orbit that can be resolved. [4]

(b) The dish is a concave mirror with a radius of curvature of 1.5 m. Where is the best place to install the radio-frequency receiver? [2]

(c) Determine the maximum diameter of the receiver such that resolving neighbouring satellites is still feasible. [2]

(d) Much smaller satellite dishes are often installed on caravans, and nonetheless the same angular resolution can be achieved if two coupled dishes are used. Describe how to arrange these for doing so. Determine their optimum separation such that neighbouring satellites can be distinguished easily. [5]

(e) The signal captured by the two small dishes travels along coaxial cables to an electronic device that adds the amplitudes and then measures the overall signal strength. One of the cables is 3.125 mm longer than the other. What happens if the two cables get accidentally swapped? (the radio frequency signal travels at a velocity of c/2 along these cables) Make a sketch comparing the signal strength at the device as a function of the satellite position when the cables are connected in each of the two configurations. [7]

8. (a) A long insulating cylinder of radius R_1 has a cylindrical hole at a distance a from the centre of the cylinder and parallel to the axis of the cylinder. The radius of the hole is $R_2 < (R_1 - a)$. There is a uniform fixed charge per unit volume ρ throughout the cylinder, except in the hole where there is no charge. Show in detail how to evaluate the electric field inside the cylinder and inside the hole as a function of $\mathbf{r}$, the position of interest in a plane perpendicular to the cylinder (you may neglect edge effects at the ends of the cylinder). [8]

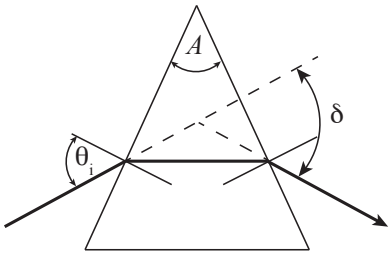
(b) Consider a metallic sphere of negligible radius carrying a charge Q, placed a distance D above an infinite grounded conducting plane. Evaluate the electric field in the region of space outside the sphere and above the conducting plane. [6]

Hence determine the induced charge density on the plane as a function of the distance r from a point on the plane to the point on the plane that is directly below the sphere. [3]

Show that the total induced charge on the conducting plane is $-Q$. [3]

9. State Huygen's Principle, and use it to derive Snell's law of refraction. [3]

The diagram below shows an isosceles prism of apex angle A and refractive index n immersed in air. The prism is illuminated such that the beam inside the prism is parallel to the base.



Use Snell's law to show that

$$n = \frac{\sin\frac{1}{2}(\delta + A)}{\sin\frac{1}{2}A},$$

where δ is the deviation of the beam from its original direction. [7]

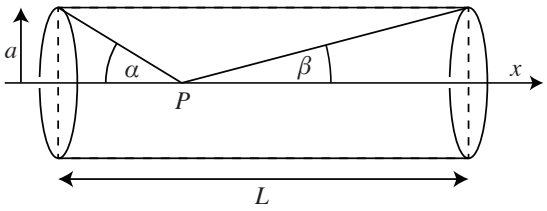
For the case of a dense flint glass prism in air with $A = 55°$, make a reasoned calculation of the angle of incidence required to achieve the above conditions for a beam of white light, and calculate the angular spread of the dispersed beam produced. [7]

What consequence might the above have on the quality of a white light image produced by a simple convex lens? [3]

[The refractive index of dense flint glass at 370 nm is 1.94, at 535 nm is 1.86 and at 700 nm is 1.83]

10. Use the Biot-Savart law to derive an expression for the magnetic flux density on the axis of a circular current loop of radius a carrying current I, as a function of the distance x from the plane of the loop. [6]

A solenoid of length L and radius a has N turns and carries current I.



Show that the magnitude of the flux at a point P on the axis of the solenoid is given by

$$B = \frac{\mu_0 N I}{2L}(\cos\alpha + \cos\beta)$$

where α and β are the angles subtended at P by the radii at the ends of the solenoid. [8]

Determine the ratio of the magnetic flux density at the end of the solenoid to the magnetic flux density in its centre. Simplify this in the limiting case $L \gg a$. [6]