A10288W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday, 8 June 2021

Opening Time 9.30 am UK time

You have $2\frac{1}{2}$ hours to complete the paper and up to 30 mins technical time to upload your answer file.

Answer **all** *of Section A and* **three** *questions from Section B.*

Write your candidate number, the paper name, paper code and a list of the questions that you have answered on the first page, and submit all of your answers as a single PDF file.

The use of approved calculators is permitted.

A list of physical constants and conversion factors will be available within Inspera during the examination.

The numbers in the margin indicate the weight that the examiners expect to assign to each part of the question.

Section A

1. Two spheres of negligible diameter, equal mass m and charge Q are suspended from the same point with thin massless rods of equal length l. Once the arrangement has reached steady state, what is the angle ϕ the rods have from the vertical? Evaluate ϕ for $m = 0.05\,\mathrm{kg}$, $l = 1\,\mathrm{m}$, $Q = 10^{-8}\,\mathrm{C}$. [2]

How large is ϕ if a conductive vertical plate, carrying a surface-charge density of $\sigma = 1.5 \times 10^{-5}\,\mathrm{Cm}^{-2}$, is placed symmetrically between the two charges? [3]

2. An electrical circuit in a black box consists of a resistor R and a capacitor C. If a DC voltage is applied, an overall resistance of $100\,\Omega$ is found. This drops to a resistance of $20\,\Omega$ if an AC voltage of $50\,\mathrm{Hz}$ is applied. Sketch the circuit and derive the values of R, C and Z. [3]

Now we consider a black-box circuit with input voltage $U_1(t)$ and output $U_2(t)$ that shows maximum transmission $|\langle U_2\rangle|/|\langle U_1\rangle|$ for an AC voltage of angular frequency $\omega_0 = 75\,\mathrm{rad\,s}^{-1}$, whilst $U_2/U_1 = 0.01$ is found if a DC voltage is applied. Describe and sketch the circuit assuming it consists of a resistor R, a capacitor C and an inductor L of internal resistance of $R_L = 1\,\Omega$. Derive the values of R and C (for the latter, disregard R_L and assume the inductor has an inductance of $L = 0.1\,\mathrm{H}$). [4]

3. Write down Maxwell's equations in differential form. Show that in a region of zero charge and current density, there exist travelling wave solutions for the fields and deduce an expression for the speed of these waves. Argue why any linear superposition of these travelling waves is also a possible solution. [3]

How does the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ relate to the intensity of a linearly polarised plane travelling wave? Use the average magnitude of the Poynting vector to express the radiation pressure of the wave on a mirror that reflects the wave back into itself. [3]

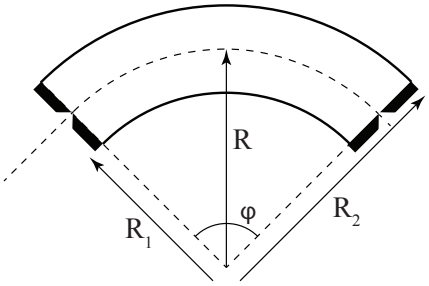
4. A data projector is used to enlarge an internal transparent liquid crystal display (LCD) of 15 mm diagonal size to a projection screen placed 2 m away from the LCD. What focal length of the projection lens is required to obtain an image diagonal of 1 m, and where should it be positioned? [3]

Illumination of the internal LCD is achieved with a point-like light source, placed on the optical axis 100 mm from the LCD, in combination with another lens of 30 mm diameter somewhere in between. If you assume the diameter of the projection lens is very small, then where should this second lens be placed to achieve equally distributed maximum brightness in the image, and what focal length should it have? [4]

5. Draw a simple circuit diagram in which a pair of ideal operational amplifiers is used to amplify and then integrate the input voltage V_{in}. Derive how the output voltage V_{out} depends on the input voltage V_{in} and the values of any other components used in your circuit. [3]

Modify the circuit to compensate for a non-zero offset in the input voltage. [2]

6. A capacitor consisting of two concentric sections of conductive cylinders with radii R_1 and R_2 and opening angle ϕ is put into vacuum. Each side is closed off by non-conductive apertures with small slits at $R = (R_1 + R_2)/2$.



What voltage U is required between the two cylindrical plates such that electrons of tangential velocity v_0 travel through both slits? [4]

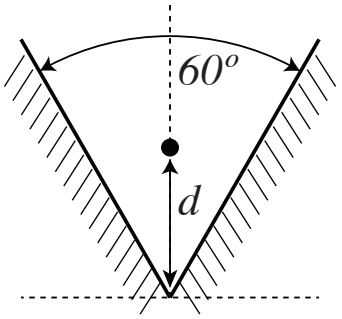
7. Explain how the Rayleigh resolution criterion in optics relates to the width of a single diffracting slit. Use this to determine the angular resolution limit of a mobile-phone camera with a lens diameter of 1.25 mm at $\lambda = 500$ nm. [3]

An image is formed 5 mm behind this lens on a CCD sensor that has an active area of 4 mm x 3 mm. Estimate the minimum number of photosensitive elements (pixels) the sensor needs to have to make full use of the optical resolution. [3]

Section B

8. A charge q is positioned a distance d above a grounded infinite conducting plane. A second charge of magnitude $-q$ is placed a distance $2d$ above that plane and along the line which is perpendicular to the plane and passes through the first charge. Derive an expression for the electric field at the point midway between the two charges. [6]

Two grounded semi-infinite conducting planes meet with an angle of 60 degrees between them as shown in the figure below. A charge q is placed at a point midway between these planes and at a distance d from their line of intersection. Draw an appropriate arrangement of image charges that will reproduce the desired potential. [6]



Derive the electrostatic force felt by the original charge. [8]

9. A parallel beam of monochromatic light of wavelength λ illuminates two narrow parallel slits in a plane parallel to the incoming wavefronts and separated by s. The interference pattern is observed on a screen parallel to the original wave fronts at a distance d from the slits, where $d \gg s^2/\lambda$.

(a) Derive an expression for the intensity distribution observed on the screen as a function of the position x in the plane of the screen. [4]

(b) How does this expression change if a lens of focal length $f < d$ is placed at a distance f in front of the screen? [4]

(c) Assume a slit separation of $s = 1.0\,\text{mm}$ and the screen at distance $d = 1.0\,\text{m}$ (without the lens). If one slit is now covered by a 0.1 mm thick glass slide of refractive index $n = 1.5$, in what direction and how far does the central fringe move? [5]

(d) The monochromatic light source is replaced by a source emitting two wavelengths of 520 nm and 540 nm. With both slits uncovered, determine the number of fringes one may clearly observe to both sides of the central maximum, and sketch the intensity distribution as a function of the diffraction angle from -1^o to $+1^o$. [7]

10. A circuit consists of a resistor R, an inductor L and a capacitor C connected in series to a voltage source V_{in} of amplitude V_0 and angular frequency ω. What is the impedance of the circuit as a function of ω? [3]

Find an expression for the relationship between the output voltage V_{out}, measured across the capacitor, and V_{in}, determining both the relative amplitude and phase ϕ as a function of frequency. [7]

With the quality factor Q of the circuit being defined as $|V_{\text{out}}/V_{\text{in}}|$ at $\omega_0 = 1/\sqrt{LC}$, determine Q in terms of R, L and C. [5]

Show that V_{out} is maximum at the resonance frequency

$$\omega_{\text{res}} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

and discuss the minimum value of Q required for a resonance to occur. [5]

11. State the laws of electromagnetic induction explaining clearly all the terms. [3]

A long air-cored solenoid of length D and radius a has n turns per unit length. Derive an expression for its self-inductance, assuming $D \gg a$. [5]

The solenoid is connected to a current source $I = I_0 \cos \omega t$. Derive the voltage induced across the solenoid as a function of time and determine the average electrical power consumption. [4]

A secondary coil consisting of a single turn, wound closely round the exterior of the solenoid, has self-inductance L_s and is in series with a lamp of resistance R. Obtain expressions for the magnitude and phase of the current flowing through the lamp. Furthermore calculate the average electrical power consumed by that lamp. [5]

Explain why this differs from the power consumption of the solenoid alone, and discuss qualitatively how this discrepancy can be explained. [3]