A10287W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday, 6 September 2021

Opening Time 9.30 a.m. (BST)

You have $2\frac{1}{2}$ hours writing time to complete the paper and up to 30 mins technical time to upload your answer file.

Answer **all** *of Section A and* **three** *questions from Section B.*

Write your candidate number, the paper name, paper code and a list of the questions that you have answered on the first page, and submit all of your answers as a single PDF file.

The use of approved calculators is permitted.

A list of physical constants and conversion factors will be available within Inspera during the examination.

The numbers in the margin indicate the weight that the examiners expect to assign to each part of the question.

Section A

1. A solid cylinder of mass m and radius r is at the top of a ramp, which is a height h above the horizontal ground. It rolls down the ramp to the bottom without slipping and collides elastically with a stationary particle of mass $2m$ suspended on a massless rod of length h, along the line of their centres. What is the maximum angle that the suspended mass will reach in the subsequent swing? [7]

2. A particle of mass m is travelling with initial speed u along the x- axis when it collides elastically and obliquely with a more massive stationary particle of mass M. Show that the angle of deflection θ of the mass m is related to the angle of deflection ϕ of the mass M by

$$\tan\,\theta = \left(\frac{\sin 2\phi}{\frac{m}{M} - \cos 2\phi}\right).$$

[8]

3. A uniform disk of mass M and radius R rotates without friction about a vertical axis with angular velocity ω_0. Sand falls vertically onto the disk at a constant mass rate α and piles up at a radial distance r from the axis. Find expressions for the angular velocity as a function of time and its rate of change, and hence find an expression for the rotational energy. At what time does the rotational energy reach half the initial value? [8]

4. Two particles have masses m_1 and m_2 and position vectors $\mathbf{r_1}$ and $\mathbf{r_2}$ relative to the centre of mass respectively, together with a potential $V(r)$. Write down the Lagrangian of this system and show how it may be expressed in terms of the reduced mass $\mu = \frac{m_1 m_2}{m_1+m_2}$ and the separation of the masses $\mathbf{r}$. What is the advantage of expressing the Lagrangian in terms of the reduced mass? [6]

5. At what speed is the kinetic energy of a relativistic proton greater by 2% than a classical calculation would indicate? What is its total energy at that speed? [5]

[The rest mass energy of a proton is 938 MeV]

6. A Spacecraft A is launched from a stationary space station at speed $0.5\,c$ towards spacecraft B, which is travelling at speed $0.4\,c$ in the space station's frame directly away from spacecraft A. A probe with speed $0.5\,c$ (in A's frame) is launched from A and travels directly towards B. The probe transmits a signal at a frequency of 100 MHz. To what frequencies should observers on the space station and on spacecraft B tune their receivers to collect the signal? [6]

Section B

7. A two-stage rocket has a total launchpad mass of 5.4×10^5 kg and is launched vertically from the surface of the Earth. The mass of the second stage and payload is 1.0×10^5 kg. The first stage rocket ejects gas at a constant rate of $2.5 \times 10^3\,\mathrm{kg\,s^{-1}}$ at a velocity of $2.5 \times 10^3\,\mathrm{m\,s^{-1}}$. Once the fuel used in the first stage launch (4.0×10^5 kg) is exhausted, the first stage separates from the rest of the rocket and eventually returns to Earth for re-use; an additional mass of fuel is reserved in the first stage rocket and used during re-entry to control the speed during descent. Once the first stage fuel has been used up, the mass remaining in the 1st stage rocket is 4.0×10^4 kg, consisting of the rocket casing (1.4×10^4 kg) and the fuel reserved for re-entry (2.6×10^4 kg).

Ignoring the effects of air resistance, find an expression for the rocket speed at the point that the first stage fuel is exhausted, stating any assumptions made, and calculate its magnitude. [6]

Find an expression for and calculate the maximum height reached by the first stage. [8]

After reaching maximum height, the first stage rocket falls back towards the Earth, until it reaches an altitude of 100 km. At that point, the re-entry rocket is ignited and the reserved fuel is used to slow the descent of the first stage rocket. The time taken for the 2.6×10^4 kg of fuel to be expended is 92 s, and gas is emitted at a constant rate with an ejection speed of $2.5 \times 10^3\,\mathrm{m\,s^{-1}}$. What is the speed of the first stage rocket at burnout of the fuel reserve? [6]

The masses of the rocket parts are:

Total mass at launch	5.4×10^5 kg
Fuel expended in first stage launch.	4.0×10^5 kg
First stage rocket casing	1.4×10^4 kg
Fuel reserved for first stage re-entry	2.6×10^4 kg

8. State the parallel and perpendicular axis theorems, defining carefully all terms. [3]

Derive the moment of inertia of a uniform cylindrical rod with mass m, length l and radius r about its centre and perpendicular to its length, and about its end. Find the relationship between the cylinder's length and radius for the case that the moment of inertia about its centre is equal to the moment of inertia about its principal axis. [7]

A model hammer is represented by a compact mass M, attached to a long, solid cylindrical rod of mass m, radius r and length l such that the centre of the mass M coincides with one end of the rod. The hammer is suspended about a frictionless pivot at the other end of the rod. Find an expression for the forces acting when the hammer is displaced from the vertical position, and hence an expression for the period of oscillation.

For the case that $m = M/10$ and $r = l/20$, compare the period of small oscillations with that of a simple pendulum with mass M and length l. [10]

9. a) A supply ship attempts to dock with a space station in a circular orbit around the Earth, with orbital radius r_o. The docking manoeuvre fails and increases the kinetic energy of the space station by 1% while leaving the angular momentum unchanged. Find an expression for the radii of the new space station orbit at perigee and apogee (the nearest and furthest positions in the orbit). [8]

b) The initial orbit of the space station was 500 km above the surface of the Earth. What fractional change in energy would produce a new orbit with a closest approach to the surface of the Earth of 100 km? Take the Earth to be spherical with a radius of 6400 km. [6]

c) Consider a spacecraft positioned along a line between the Earth and the Moon. Find the point where the net gravitational force on the spacecraft from the two bodies is zero, and thus find an expression for its distance from the centre of mass of the Earth-Moon system. Calculate its value. Take the mass ratio of the Earth and Moon to be 81 and their separation to be 3.84×10^8 m. You may ignore any gravitational forces from other solar system bodies. [6]

10. State the relativistic expressions for the energy E and momentum p of a particle of rest mass m and speed v, and show that the quantity $E^2 - p^2c^2$ is invariant in all inertial frames. [3]

A particle with rest mass m_0 and velocity $0.6\,c$ collides with a stationary particle of mass $2m_0$ in a laboratory, forming a single new particle with rest mass m'. Find the mass m' and the speed v' of the new particle. [10]

This new particle decays into two particles each with rest mass $m_0/2$ travelling along the line of flight of particle m'. What are their velocities and energies in the laboratory frame? [7]