A10287W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday, 7 June 2021

Opening Time 9.30 am UK time

You have $2\frac{1}{2}$ hours writing time to complete the paper and up to 30 mins technical time to upload your answer file.

Answer **all** *of Section A and* **three** *questions from Section B.*

Write your candidate number, the paper name, paper code and a list of the questions that you have answered on the first page, and submit all of your answers as a single PDF file.

The use of approved calculators is permitted.

A list of physical constants and conversion factors will be available within Inspera during the examination.

The numbers in the margin indicate the weight that the examiners expect to assign to each part of the question.

Section A

1. Assuming coplanar, circular motion about the centre of the Sun, calculate the orbital periods of Jupiter and Saturn in Earth years, given that their orbital radii are 5.2 and 9.5 Astronomical Units respectively. Find the interval in Earth years between successive conjunctions (times of zero angular separation) of the two planets as viewed from the position of the Sun. Without detailed calculation, discuss the interval between conjunctions when observed from the surface of the Earth. [8]

2. A mass is released at rest from a tower, falling under gravity and subject to a resistive force $\mathbf{F} = -mk\mathbf{v}^2$, where m is the mass of the projectile, $\mathbf{v}$ is its velocity and k is a constant. Find an expression for the maximum velocity and find the distance travelled at which its velocity reaches half of that value. [5]

3. A potential energy function is given by

$$U(x) = \frac{-kx}{(a^2 + x^2)},$$

where k and a are constants. Sketch the force and potential as a function of position, x. Find their values at the positions of stable equilibrium, and define the regions of attraction and repulsion. [8]

4. A compact mass m is suspended at the end of a massless rigid rod of length r fixed at the origin. The mass is free to move in all three spatial dimensions. Write down expressions for the Cartesian coordinates in terms of the polar angle, θ, and the azimuthal angle, ϕ, and hence find an expression for the Lagrangian of the system. Find the equations of motion and identify any cyclic coordinates. [8]

5. Two events separated by a distance of 1 km are measured to occur 1 μs apart. In a different inertial frame, the events are observed to occur simultaneously. What do you conclude about the motion of the second frame? Find the maximum time interval possible between the events in the first frame for them to occur simultaneously in a second inertial frame. [4]

6. At time $t = 0$, two spacecraft pass close to the Earth, moving directly towards the nearest star, Proxima Centauri, a distance of 4.2 light years away. Spacecraft A has a velocity of $0.25\,c$ relative to the Earth, while the velocity of spacecraft B relative to spacecraft A is $0.5\,c$. What is the velocity of spacecraft B as measured by an observer on Earth? When they pass by Proxima Centauri, the spacecraft send radio signals back to the Earth. As measured by a clock on the Earth, when will spacecraft A receive the signal sent by spacecraft B and when will the signals transmitted by the two spacecraft as they pass Proxima Centauri be received at the Earth? [7]

Section B

7. Explain what is meant by the inertia of an object and how its moment of inertia is calculated. Find an expression for the moment of inertia of a uniform thin rod with mass m and length l about its centre of mass and about one end. [4]

A uniform thin rod of mass m and length l is suspended at one end from a frictionless pivot. A particle also of mass m travelling horizontally at speed v_0 collides elastically with the lower end of the rod along their line of centres. Describe the subsequent motion of the rod and the particle. Calculate the speed of the particle after the collision and find the condition that the rod swings up so that it just reaches the horizontal. [10]

Describe and find an expression for the motion of the rod after it reaches the horizontal. Compare this to the oscillations of a simple pendulum of length $l/2$. [6]

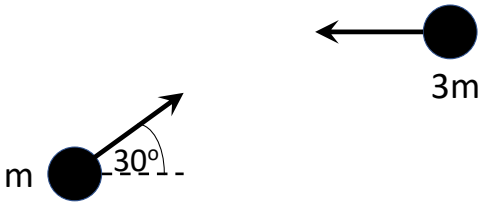
8. Write down the expression for the energy of a satellite in a non-circular orbit around a massive body, explaining all of the terms. [2]

A spacecraft bound for Mars is moved from a circular orbit of radius 1 Astronomical Unit (AU) about the Sun into an elliptical orbit with maximum and minimum distances of 1.52 and 1.0 AU, corresponding to the orbital radii of Mars and Earth. Find the speed of the spacecraft at the minimum and maximum distances of the elliptical orbit. What is the difference in the speed of the spacecraft at the minimum distance of the elliptical orbit compared to its circular orbit around the Sun? [10]

When the spacecraft reaches the furthest point in its orbit, a rocket fires to move it into a circular orbit at the distance of Mars. What is the change in orbital speed required and in what direction should the rocket thruster be fired? In order to achieve a successful rendezvous with Mars, what are the relative orbital positions of Mars and the spacecraft when it moves into the elliptical orbit? [8]

[You may assume that the orbits of the planets around the Sun are circular and that all orbits are in the same plane.]

9. Particle A of mass $3\,m$ travels in the negative x direction with non-relativistic speed u. Particle B has mass m and travels on a trajectory that makes an angle of 30 degrees to the x-axis. It collides elastically with particle A. What speed must particle B have in order to bring the motion of particle A to a halt in the x direction? What are the velocities and energies of the particles after the collision? [12]



A second collision has particles with the same initial velocities and masses, but the collision is inelastic such that the two particles coalesce when they collide. What are the velocities and energies of the particles after this collision? Explain the difference with the first collision. [8]

10. A particle of rest mass $4m$ at rest in a laboratory decays, forming two particles with rest masses of m and $2m$ respectively. What are the speeds of the two new particles in the laboratory frame? [12]

Explain what is meant by the proper mean lifetime of a particle. The particle with rest mass m has a proper mean life time of $2 \times 10^{-6}\,\mathrm{s}$. What is the probability that it will travel further than 1000 m in the laboratory frame before it decays? [8]