Trinity Term Assessment Paper

Paper CP2: PHYSICS 2

Tuesday 9 June 2020

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

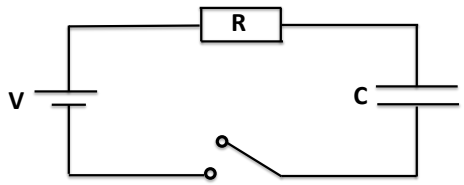
1. Derive an expression for the force per unit length between two parallel and infinitely long straight wires, each carrying a constant current I and separated by a distance r. Under what circumstances is the force repulsive? One of the long wires lies on the ground with the other one placed parallel to and directly above it. The current flowing through the wires produces a repulsive force, and the wires have a mass per unit length λ. Find an expression for the separation of the wires at which the gravitational and magnetic forces are balanced. [5]

2. A stationary charge $+Q$ is fixed at position (x, y) = (-d, 0) in a two dimensional space. A charge $-Q$ is brought from a large distance to a position $(x, y) = (+d, 0)$. What is the potential energy of this configuration? With these two charges in place, what is the **E**-field at positions $(x, y) = (\pm 2d, 0), (0, 0)$ and $(0, \pm 2d)$? [7]

3. Calculate the potential energy, that is the work required for its assembly against electro-static forces, of a solid sphere of radius R containing a uniform charge distribution with a total charge Q. [5]

4. A capacitor consists of two large parallel rectangular plates, separated by a distance x_o. The plates are charged to potentials of V_1 and V_2 respectively. The region between the plates is filled with a fixed, uniform space charge distribution of volume charge density ρ. Find and sketch the magnitude of the electric field and electric potential as a function of position x between the plates. [6]

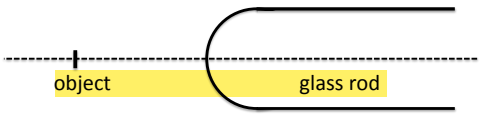
5. In the circuit below, a capacitor C is wired in series with a resistor R and a battery with potential difference V via a switch. Initially the switch is open and the capacitor is uncharged. The switch is closed at time $t = 0$. Find an expression for the voltage across the capacitor as a function of time. [4]



The resistor has resistance 100 kΩ and the capacitor has capacitance 1 μF. The battery is replaced by a square wave signal generator with frequency f and voltage range ±12 V. Without detailed calculation, describe and sketch the voltage across the capacitor when $f = 100$ Hz, 10 Hz and 0.1 Hz after several periods have elapsed. [4]

6. An astronomer suspects that a star at a distance of 4×10^{17} m from the Earth is actually a binary system with an orbital separation of 1×10^{11} m. What diameter telescope is required to confirm this by imaging it at wavelength $\lambda = 410$ nm? [3]

7. A cylindrical glass rod has a hemispherical end with a radius of curvature $R = 20$ mm and a refractive index $n_g = 1.60$. A small object is placed 60 mm in front of the hemispherical end on the axis of the rod.



In the paraxial approximation, find an expression for the position of the image of the object. Find the magnification and position of the image when the rod is used in air with refractive index $n_0 = 1.00$, and in water with refractive index $n_w = 1.33$. [6]

Section B

8. State the laws of electromagnetic induction, clearly explaining the direction of the induced e.m.f. Starting from the expression for the Lorenz force on a moving charge, find an expression for the force resulting from a current flowing in a long wire placed within a magnetic field. [6]

Two long parallel conducting rods are inclined to the horizontal at an angle ϕ and separated by a distance s. A conducting bar of mass m rests perpendicular to and across the rods at the top. The rods are connected at the bottom by a large resistance R and a uniform, vertical magnetic field of flux density B fills the whole region. The bar is released and slides without friction down the rods, remaining perpendicular to them and maintaining contact with them.

Find:
(i) the current induced,
(ii) the equation of motion of the bar,
(iii) the terminal velocity of the bar,
(iv) the rate at which work is done by the bar sliding on the rods and compare it to the power dissipated by the resistor. [10]

In order to reverse the motion of the bar and send it up the slope, the system is modified by replacing the resistor with a wire and passing a large current up one rod, through the bar and down the other rod so that the bar moves with uniform speed. Find expressions for the forces acting on the bar and hence find an expression for the current required to maintain a uniform speed. In what direction must the current flow through the bar? [4]

9. State Maxwell's equations for electric and magnetic fields in vacuum in the differential form. Starting from these expressions, use the divergence theorem to derive Gauss' law and hence derive Coulomb's law for the force between two point electric charges. [5]

A capacitor consists of two thin concentric conducting spherical shells with radii a and b, with $b > a$, in vacuum, and a charge q on the inner shell. Find an expression for the potential between the shells, and hence show that the capacitance is [5]

$$C = 4\pi\epsilon_0 \frac{ab}{b-a} \ .$$

The inner shell is raised to a potential V and then isolated and the outer shell removed. Find an expression for the residual electric field and hence determine the potential on the surface of the remaining sphere and at the position of the sphere that was removed.

An isolated, uncharged conducting spherical shell of radius $2a$ is connected to the remaining sphere with a long wire. What is the ratio of the electric fields on the surfaces of the two spherical shells? [10]

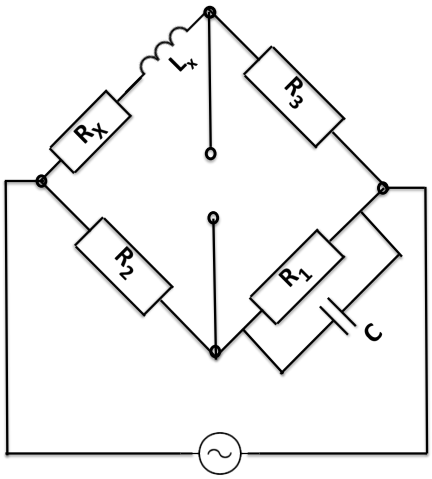
10. From consideration of Huygens' construction for a plane wavefront passing from a medium of refractive index n_1 to a medium of lower refractive index n_2, find an expression for the angle of refraction. What is the condition for total internal reflection? [4]

An optical fibre has a core with refractive index n_1 surrounded by a coaxial cladding layer with a lower refractive index n_2, and is used in air. Light enters the end of the fibre at an angle of incidence θ_i to its axis. Find an expression for the maximum angle of incidence for lossless light transmission along the fibre in terms of the refractive indices of the layers. Take the refractive index of air to be n =1.00. [4]

Signals consisting of very short pulses of light from two lasers operating at the same wavelength are incident on the axis of the fibre at the angle of incidence calculated above and at half that angle respectively. The refractive indices of the core and cladding are 1.5 and 1.45 respectively. Calculate the travel times of the signals along a fibre of length 2 km.

Describe the effect on the transmission of light through a fibre when it is bent into a tight circular loop. Calculate the radius of curvature R (the radius of curvature of the loop to the centre of the fibre) at which light entering a fibre of core radius $r = 80\,\mu$m at an angle of incidence of 20° is not transmitted. [12]

11. Write down expressions for the impedances of resistors, capacitors and inductors. Explain why complex numbers are useful in the analysis of electrical circuits containing these components and why the impedance of resistors can be represented by real numbers while capacitors and inductors require imaginary components. [6]



A component with resistive and inductive properties is to be measured by inserting it into a bridge as illustrated in the figure, where its inductance and resistance are represented by L_{x} and R_{x} respectively. Describe how such a bridge can be used to measure these quantities, and derive expressions for the resistance and inductance in terms of the values of the resistors and capacitor shown in the figure. What are the advantages of this bridge configuration? [10]

When the component is inserted, the bridge is found to be far from the balance point. What practical alterations to the bridge shown in the figure would allow the measurements to be made? [4]