**Trinity Term Assessment Paper**

**Paper CP1: PHYSICS 1**

**Monday 8 June 2020**

**Time allowed: $2\frac{1}{2}$ hours**

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*Answer* **all** *of Section A and* **three** *questions from Section B.*

*Start the answer to each question on a new page.*

*The use of approved calculators is permitted.*

*A list of physical constants, mathematical formulae and conversion factors accompanies this paper.*

*The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.*

**Do NOT turn over until told that you may do so.**

## Section A

**1.** An explosion occurs, which produces a spherical shock wave with a radius $R$ that depends on the air density $\rho$, the time since the explosion $t$, and the energy released $E$. Use dimensional analysis to determine the exponents $\alpha$, $\beta$, and $\gamma$ in the formula

$$R = C\rho^{\alpha}t^{\beta}E^{\gamma}\,,$$

where $C$ is a dimensionless constant. [3]

**2.** A projectile is fired from the ground with initial speed $u_0$ at an angle $\alpha$ to the horizontal. Assuming an air resistance linearly proportional to speed, and an acceleration due to gravity $g$, show that the projectile reaches its highest point after a time

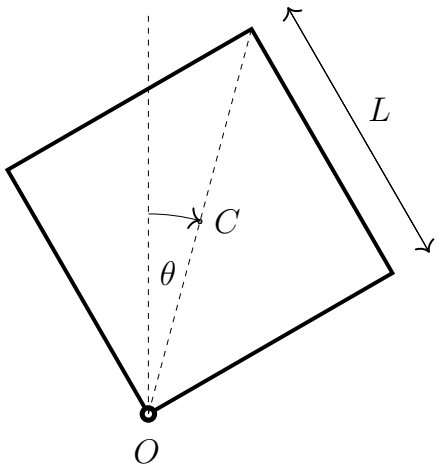
$$\frac{v_t}{g}\ln\left(\frac{u_0\sin\alpha}{v_t}+1\right)\,,$$

and give an expression for the constant $v_t$. [5]

**3.** An asteroid is moving towards an isolated planet of mass $M$ and radius $R$, from a long distance away, with initial speed $v_0$ and impact parameter $d$. Find an expression for the minimum value of $v_0$ such that the asteroid does not hit the planet. [5]

**4.** A uniform square lamina has a mass $M$ and sides of length $L$. Determine the moment of inertia about an axis perpendicular to the plane of the lamina and passing through a corner.

The lamina is now free to rotate in a vertical plane about a fixed smooth horizontal axis passing through one of its corners ($O$) and perpendicular to the lamina. The centre of the lamina ($C$) is initially held vertically above $O$, and is then infinitesimally displaced from rest. Find an expression for the angular speed of the lamina, $\dot{\theta}$, as a function of the angle of rotation $\theta$, as shown in the figure. [8]



**5.** An open railway carriage of mass $M_0$ is freely running on smooth horizontal rails under rain falling vertically down at a constant rate, which it catches and retains. Given that $u_0$ is the initial speed of the carriage and $k$ is the constant mass of rain falling into the carriage per unit time, find an expression for the distance travelled in a time $t$ after the rain starts to fall. (You may assume that the carriage is never completely filled with rain). [6]

**6.** A particle of rest mass $m$ with relativistic velocity $v\hat{\mathbf{x}}$ collides head-on with a second particle of rest mass $m/2$, travelling with velocity $-v\hat{\mathbf{x}}$. Given that the two particles coalesce, find the rest mass of the resulting particle. Verify that in the limit $v \ll c$, the result reduces to the classical expectation. [6]

**7.** A train of 100 m proper length travels at a speed $0.99c$ in the frame of the tracks on which it is running. It passes through a tunnel of length 90 m. Is the train ever completely inside the tunnel? Explain your answer in both the frame of the railway tracks and the frame of the train. Also explain how your answers are consistent. [7]

## Section B

**8.** A lander probe is moving vertically downwards towards the surface of Mars with an initial speed $v_0$ of $100\,\mathrm{ms}^{-1}$. A retrorocket is switched on to control the lander's descent at time $t = 0$. The retrorocket ejects mass at a constant rate $\alpha$ along its line of flight with a constant ejection velocity $u$ relative to the lander. Show that the velocity of the lander $v$ towards the surface can be expressed as:

$$v = v_0 + gt + u\ln[(M_0 - \alpha t)/M_0],$$

where $M_0$ is the mass of the lander and fuel at $t = 0$, and $g$ is the Martian gravitational acceleration. [8]

The initial mass of the lander and fuel is 500 Kg, the rocket ejection rate is $2\,\mathrm{kgs}^{-1}$, the gravitational acceleration may be assumed constant at $3.73\,\mathrm{ms}^{-2}$, and the descent lasts for 100 seconds. What is the range of fuel velocities required to produce a soft landing, with a landing speed at the surface between 0 and $1\,\mathrm{ms}^{-1}$? [4]

An error occurs when converting from imperial to metric units, and the fuel velocity is actually 10% higher than calculated above. Calculate the speed at burnout of the retrorocket and describe the motion from $t = 0$. Sketch a graph of the height of the lander as a function of time from $t = 0$ for this case, given that the retrorocket operates for 100 s. [8]

**9.** Compare and contrast the relative merits of the centre of momentum and laboratory reference frames when considering particle collisions. [3]

A non-relativistic particle of mass $M_1$ and initial speed $u_1$ collides *inelastically* and obliquely with a second particle of mass $M_2$, which is initially at rest. Write down expressions for the velocity of the centre of momentum frame, and the velocities of $M_1$ and $M_2$ in the centre of momentum frame before the collision. Show that the ratio of the final-to-initial speeds of *each* particle in the centre of momentum frame is equal to the coefficient of restitution, $e$. [5]

Following the collision, $M_1$ has a speed $v_1$ and a scattering angle $\theta_1$ (measured with respect to the initial direction of $M_1$) in the laboratory frame. Show that the ratio of the final-to-initial speeds of $M_1$ in the laboratory, is given by

$$\frac{v_1}{u_1} = \frac{M_1}{M_1 + M_2}\left[\cos\theta_1 \pm \sqrt{\cos^2\theta_1 - 1 + e^2\left(\frac{M_2}{M_1}\right)^2}\right].$$

[5]

Making use of the above equation:

(i) calculate the possible values of $v_1$ for the case of a perfectly *elastic* and equal mass collision, and comment on the solutions;

(ii) determine the speed $v_1$ for a perfectly *elastic* collision when $M_2 \gg M_1$;

(iii) describe the motion of $M_1$ and $M_2$ for the case of a totally *inelastic* collision;

(iv) show that for a perfectly *elastic* collision with $M_1 > M_2$, the scattering angle $\theta_1$ must take on a maximum value between $0 \leq \theta_1^{\max} < \frac{\pi}{2}$, depending on the relative values of $M_1$ and $M_2$. [7]

**10.** What is meant by an *invariant* quantity in the context of special relativity? Illustrate your answer by discussing the relationship between total energy, momentum and rest mass for a particle in special relativity. [5]

In the jet of a quasar, an ultra-relativistic electron collides head-on with a very low energy photon of the cosmic microwave background. No new particles are created. Show that the energy of the photon after the collision is given by

$$E = \frac{(E_1 + p_1 c)E_2}{(E_1 - p_1 c) + 2E_2}$$

where $E_1$ and $p_1$ are respectively the energy and the magnitude of the momentum of the incident electron, and $E_2$ is the energy of the initial photon. [10]

In the case where the initial wavelength of the photon is $\lambda = 2\,\text{mm}$ and the electron is travelling at $v = 0.99\,c$, find the photon wavelength after the collision. [5]

**11.** A binary star system with an orbital period $P$ consists of two stars of mass $M_1$ and $M_2$ separated by distance $r$. Obtain an expression for $r$ in terms of the stellar masses and the separations of the stars from the centre of mass point, $r_1$ and $r_2$. Show that

$$P^2 = \frac{4\pi^2 r^3}{G(M_1 + M_2)}$$

where $G$ is the gravitational constant. [6]

A gravitational wave observatory has detected a binary system consisting of two black holes each of mass $30\,M_\odot$ in circular orbits around the centre of mass. The orbital period is found to be 0.1 seconds What is the orbital separation of the black holes? Compare this separation to an estimate of the Schwartzschild radius of a $30\,M_\odot$ black hole calculated using an escape speed of $c$. [6]

The binary system emits gravitational radiation which causes the system to lose energy at a rate

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{-kG^4M^5}{r^5}$$

where $k$ is a constant. By considering the total energy of the system show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{-2kG^3M^3}{r^3}$$

Evaluate the change in orbital separation after one orbit and describe what will happen to the system. Take the value of $k$ to be $5 \times 10^{-42}\,\mathrm{m}^{-5}\,\mathrm{s}^5$ and ignore any relativistic effects. [8]