A10287W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 2 September 2019, 9.30 am – 12.00 noon

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. An explosion occurs, which produces a spherical shock wave with a radius R that depends on the air density ρ, the time since the explosion t, and the energy released E. Use dimensional analysis to determine the exponents α, β, and γ in the formula

$$R = C\rho^{\alpha}t^{\beta}E^{\gamma}\,,$$

where C is a dimensionless constant. [3]

2. A projectile is fired from the ground with initial speed u_0 at an angle α to the horizontal. Assuming an air resistance linearly proportional to speed, and an acceleration due to gravity g, show that the projectile reaches its highest point after a time

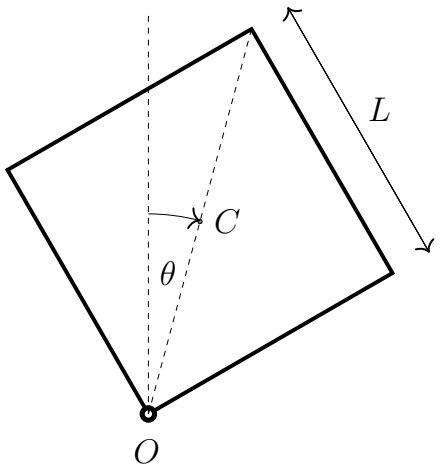
$$\frac{v_t}{g}\ln\left(\frac{u_0\sin\alpha}{v_t}+1\right)\,,$$

and give an expression for the constant v_t. [5]

3. An asteroid is moving towards an isolated planet of mass M and radius R, from a long distance away, with initial speed v_0 and impact parameter d. Find an expression for the minimum value of v_0 such that the asteroid does not hit the planet. [5]

4. A uniform square lamina has a mass M and sides of length L. Determine the moment of inertia about an axis perpendicular to the plane of the lamina and passing through a corner.

The lamina is now free to rotate in a vertical plane about a fixed smooth horizontal axis passing through one of its corners (O) and perpendicular to the lamina. The centre of the lamina (C) is initially held vertically above O, and is then infinitesimally displaced from rest. Find an expression for the angular speed of the lamina, $\dot{\theta}$, as a function of the angle of rotation θ, as shown in the figure. [8]



5. An open railway carriage of mass M_0 is freely running on smooth horizontal rails under rain falling vertically down at a constant rate, which it catches and retains. Given that u_0 is the initial speed of the carriage and k is the constant mass of rain falling into the carriage per unit time, find an expression for the distance travelled in a time t after the rain starts to fall. (You may assume that the carriage is never completely filled with rain). [6]

6. A particle of rest mass m with relativistic velocity $v\hat{\mathbf{x}}$ collides head-on with a second particle of rest mass $m/2$, travelling with velocity $-v\hat{\mathbf{x}}$. Given that the two particles coalesce, find the rest mass of the resulting particle. Verify that in the limit $v \ll c$, the result reduces to the classical expectation. [6]

7. A train of 100 m proper length travels at a speed $0.99c$ in the frame of the tracks on which it is running. It passes through a tunnel of length 90 m. Is the train ever completely inside the tunnel? Explain your answer in both the frame of the railway tracks and the frame of the train. Also explain how your answers are consistent. [7]

Section B

8. Compare and contrast the relative merits of the centre of momentum and laboratory reference frames when considering particle collisions. [3]

A non-relativistic particle of mass M_1 and initial speed u_1 collides *inelastically* and obliquely with a second particle of mass M_2, which is initially at rest. Write down expressions for the velocity of the centre of momentum frame, and the velocities of M_1 and M_2 in the centre of momentum frame before the collision. Show that the ratio of the final-to-initial speeds of *each* particle in the centre of momentum frame is equal to the coefficient of restitution, e. [5]

Following the collision, M_1 has a speed v_1 and a scattering angle θ_1 (measured with respect to the initial direction of M_1) in the laboratory frame. Show that the ratio of the final-to-initial speeds of M_1 in the laboratory, is given by

$$\frac{v_1}{u_1} = \frac{M_1}{M_1 + M_2}\left[\cos\theta_1 \pm \sqrt{\cos^2\theta_1 - 1 + e^2\left(\frac{M_2}{M_1}\right)^2}\right] .$$

[5]

Making use of the above equation:

(i) calculate the possible values of v_1 for the case of a perfectly *elastic* and equal mass collision, and comment on the solutions;

(ii) determine the speed v_1 for a perfectly *elastic* collision when $M_2 \gg M_1$;

(iii) describe the motion of M_1 and M_2 for the case of a totally *inelastic* collision;

(iv) show that for a perfectly *elastic* collision with $M_1 > M_2$, the scattering angle θ_1 must take on a maximum value between $0 \leq \theta_1^{\max} < \frac{\pi}{2}$, depending on the relative values of M_1 and M_2. [7]

9. Write down expressions for the total energy E and linear momentum p of a relativistic particle with rest mass m_0 and speed u. Write down the equivalent expressions for a photon of wavelength λ. [4]

Consider the elastic scattering of a photon by a stationary unbound electron of rest mass m_e. Show that the change in wavelength of the photon when it is scattered through an angle θ is given by

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)\,,$$

where c is the speed of light in vacuum and h is Planck's constant. [10]

Determine the final energy of a photon of initial energy 1 MeV after a single such scattering through 180° in the laboratory frame. At what speed and in which direction would an observer need to move in order to measure the scattered photon as having its original laboratory-frame energy of 1 MeV? Assume the observer travels along the same axis as the photon. [6]

[The rest mass energy of the electron is 0.511 MeV.]

10. A mass m is free to move on a frictionless horizontal table and is connected by a light inextensible string of length l, which passes through a small hole in the table, to a second mass M which hangs below. Assume that M moves only in a vertical line, that in general m has both radial and tangential components of velocity, and that the string always remains taut.

(a) Show that the Lagrangian for this system in terms of polar coordinates (r, θ), centred on the hole, can be written as

$$L = \frac{1}{2}(M+m)\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + Mg(l-r)\,.$$

Compute the Hamiltonian, and state whether it is conserved. Identify any other conserved quantities. [6]

(b) Find the Euler-Lagrange equations of motion and show that

$$(M+m)\ddot{r} = \frac{J^2}{mr^3} - Mg\,,$$

giving an expression for J. Find the value of $r = r_0$ in terms of J, M, m and g, for which m executes circular motion. [4]

(c) Consider the case where the mass m is initially a distance $l/2$ from the hole and moves with a speed v_0 directed perpendicular to the string.

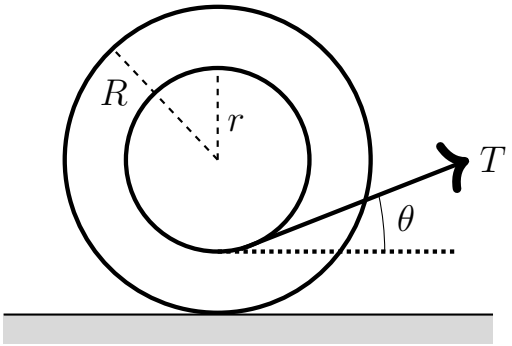
(i) Explain whether the mass m on the table can ever reach the hole.

(ii) Determine the condition on v_0 in terms of M, m, l and g, such that the hanging mass M *just* reaches the table.

(iii) Now let the value of v_0 be such that the mass m is executing circular motion at $r_0 = l/2$. It is then given a small perturbation so that $r(t) = r_0 + \epsilon(t)$. Show that $r(t)$ executes simple harmonic motion about r_0, and determine the angular frequency of the oscillations in terms of M, m, l and g. [10]

11. (a) Write down expressions for the angular momentum and torque about an axis through the origin, in terms of the position vector, $\mathbf{r}$, and velocity, $\mathbf{v}$, of a particle of mass m. Making use of Newton's equations, find an equation relating torque and angular momentum. [5]

(b) A spool consists of a cylindrical axle of radius r attached to circular end caps of larger radius R, which roll on horizontal ground. The spool has a total mass M and moment of inertia I about its principal axis. A light inextensible string is wrapped around the axle and is pulled with tension T at an angle θ to the horizontal, as shown in the figure. Assume that the friction between the spool and the ground is large enough that only rolling motion, without slipping, is possible.



(i) Consider first the case where the angle $\theta = 0$. Show that the magnitude of the total frictional force F is given by

$$F = T\left(\frac{I + MrR}{I + MR^2}\right),$$

and determine the direction in which the spool moves. [6]

(ii) Consider now the case where $\theta \neq 0$. Show that there is a critical angle θ_c for which the spool does not move, and find an expression for this angle.

For $\theta = \theta_c$, and given a coefficient of static friction μ, find an expression for the maximum value of T for which equilibrium is maintained.

Comment briefly on the motion of the spool for angles $\theta > \theta_c$. [9]