A10427W1

SECOND PUBLIC EXAMINATION

Honour School of Physics and Philosophy Part A

A2P: ELECTROMAGNETISM

TRINITY TERM 2018

Tuesday 5 June, 2.30 pm – 4.10 pm

Time allowed: 1 hour 40 minutes

Answer **all** *of Section A and* **two** *questions from Section B.*

For Section A start the answer to each question on a new page.
For Section B start the answer to each question in a new book.

The use of approved calculators is permitted.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Write down an expression for the electric field at a displacement $\mathbf{r}$ from a charge of magnitude Q. [2]

A vertical dipole consists of a charge of $+30\,\mathrm{C}$ held at a height of $8\,\mathrm{km}$ above an infinite conducting plane and a second charge of $-30\,\mathrm{C}$ held directly below the first charge but at a height of $5\,\mathrm{km}$ above the same infinite conducting plane. Calculate the electric field directly below the vertical dipole at a height of 0 km. [5]

2. Write down Ampère's law in the case of non-time-varying fields and explain each of the terms. Using this law derive an expression for the force per unit length between two parallel and infinitely long straight wires, each carrying a constant current I in the same direction and separated by a distance r. In which direction does the force act? [6]

3. Explain why a displacement current term has to be added to Ampère's law in the case of time-varying fields. [3]

A parallel-plate capacitor consists of two circular plates, each of radius a and separated by a distance d, where $d \ll a$. The space between the plates is filled with air. An alternating potential difference of $V_0 \exp(-j\omega t)$ is applied across the capacitor. Find an expression for the magnitude of the magnetic field between the plates. [5]

4. Estimate the electric field strength $100\,\mathrm{km}$ from a $100\,\mathrm{kW}$ television transmitter assuming isotropic radiation. [6]

Section B

5. State Gauss's Law in both integral and differential form and use it to derive an expression for the capacitance C of an isolated conducting sphere of radius R which carries a charge Q on its surface. [7]

Calculate the potential energy, W, of the system. [3]

A long cylindrical capacitor of length l consists of an outer conductor of radius a and an inner conductor of radius b, where $l \gg a$. The outer conductor is earthed, and the inner conductor is hollow, insulated and uncharged. The gaps between both conductors are air-filled. A sphere of radius R is charged to a potential V far from any other bodies and is then inserted inside the inner conductor of the cylindrical capacitor without touching it.

(i) Draw a clearly labelled diagram showing the field lines and explain the steps in your reasoning. [4]

(ii) Calculate the electric field strength at a radius $b < r < a$, in terms of the potential V and the radius R of the sphere. [3]

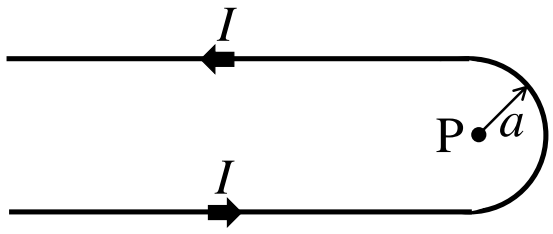
(iii) Hence find the potential of the inner cylinder. [3]

6. Write down the Biot-Savart law, clearly explaining all the terms. [2]

A current I is passed through a circular coil of radius a. Use the Biot-Savart law to calculate the magnitude and direction of the **B**-field at a point on the axis of the coil, a distance x from its centre. [5]

Calculate the magnitude and direction of the **B**-field produced by a thin wire of length $2b$ carrying a current I, at a point on the perpendicular plane that bisects the wire, at a distance a from the centre of the wire. [5]

In the figure below, a long straight wire has been bent into a hairpin shape:



Calculate the magnitude and direction of the magnetic field **B** at the centre P of the semicircle when a current I flows through the wire in the direction indicated. [8]

7. Write down an expression for the force $\mathbf{F}$ acting on a particle of charge q moving with a velocity $\mathbf{v}$ in the presence of an electric field $\mathbf{E}$ and a magnetic flux density $\mathbf{B}$. [2]

A horizontal parallel-plate capacitor has a plate separation of 0.05 m and a voltage between the plates of 500 V. A particle with a charge/mass ratio of $4.2 \times 10^6\,\mathrm{C\,kg^{-1}}$ enters the parallel-plate capacitor in a direction perpendicular to its electric field. After travelling a horizontal distance of 0.2 m, the particle has been deflected vertically by 17 mm. Calculate the magnitude and direction of a uniform magnetic field that must be applied to ensure that instead the particle is not deflected. [8]

A particle with the same charge/mass ratio as before now enters a region with no electric field but with a uniform magnetic field with magnetic flux density components given by $B_x = 0, B_y = 0$ and $B_z = B$. If the particle enters the field at an angle θ to the z-axis, determine the equations of motion for the particle in x, y and z and show that in general it moves in a helix.

If the magnetic flux density is 5 T and the particle enters the region with a velocity of $2.1 \times 10^6\,\mathrm{m\,s^{-1}}$ and at an angle of 81° to the magnetic field, calculate the radius of curvature of the resulting motion. [10]