7D (Version: 2018)
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HONOUR SCHOOL OF PHYSICS

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PRELIMINARY EXAMINATION IN PHYSICS

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SOME USEFUL DATA AND MATHEMATICAL FORMULAE

Fundamental constants

Constants are given to 4 or 5 significant figures (data from NIST CODATA 2014).

Electron rest mass	m_e	$9{\cdot}109 \times 10^{-31}\,\mathrm{kg}$
Proton rest mass	M_p	$1{\cdot}6726 \times 10^{-27}\,\mathrm{kg}$
Electronic charge	e	$1{\cdot}6022 \times 10^{-19}\,\mathrm{C}$
Speed of light in free space	c	$2{\cdot}9979 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck's constant	h	$6{\cdot}626 \times 10^{-34}\,\mathrm{J\,s}$
$h/2\pi =$	$\hbar$	$1{\cdot}0546 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar c$	$197{\cdot}33$ MeV fm
Boltzmann's constant	k_B	$1{\cdot}3806 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Molar gas constant	R	$8{\cdot}314\ \mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro's number	N_A	$6{\cdot}022 \times 10^{23}\,\mathrm{mol^{-1}}$
Standard molar volume		$22{\cdot}414 \times 10^{-3}\,\mathrm{m^3\,mol^{-1}}$
Bohr magneton	μ_B	$9{\cdot}274 \times 10^{-24}\,\mathrm{A\,m^2}$ *or* $\mathrm{J\,T^{-1}}$
Nuclear magneton	μ_N	$5{\cdot}051 \times 10^{-27}\,\mathrm{A\,m^2}$ *or* $\mathrm{J\,T^{-1}}$
Bohr radius	a_0	$5{\cdot}292 \times 10^{-11}\,\mathrm{m}$
Fine structure constant $e^2/(4\pi\varepsilon_0\hbar c) =$	α	$(137{\cdot}04)^{-1}$
Thomson cross section	σ_T	$6{\cdot}6525 \times 10^{-29}\,\mathrm{m^2}$
Compton wavelength of electron $h/(m_\mathrm{e}c) =$	λ_C	$2{\cdot}4263 \times 10^{-12}\,\mathrm{m}$
Rydberg's constant	R_∞	$1{\cdot}0974 \times 10^{7}\,\mathrm{m^{-1}}$
	$R_\infty hc$	$13{\cdot}606$ eV
Stefan's constant	σ	$5{\cdot}6704 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Gravitational constant	G	$6{\cdot}6741 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Proton magnetic moment	μ_p	$2{\cdot}7928\mu_\mathrm{N}$
Neutron magnetic moment	μ_n	$-1{\cdot}9130\mu_\mathrm{N}$

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Other data and conversion factors

1 angstrom	Å	$10^{-10}\,\mathrm{m}$
1 fermi	fm	$10^{-15}\,\mathrm{m}$
1 barn	b	$10^{-28}\,\mathrm{m}^2$
1 cm^{-1}		$10^{2}\,\mathrm{m}^{-1}$
1 pascal	Pa	$1\,\mathrm{N\,m}^{-2}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m}^{-1}$
Permittivity of free space	ε_0	$8{\cdot}854 \times 10^{-12}\,\mathrm{F\,m}^{-1}$
1 electron volt	eV	$1{\cdot}6022 \times 10^{-19}\,\mathrm{J}$
	eV/hc	$8{\cdot}065 \times 10^{5}\,\mathrm{m}^{-1}$
	eV/k_B	$1{\cdot}1604 \times 10^{4}\,\mathrm{K}$
1 unified atomic mass unit (^{12}C scale)	u	$931{\cdot}5\,\mathrm{MeV}/c^2 = 1{\cdot}6605 \times 10^{-27}$ kg
Wavelength of 1 eV photon		$1{\cdot}23984 \times 10^{-6}\,\mathrm{m}$

Particle physics data

Masses (the symbols have their conventional meanings) in MeV/c^2 (PDG 2018):
$\mathrm{e}^{\pm}$ 0·5110, $\mu^{\pm}$ 105·66, $\tau^{\pm}$ 1777, π^0 134·98, $\pi^{\pm}$ 139·57, $\mathrm{K}^{\pm}$ 493·7, K^0 497·6, η 547·9,
D^0 1865, $\mathrm{D}^{\pm}$ 1870, p 938·3, n 939·6, Λ^0 1116, Σ^+ 1189, Σ^0 1193, Σ^- 1197,
Ξ^0 1315, Ξ^- 1322, Ω^- 1672, B^0 5280, $\mathrm{B}^{\pm}$ 5279, Z^0 91188, $\mathrm{W}^{\pm}$ 80390, H^0 125180.

Quark	Charge	I_3	S	C	B	T	Mass (GeV/c^2)
u	$+\frac{2}{3}$	$+\frac{1}{2}$	0	0	0	0	< 1
d	$-\frac{1}{3}$	$-\frac{1}{2}$	0	0	0	0	< 1
c	$+\frac{2}{3}$	0	0	+1	0	0	1·27
s	$-\frac{1}{3}$	0	−1	0	0	0	< 1
t	$+\frac{2}{3}$	0	0	0	0	+1	173·0
b	$-\frac{1}{3}$	0	0	0	−1	0	4·18

Astrophysical data

1 astronomical unit	AU	$1{\cdot}496 \times 10^{11}\,\mathrm{m}$
1 parsec	pc	$3{\cdot}086 \times 10^{16}\,\mathrm{m}$
Luminosity of Sun	$L_\odot$	$3{\cdot}828 \times 10^{26}\,\mathrm{W}$
Mass of Sun	$M_\odot$	$1{\cdot}988 \times 10^{30}\,\mathrm{kg}$
Radius of Sun	$R_\odot$	$6{\cdot}957 \times 10^{8}\,\mathrm{m}$

Geophysical data

Mean radius of Earth	6371 km
Molar mass of dry air	$28{\cdot}96 \times 10^{-3}\,\mathrm{kg}$
Molar heat capacity of dry air	
at constant pressure	$29{\cdot}13\,\mathrm{J\,K}^{-1}$
at constant volume	$20{\cdot}83\,\mathrm{J\,K}^{-1}$
1 standard atmosphere	$1{\cdot}013 \times 10^{5}\,\mathrm{Pa}$
Acceleration due to gravity	$9{\cdot}807\,\mathrm{m\,s}^{-2}$
Solar constant	$1361\,\mathrm{W\,m}^{-2}$

Trigonometrical identities

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta)$$
$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta)$$
$$\cos\alpha - \cos\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\sin\tfrac{1}{2}(\beta-\alpha)$$

In a triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Indefinite and definite integrals

Indefinite (with $a > 0$):

$$\int \frac{\mathrm{d}x}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$
$$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$$
$$\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \sinh^{-1}\frac{x}{a}$$
$$\int \frac{\mathrm{d}x}{\sqrt{x^2-a^2}} = \begin{cases} \cosh^{-1}\dfrac{x}{a} & \text{if } x > a \\ -\cosh^{-1}\dfrac{-x}{a} & \text{if } x < -a \end{cases}$$
$$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

Definite:

$$\int_0^{\pi/2} \sin^m x\cos^n x\,\mathrm{d}x = \frac{m-1}{m+n}\int_0^{\pi/2}\sin^{m-2}x\cos^n x\,\mathrm{d}x = \frac{n-1}{m+n}\int_0^{\pi/2}\sin^m x\cos^{n-2}x\,\mathrm{d}x$$

$$I_n \equiv \int_0^\infty x^n \mathrm{e}^{-ax^2}\mathrm{d}x:\ I_0 = \frac{1}{2}\sqrt{\frac{\pi}{a}},\ I_1 = \frac{1}{2a},\ I_{n+2} = \frac{(n+1)I_n}{2a}; \qquad \int_0^\infty \frac{x^{1/2}}{\mathrm{e}^x - 1}\mathrm{d}x = 2{\cdot}32$$

Vector identities

In a right handed system $\quad \mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

Vector triple product $\quad \mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$

Vector calculus

If U is a scalar field and $\mathbf{F}$ is a vector field:

Cartesian coordinates:

$$\boldsymbol{\nabla} U = \operatorname{grad} U = \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right)$$

$$\boldsymbol{\nabla} \cdot \mathbf{F} = \operatorname{div} \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\boldsymbol{\nabla} \times \mathbf{F} = \operatorname{curl} \mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}, \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)$$

$$\nabla^2 U = \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} U) = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

$$\nabla^2 \mathbf{F} = \left(\nabla^2 F_x, \nabla^2 F_y, \nabla^2 F_z \right)$$

Cylindrical coordinates:

$$\boldsymbol{\nabla} U = \left(\frac{\partial U}{\partial r}, \frac{1}{r} \frac{\partial U}{\partial \phi}, \frac{\partial U}{\partial z} \right)$$

$$\boldsymbol{\nabla} \cdot \mathbf{F} = \frac{1}{r} \frac{\partial \left(r F_r \right)}{\partial r} + \frac{1}{r} \frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

$$\boldsymbol{\nabla} \times \mathbf{F} = \left(\frac{1}{r} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}, \frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}, \frac{1}{r} \left[\frac{\partial \left(r F_\phi \right)}{\partial r} - \frac{\partial F_r}{\partial \phi} \right] \right)$$

$$\nabla^2 U = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 U}{\partial \phi^2} + \frac{\partial^2 U}{\partial z^2}$$

Spherical polar coordinates:

$$\boldsymbol{\nabla} U = \left(\frac{\partial U}{\partial r}, \frac{1}{r} \frac{\partial U}{\partial \theta}, \frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} \right)$$

$$\boldsymbol{\nabla} \cdot \mathbf{F} = \frac{1}{r^2} \frac{\partial \left(r^2 F_r \right)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial \left(\sin \theta F_\theta \right)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial F_\phi}{\partial \phi}$$

$$\boldsymbol{\nabla} \times \mathbf{F} = \left(\frac{1}{r \sin \theta} \left[\frac{\partial \left(\sin \theta F_\phi \right)}{\partial \theta} - \frac{\partial F_\theta}{\partial \phi} \right], \right.$$
$$\frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial F_r}{\partial \phi} - \frac{\partial \left(r F_\phi \right)}{\partial r} \right],$$
$$\left. \frac{1}{r} \left[\frac{\partial \left(r F_\theta \right)}{\partial r} - \frac{\partial F_r}{\partial \theta} \right] \right)$$

$$\nabla^2 U = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial U}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial U}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 U}{\partial \phi^2}$$

Note that:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial U}{\partial r} \right) \equiv \frac{1}{r} \frac{\partial^2}{\partial r^2} (rU)$$

Useful identities:

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{F}) = \boldsymbol{\nabla} (\boldsymbol{\nabla} \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$$

$$\boldsymbol{\nabla} \cdot (\varphi \mathbf{F}) = \mathbf{F} \cdot (\boldsymbol{\nabla} \varphi) + \varphi (\boldsymbol{\nabla} \cdot \mathbf{F})$$

$$\boldsymbol{\nabla} \times (\varphi \mathbf{F}) = \varphi (\boldsymbol{\nabla} \times \mathbf{F}) - \mathbf{F} \times (\boldsymbol{\nabla} \varphi)$$

$$\boldsymbol{\nabla} (\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \boldsymbol{\nabla}) \mathbf{G} + (\mathbf{G} \cdot \boldsymbol{\nabla}) \mathbf{F} + \mathbf{F} \times (\boldsymbol{\nabla} \times \mathbf{G}) + \mathbf{G} \times (\boldsymbol{\nabla} \times \mathbf{F})$$

$$\boldsymbol{\nabla} \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\boldsymbol{\nabla} \times \mathbf{F}) - \mathbf{F} \cdot (\boldsymbol{\nabla} \times \mathbf{G})$$

$$\boldsymbol{\nabla} \times (\mathbf{F} \times \mathbf{G}) = (\mathbf{G} \cdot \boldsymbol{\nabla}) \mathbf{F} - (\mathbf{F} \cdot \boldsymbol{\nabla}) \mathbf{G} + \mathbf{F} (\boldsymbol{\nabla} \cdot \mathbf{G}) - \mathbf{G} (\boldsymbol{\nabla} \cdot \mathbf{F})$$