A10288W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday 4 September 2018, 9.30 am – 12.00 noon

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

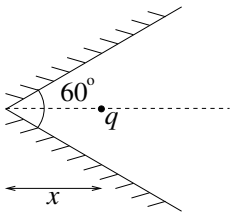
A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Two semi-infinite conducting plane sheets meet at an angle of 60°. A particle of charge q is placed in the plane of symmetry a distance x from the line at which the two planes meet, as shown in the diagram below. Find an expression for the force acting on the particle. [6]



2. A long metal wire of radius a carries a current I that is uniformly distributed over its cross-section. Derive expressions for the magnetic field strength B as a function of distance r from the centre of the wire both inside and outside the wire, and sketch B as a function of r. [5]

3. Show that the capacitance per unit length of a long air-filled coaxial cable is

$$C = \frac{2\pi\epsilon_0}{\ln(b/a)},$$

where a and b are the radii of the inner and outer conductors respectively. Find an expression for the potential difference V between the two conductors in terms of $E_{\max}$, the maximum electric field strength between the conductors. If the breakdown electric field strength of air is $3 \times 10^6\,\mathrm{V\,m^{-1}}$ and $b = 10\,\mathrm{mm}$, what value should be chosen for a to maximise the potential difference V, and what is this maximum value of V? [8]

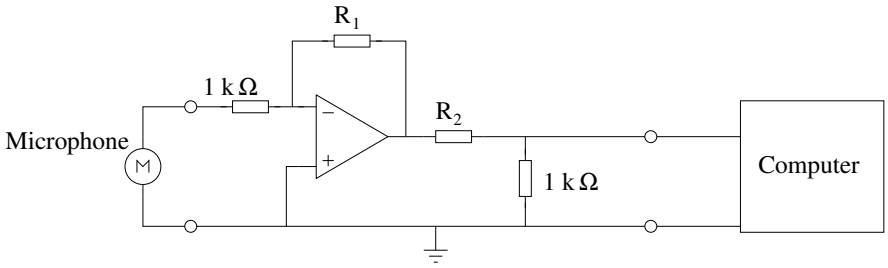
4. A long solenoid with $n = 10^4$ turns per metre and cross-sectional area $A = 10^{-3}\,\mathrm{m^2}$ carries a current $I = 2\,\mathrm{A}$. A second coil with $N = 100$ turns is wound outside the solenoid. Calculate

(i) the magnetic field inside the solenoid,
(ii) the flux through the cross-section of the solenoid,
(iii) the flux linkage of the second coil, and
(iv) their mutual inductance. [4]

5. An object 1.0 cm high is placed (a) 15 cm and (b) 6 cm from a converging lens with a focal length of 10 cm. In each case draw a ray diagram showing the situation and calculate where the image is formed and what size it is. [5]

6. Write down Maxwell's equations in a vacuum in differential form. Use them to show that there is a wave equation for electric and magnetic fields, and deduce the speed of the waves. [6]

7. A microphone that produces a typical voltage of about 5 mV is to be interfaced to a computer input port using the circuit sketched below using an ideal op-amp. The computer input port requires signals of about 0.5 V, has very high impedance, and would be damaged by signals outside the range $\pm 5\,\mathrm{V}$. The op-amp has a maximum output voltage of $\pm 15\,\mathrm{V}$. Calculate values of the resistors R_1 and R_2 to provide useful operation of the interface without risking damage to the computer. [6]



Section B

8. State the Biot-Savart Law, explaining all the terms. [2]

A current I flows in a circular loop of radius a. Show that the magnetic field on the axis of the loop a distance x from its centre is given by

$$B = \frac{\mu_0 I}{2} \frac{a^2}{(x^2 + a^2)^{3/2}}.$$

[4]

Find expressions for the magnetic field strength in the limiting cases (i) $x \ll a$ and (ii) $x \gg a$. Sketch the form of the curve $B(x)$. [6]

A thin insulating disc of radius a carries a total charge Q distributed evenly over its surface. The disc is rotating in its own plane about its centre at an angular velocity ω. Calculate the magnetic field at the centre of this disc, and show that the magnetic dipole moment of the disc is

$$M = \frac{\omega Q a^2}{4}.$$

[8]

9. Write down the expression for the force on a charge moving in an electric and magnetic field.

A straight conducting rod of length l is moving in a uniform magnetic field B at a velocity v. The magnetic field is oriented perpendicular to the rod. Show that there is a potential difference $V = Blv_\perp$ induced along the rod, where $v_\perp$ is the component of the velocity perpendicular to the B field. Indicate on a sketch the sign of the potential difference. [5]

A swing consists of a metal rod of length l that is suspended at both ends by insulating ropes of length R. It swings with small oscillations under gravity in the $y-z$ plane (see sketch below) such that $\theta = \theta_0 \cos(\omega t)$. If there is a uniform magnetic field of strength B in the y direction, show that there is a sinusoidal voltage

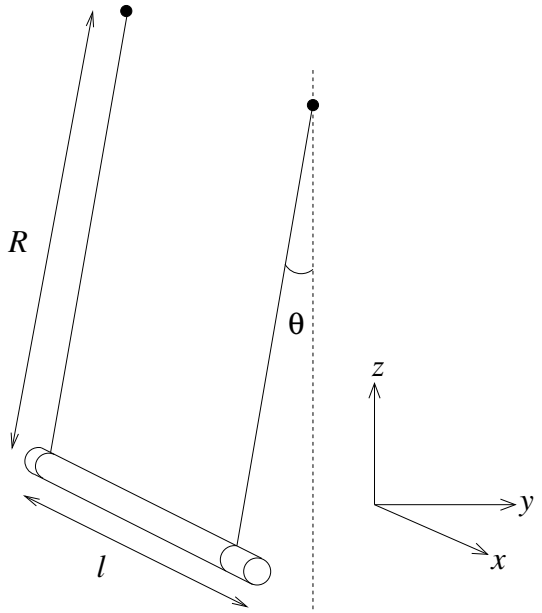
$$V = \frac{BlR\,\theta_0^2\,\omega}{2} \sin 2\omega t$$

induced across the rod. [9]

If the magnetic field is now oriented in the z direction, calculate the new expression for the voltage across the rod.

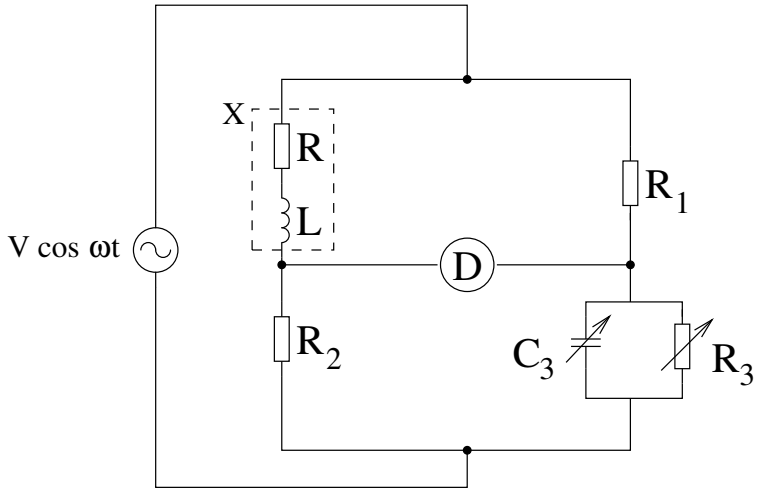
If $l = 0.1\,\text{m}$ and $R = 1\,\text{m}$, and the amplitudes of the voltages in the two cases are 8 mV and 33 mV respectively, calculate the magnitude of the magnetic field B. [6]

[The angular frequency of a simple pendulum of length R is $\omega = \sqrt{g/R}$.]



10. Explain the essential features of a capacitor and an inductor. Write down the differential equations that relate the current through and voltage across each of these components. Hence derive an expression for the complex impedance of each component when driven by a sinusoidal voltage of angular frequency ω. [8]

The circuit below is used to measure the unknown inductance L and series resistance R of component X. The variable resistance R_3 and variable capacitance C_3 are adjusted until the circuit is balanced, i.e. the voltage detector D reads zero. Resistances R_1 and R_2 are fixed and known.



Derive formulas for the values of the unknown quantities R and L when the circuit is balanced. [8]

If $R_1 = R_2 = 300\,\Omega$, and the bridge is balanced when $C_3 = 1.0\,\mu$F and $R_3 = 9.0\,\text{k}\Omega$, find the values of R and L. [2]

In this case comment on the practical limits on the frequency ω of the voltage source that is used. [2]

11. A mobile phone base station has two antennas separated by a distance d, transmitting identical signals of wavelength λ. Each antenna radiates equally in all directions. Show that the signal intensity at a point P far from the transmitters is proportional to $\cos^2(kds/2)$ where $k = 2\pi/\lambda$, $s = \sin\theta$, and θ is the angle from the antennas to P as shown in the diagram. [6]

A mobile phone user initially at the point O moves in the direction indicated by the arrow in the diagram until the signal strength they see falls to zero. What phase difference between the two antennas should be introduced to restore the signal strength to a maximum? What is now the expression for the intensity as a function of s? [6]

If the mobile phone user is initially a distance $L = 1$ km from the base station, and has to move a distance 100 m from point O to go from the maximum to the minimum signal intensity, calculate the separation between the antennas given that the wavelength $\lambda = 1.5\,\text{m}$. [4]

If one of the antennas was switched off, by what factor would the maximum intensity as seen by a mobile phone user at position O fall? What does this imply for the useful range of the base station using two antennas compared to one? [4]

