A10288W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday 5 June 2018, 2.30 pm – 5.00 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Write down an expression for the electric field at a displacement $\mathbf{r}$ from a charge of magnitude Q. [2]

A vertical dipole consists of a charge of $+30\,\mathrm{C}$ held at a height of $8\,\mathrm{km}$ above an infinite conducting plane and a second charge of $-30\,\mathrm{C}$ held directly below the first charge but at a height of $5\,\mathrm{km}$ above the same infinite conducting plane. Calculate the electric field directly below the vertical dipole at a height of 0 km. [5]

2. Write down Ampère's law in the case of non-time-varying fields and explain each of the terms. Using this law derive an expression for the force per unit length between two parallel and infinitely long straight wires, each carrying a constant current I in the same direction and separated by a distance r. In which direction does the force act? [6]

3. Explain why a displacement current term has to be added to Ampère's law in the case of time-varying fields. [3]

A parallel-plate capacitor consists of two circular plates, each of radius a and separated by a distance d, where $d \ll a$. The space between the plates is filled with air. An alternating potential difference of $V_0 \exp(-j\omega t)$ is applied across the capacitor. Find an expression for the magnitude of the magnetic field between the plates. [5]

4. Estimate the electric field strength $100\,\mathrm{km}$ from a $100\,\mathrm{kW}$ television transmitter assuming isotropic radiation. [6]

5. Sketch and clearly label an experimental arrangement for using a diffraction grating as a spectrometer. Comment on the conditions required for proper illumination of the diffraction grating. [4]

If a diffraction grating of 2000 lines per cm is used to analyse the spectrum of mercury, find the angular separation in the second-order spectrum of the two lines of wavelengths $579.0\,\mathrm{nm}$ and $577.0\,\mathrm{nm}$. [3]

6. Draw and label a circuit using one operational amplifier and resistors which gives an output

$$V_{\mathrm{out}} = -2(V_{\mathrm{a}} + 2V_{\mathrm{b}})$$

for two voltage inputs V_{a} and V_{b}.

Describe the behaviour of the circuit if the feedback resistor is replaced by a capacitor. [6]

Section B

7. State Gauss's Law in both integral and differential form and use it to derive an expression for the capacitance C of an isolated conducting sphere of radius R which carries a charge Q on its surface. [7]

Calculate the potential energy, W, of the system. [3]

A long cylindrical capacitor of length l consists of an outer conductor of radius a and an inner conductor of radius b, where $l \gg a$. The outer conductor is earthed, and the inner conductor is hollow, insulated and uncharged. The gaps between both conductors are air-filled. A sphere of radius R is charged to a potential V far from any other bodies and is then inserted inside the inner conductor of the cylindrical capacitor without touching it.

(i) Draw a clearly labelled diagram showing the field lines and explain the steps in your reasoning. [4]

(ii) Calculate the electric field strength at a radius $b < r < a$, in terms of the potential V and the radius R of the sphere. [3]

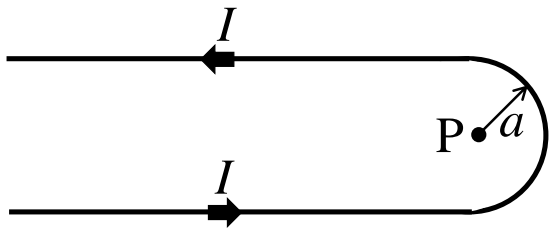
(iii) Hence find the potential of the inner cylinder. [3]

8. Write down the Biot-Savart law, clearly explaining all the terms. [2]

A current I is passed through a circular coil of radius a. Use the Biot-Savart law to calculate the magnitude and direction of the **B**-field at a point on the axis of the coil, a distance x from its centre. [5]

Calculate the magnitude and direction of the **B**-field produced by a thin wire of length $2b$ carrying a current I, at a point on the perpendicular plane that bisects the wire, at a distance a from the centre of the wire. [5]

In the figure below, a long straight wire has been bent into a hairpin shape:



Calculate the magnitude and direction of the magnetic field **B** at the centre P of the semicircle when a current I flows through the wire in the direction indicated. [8]

9. A voltage source produces a voltage $V_0 \cos(\omega t)$. For each of the components listed below, state its complex impedance and sketch the voltage across and the current through each component for several cycles of the sinusoidal waveform when they are connected across the terminals of the voltage source.

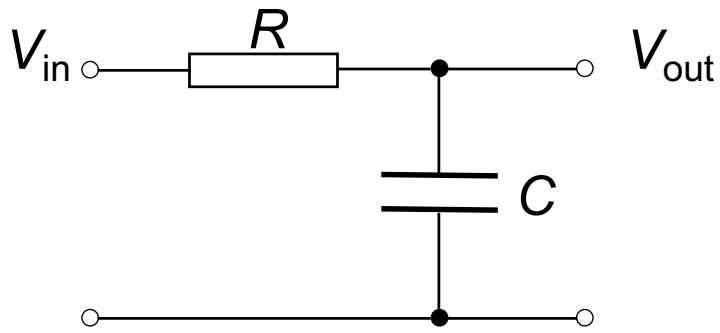
(i) A resistor of resistance R.

(ii) A capacitor of capacitance C.

(iii) An inductor of inductance L.

Calculate the total impedance of the system if a resistor, capacitor and inductor are connected together in series. [6]

The diagram below shows the circuit diagram for a simple low-pass filter, consisting of a resistor of resistance R and a capacitor of capacitance C.



(a) If a sinusoidal voltage of $V_0 \cos(\omega t)$ is applied to the filter, find the amplitude and phase of V_{out} relative to V_{in}. Sketch the resulting amplitude and phase as a function of ω and comment on the cases when the angular frequency, ω, tends to 0 and ∞. [4]

(b) Calculate the angular frequency for which

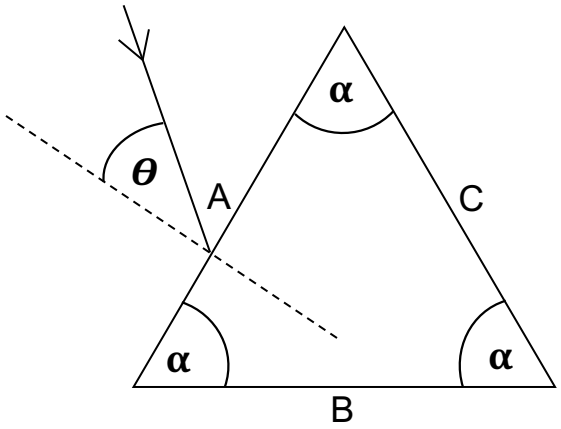
$$|V_{\text{out}}| = \frac{|V_{\text{in}}|}{\sqrt{2}}.$$

[2]

(c) Now a constant voltage V is applied to the input of the filter at time $t = 0$. If the capacitor is initially uncharged, determine an expression for the output voltage V_{out} as a function of time t. [3]

(d) If instead a square-wave voltage of fixed period $T = 2\pi/\omega$ is applied to the input, sketch V_{out} for the cases when (i) $\omega \ll 1/RC$, (ii) $\omega \gg 1/RC$ and (iii) $\omega \sim 1/RC$. [5]

10. State Huygen's Principle, and use it to derive Snell's Law of refraction. [5]



The figure above shows a prism of refractive index $n > 1$ which has all angles $\alpha = 60°$, and has faces labelled A, B and C.

Consider a light ray of green monochromatic light which enters the prism at an angle θ to the normal of face A as shown in the figure above.

(i) Sketch and comment on the possible light paths that the light ray may take through the prism. You should consider only positive angles of θ as measured clockwise from the normal to face A. [4]

(ii) If the refractive index of the prism is $n = 1.5$, for what range of angles θ does the ray undergo total internal reflection at face B of the prism? [4]

(iii) For a ray that does undergo total internal reflection at face B of the prism, calculate the angle of deviation, δ, of the ray that subsequently emerges from face C of the prism with respect to the initial direction of the ray incident on face A. Express your answer in terms of θ and α. [4]

(iv) The incident angle θ is fixed such that total internal reflection is just occurring at face B for green monochromatic light. Describe what would be observed if the monochromatic light source is replaced with a white light source incident at the same angle θ. [3]