A10287W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 3 September 2018, 9.30 am – 12.00 noon

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. A man fires a pistol at an angle θ_1 to the horizontal, and a time Δt later fires again at a lower angle θ_2. The bullets collide in mid air. Neglecting air resistance, show that the time interval between shots is

$$\Delta t = \frac{2v}{g}\frac{\sin(\theta_1 - \theta_2)}{(\cos\theta_1 + \cos\theta_2)} \ ,$$

where v is the initial speed of each bullet. [6]

2. Consider two particles with masses m_1 and m_2 and position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, interacting through a mutual potential, $U(|\mathbf{r}|)$. By expressing in terms of the relative position vector $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and the centre-of-mass coordinate $\mathbf{R}$, show that the total energy can be written as

$$E = \frac{1}{2}M\dot{\mathbf{R}}^2 + \frac{1}{2}\mu\dot{\mathbf{r}}^2 + U(|\mathbf{r}|) \ ,$$

and give expressions for M and μ.
Comment on the usefulness of expressing the energy in this form. [6]

3. Consider a simple, plane pendulum consisting of a mass m attached to a light inextensible string of length l. Let θ be the angle of the pendulum with respect to the vertical. After the pendulum is set in motion, the length of the string is shortened at a constant rate $\alpha = -\frac{\mathrm{d}l}{\mathrm{d}t}$, while the suspension point remains fixed.
Find the Lagrangian and the Hamiltonian for this system, and comment on whether energy is conserved. Find also (but do not solve) the differential equation of motion for θ. [7]

4. If a large number of people on Earth were to move to the equator, explain qualitatively how this would affect the length of a day. [3]

5. Consider a perfectly spherical balloon of radius a, made from lead foil of uniform thickness t (where $t \ll a$) and density ρ_{Pb}. The balloon is filled with helium of density ρ_{He}, and the density of the air outside the balloon is ρ_0. Neglecting air resistance, derive an expression for the minimum radius possible such that the lead balloon will not sink. Evaluate your expression numerically for $\rho_{Pb} = 11340\ \mathrm{kg\,m^{-3}}$, $\rho_{He} = 0.164\ \mathrm{kg\,m^{-3}}$, $\rho_0 = 1.225\ \mathrm{kg\,m^{-3}}$ and $t = 0.015$ mm. [6]

6. A photon of energy Q collides head-on with a moving electron of rest mass M_0, travelling relativistically in the opposite direction. What initial velocity must the electron have if the photon recoils straight back in the direction from which it came, with the same energy Q. [5]

7. Derive the formula for the relativistic Doppler effect. A relativistic taxi has identical monochromatic lights at the front and rear, as viewed in its own rest frame. To a stationary observer, the headlights appear green ($\lambda = 550$ nm) when the taxi approaches, and the rear lights appear red ($\lambda = 700$ nm) when the taxi is receding. Find the speed of the taxi, and the wavelength of the emitted light in the taxi's rest frame. (You may assume that the taxi travels at a constant speed throughout its journey). [7]

Section B

8. A raindrop falls vertically through a cloud in the Earth's gravitational field and, as it falls, its mass $m(t)$ is increased by the accretion of water vapour that is initially at rest.

Neglecting air resistance, derive from first principles a differential equation relating the downwards velocity $v(t)$ of the raindrop to the rate of increase of its mass. [6]

(a) If the raindrop has an initial mass m_0, and accumulates mass at a constant rate αm_0, show that

$$\frac{\mathrm{d}}{\mathrm{d}t}[v(1+\alpha t)] = g(1+\alpha t) \ ,$$

and find its velocity $v(t)$ and acceleration $\frac{\mathrm{d}v(t)}{\mathrm{d}t}$, for an initial speed $v_0 = 0$. [8]

(b) Now consider the case where the mass of the raindrop increases proportionally with the mass already accumulated, $\frac{\mathrm{d}m(t)}{\mathrm{d}t} = \beta m(t)$, where β is a constant. Find $v(t)$ for an initial speed $v_0 = 0$.
Compare the results for the velocities obtained from (a) and (b), illustrating your answer with sketches of $v(t)$ versus time from $t = 0$ to large values of t. [6]

9. Compare and contrast the relative merits of the centre-of-mass and laboratory reference frames when considering particle collisions. [3]

A non-relativistic particle A, of mass m and initial velocity u, has a head-on and perfectly elastic collision with a stationary particle B of mass M (where $M > m$). Find the velocity of the centre-of-mass frame, and the velocities of A and B after the collision in both the laboratory and centre-of-mass frames. [6]

After a certain time, t, particle A has a second head-on and elastic collision with a stationary particle C, which has the same mass M as particle B. Show that the condition for a second collision of A with B to occur, is

$$M > (2+\sqrt{5})m \ .$$

Find the total kinetic energy of the 3 particles following the collision between A and C, and verify that no energy has been lost. [11]

10. Write down an expression for the invariant interval in space-time. Verify that this form of the invariant interval is consistent with the Lorentz transformations. Using the form of this invariant, or otherwise, derive an expression for the time dilation between observers moving at constant relative speed. [8]

A rocket with proper length L is moving at a speed $0.8c$ according to a stationary observer on Earth. A ball is thrown from the back of the rocket to the front, at speed $0.5c$ with respect to the rocket.

How much time does this take, and what distance does the ball cover, in

(i) the rocket frame;

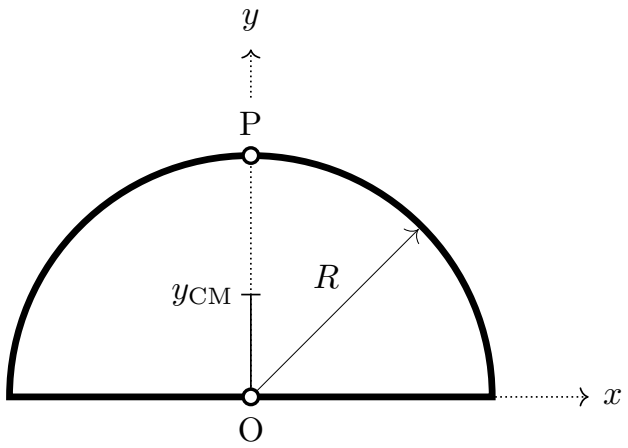
(ii) the Earth frame;

(iii) the ball frame?

Verify that the invariant interval is the same in all 3 inertial frames. [12]

11. Derive the parallel axis theorem for the moment of inertia of a body. [4]

Consider a thin, uniform, solid semicircular plate of mass M and radius R, as shown in the diagram below.



Show that the distance of the centre-of-mass of the semicircle from its straight edge is $y_{CM} = \frac{4R}{3\pi}$. [5]

By making use of the moment of inertia about the principal axis of a uniform circular disk, or otherwise, find the moment of inertia of the semicircular plate about an axis through O (the midpoint of the straight edge of the plate), and perpendicular to its plane. [2]

The semicircular plate is now pivoted at a point on its edge and caused to swing in a vertical plane. Find the period of small oscillations for the following cases:

(i) the pivot is at point O;

(ii) the pivot is on the curved edge at point P, directly opposite O.

Evaluate numerically the ratio of your answers for (i) and (ii). [9]

[The moment of inertia of a circular disk of mass m and radius a about an axis through its centre and perpendicular to its plane is $\frac{1}{2}ma^2$.]