A10287W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 4 June 2018, 2.30 pm – 5.00 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Define the Lagrangian and the Hamiltonian as used in classical mechanics. Prove that the Hamiltonian is conserved in a system for which the Lagrangian has no explicit time dependence. [6]

2. A spaceship is required to hover at a constant height just above the surface of a planet where the acceleration due to gravity is g. The exhaust expels fuel at a constant speed u, and the initial mass of the spaceship including fuel is M_0. While there is fuel remaining, what should be the rate of ejected mass $\alpha(t) = -\frac{\mathrm{d}M(t)}{\mathrm{d}t}$ to keep the ship hovering at rest at a time t? Express your answer in terms of M_0, u, g and t. [6]

3. A ball of mass M and radius r rests on top of a fixed hemisphere of radius R. There is friction between the ball and hemisphere. The ball is given an infinitesimal perturbation and rolls down the hemisphere without slipping. Assuming that $r \ll R$, find the angle to the vertical at which the ball loses contact with the hemisphere. [7]

[The moment of inertia of the ball about its principal axis is $\frac{2}{5}Mr^2$.]

4. A non-relativistic particle of mass m_1 and speed u_1 collides elastically and obliquely with another particle of mass m_2, which is initially at rest. Draw diagrams illustrating the situation after the collision in the laboratory frame and in the centre of mass frame.

Show that, after the collision, the scattering angle of m_2 in the laboratory frame, θ_2, is related to the scattering angle, θ^*, in the centre of mass frame by

$$\tan\theta_2 = \frac{\sin\theta^*}{1-\cos\theta^*} \ ,$$

where all angles are defined with respect to the initial direction of m_1. [8]

5. An object has velocity components u_x and u_y in an inertial frame S. Starting from the Lorentz transforms for space and time, show that the velocity components of the same object in a frame S', with speed v in the positive x direction relative to S, are

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} \quad \text{and} \quad u'_y = \frac{u_y}{\gamma\,(1 - u_x v/c^2)} \ ,$$

where $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$.

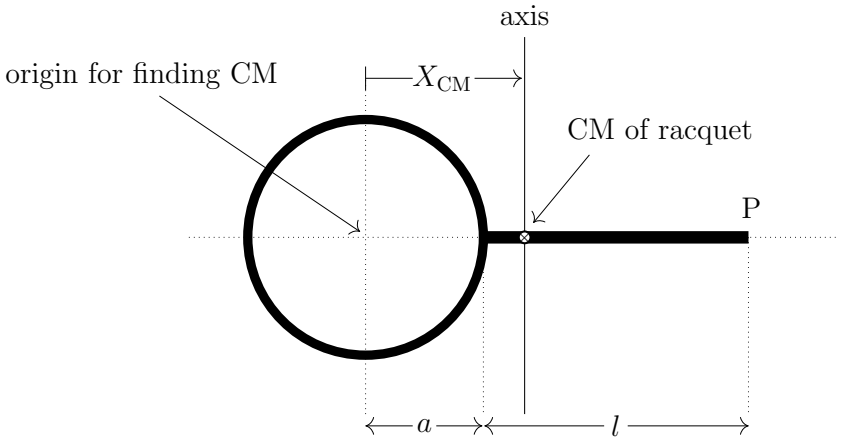
As seen from Earth, two spaceships A and B are travelling along perpendicular directions. If A is observed by a stationary Earth observer to have velocity $+0.5c$ in the x direction and B to have velocity $-0.5c$ along y, determine the speed of ship B as measured by the pilot of ship A. [8]

6. Muons of energy 6 GeV enter the atmosphere at a height of 15 km above the surface of the Earth and travel vertically downwards. Given that the muon rest mass energy is 106 MeV and the mean lifetime in its rest frame is 2.2×10^{-6} s, what proportion of the incident muons reach the ground? [5]

Section B

7. State the parallel and perpendicular axis theorems for a thin, planar, rigid body, defining the terms used. [4]

A simple tennis racquet can be modelled as a circular head that is a uniform ring of radius a and mass M_a, attached to a handle that is a uniform thin rod of length l and mass M_l, as illustrated in the diagram below.



(a) Find an expression for the distance, X_{CM}, of the centre of mass (CM) of the racquet from the centre of the head in terms of a, l, M_a and M_l. [2]

(b) Derive an expression for the moment of inertia of the racquet about the axis shown in the diagram, which passes through the centre of mass, is perpendicular to the handle and in the plane of the racquet. (You may leave your answer in terms of the symbol X_{CM}, where this is given by your answer to (a).) [4]

(c) A racquet is constructed in this simple model with $M_a = M_l = M$ and $l = 2a$. A tennis player holds the racquet at the very end of the handle (at point P), and a ball is launched perpendicularly at the flat face of the head. At what position relative to the centre of the head should the ball hit, so that the player feels no net force on her hand? Express your final answer as a fraction of a. [10]

[The moment of inertia of a thin uniform rod of mass M and length l about an axis through its centre of mass and perpendicular to the rod is $\frac{1}{12}Ml^2$.]

8. A point particle of mass m moves within a central potential $V(|\mathbf{r}|)$, where $\mathbf{r}$ is the position vector of the particle. Show that the angular momentum about the origin is a conserved quantity.

Explain how this gives rise to the concept of an *effective potential*, and give an expression for it in terms of the magnitude of the angular momentum J, the mass m and the distance from the origin $r = |\mathbf{r}|$. [6]

A satellite moves in an elliptical orbit of major axis $2a$ about the Earth. Using conservation of energy and angular momentum at the distances of closest approach (perigee) and furthest approach (apogee) to Earth, show that

$$v_{\mathrm{A}}^2 = GM\left(\frac{2}{r_{\mathrm{A}}} - \frac{1}{a}\right),$$

where r_{A} is the distance of the satellite from Earth, and v_{A} is its speed, at apogee. [6]

When at its apogee, a small retrorocket on the satellite is ignited, which reduces its speed from v to $(1-\epsilon)v$ where $\epsilon \ll 1$. Given that $r_{\mathrm{A}} = a(1+e)$, where e is the eccentricity of the original orbit, and neglecting any change to the satellite's mass, show that the major axis of the orbit is reduced by an amount

$$4\epsilon a\frac{(1-e)}{(1+e)}\ .$$

[8]

9. Explain what is meant by an invariant quantity in the context of special relativity. Illustrate your answer with the relationship between total energy, momentum and rest mass for a particle in special relativity. [3]

In the laboratory frame, a particle of rest mass M and total energy E collides with a stationary antiparticle, also of rest mass M, and annihilates. Calculate the energy available for the creation of new particles. [5]

Now consider a particle of rest mass M and kinetic energy T in the laboratory, which collides *elastically* with an identical particle that is initially stationary. After the collision, the two particles travel with paths inclined to the original track of the first particle by $\pm\theta$. By considering the motion of the particles in the laboratory frame, or otherwise, show that

$$\cos^2\theta = \frac{T+2Mc^2}{T+4Mc^2}\ .$$

Find the angle between the two particles, after the collision, for the two cases:

(i) $T \gg Mc^2$,

(ii) $T \ll Mc^2$,

and briefly comment on the results. [12]

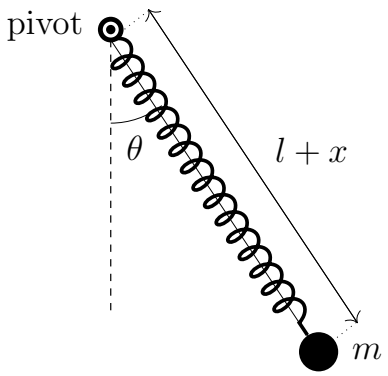
10. (a) Consider a mass m on the end of a light spring of natural length l and spring constant k, which hangs vertically in the Earth's gravitational field. Let $l + x(t)$ be the total length of the spring, and assume the mass can move only up and down in the vertical direction.

Show that, for a particular choice of reference for zero gravitational potential energy, the Lagrangian for this system is

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 + mg(l + x) \ ,$$

and find the Euler-Lagrange equation of motion. Verify that it is consistent with that obtained using Newton's 2nd Law. [5]

(b) Now consider the same spring and mass system, but allow the mass to also swing from side to side, as shown in the diagram. Let the angle of the spring with the vertical be $\theta(t)$, and let the total length of the spring again be $l + x(t)$. Assume that the motion is confined to a vertical plane, and that the spring stays in a straight line.



Write down the Lagrangian, and find the equations of motion for x and θ. [9]

Show that there are two equilibrium positions at $(x, \theta) = (\frac{mg}{k}, 0)$ and $(-\frac{mg}{k}, \pi)$.

For the stable equilibrium, approximate the equations of motion to 1st order in x and θ, and show that the small oscillations of the pendulum on a spring decouple into two independent modes. Give a physical interpretation for each mode. [6]