A10427W1

SECOND PUBLIC EXAMINATION

Honour School of Physics and Philosophy Part A

A2P: ELECTROMAGNETISM

TRINITY TERM 2017

Tuesday, 6 June, 2.30 pm – 4.10 pm

Time allowed: 1 hour 40 minutes

Answer **all** *of Section A and* **two** *questions from Section B.*

For Section A start the answer to each question on a new page.
For Section B start the answer to each question in a new book.

The use of approved calculators is permitted.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. An old-fashioned television set has a screen 50 cm across and uses an evacuated cathode ray tube: an electron gun behind the screen fires electrons with initial energy 100 eV in the direction of the centre of the screen. Before reaching the screen these electrons pass between a pair of parallel conducting plates located 30 cm behind the screen and at right angles to the screen. If the plates are squares of side 1 cm, and are separated by 1 cm, calculate the range of voltages between the two plates which are required to produce a display across the full width of the television screen. [You may ignore edge effects for the plates and the influence of gravity.] [5]

2. A wire forms a closed circular ring of enclosed area A, resistance R and self-inductance of L. A spatially-uniform external magnetic field is applied normal to the plane of the ring, and the amplitude of the magnetic flux density varies with time as $B = B_0 \cos \omega t$, where B_0 and ω are constants. Find expressions for the e.m.f. and the current induced in the ring due to the applied field. Find an expression for the time-average of the power dissipated in the ring. [6]

3. Two infinite sheets of electric charge, one with surface charge density $+\sigma$ and the other with surface charge density $-\sigma$, intersect in the centre at right angles. Find the magnitude and direction of the **E**-field, and sketch the **E**-field lines. [5]

4. What are the fluxes of **E**-field ($\iint_{\text{Area}} \mathbf{E} \cdot \mathrm{d}\mathbf{A}$) through each of the faces of a cube of side a in the following cases:
i) if a point charge q is placed at the centre of the cube;
ii) if a point charge q is placed at one of the corners of the cube?
[Hint: you do not need to evaluate the surface integral directly if you consider the symmetries.] [4]

5. Starting from Maxwell's equations in a vacuum, show that the **E**-field of an electromagnetic wave obeys a wave equation, and find the velocity of propagation. If the **E**-field is polarized along the x-axis and varies as $\mathbf{E} = E_0 \cos(k\,z - \omega\,t)\,\hat{\mathbf{x}}$, where $E_0 = 100\,\mathrm{V\,m^{-1}}$, derive an expression for the magnetic flux density vector **B** and calculate the amplitude of the **B**-field. [7]

Section B

6. A solid metal sphere of radius a carries a charge Q. Find expressions for the electric field, $\mathbf{E}$, at $r > a$ and $r < a$, where r is the distance from the centre of the sphere. [2]

Calculate:
(i) the capacitance of the sphere;
(ii) the energy density of the electric field at a distance r (where $r > a$);
(iii) the total energy of the E-field. [7]

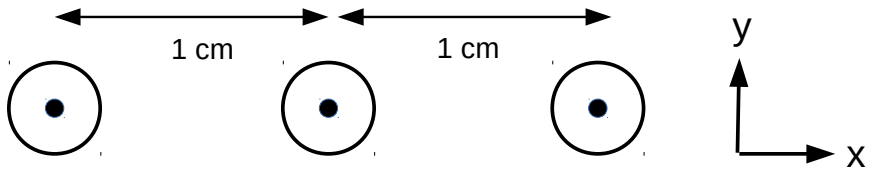
Calculate the work done in charging the sphere by bringing infinitesimal charges from infinity, and compare this result with your answer to (iii) above. [3]

The sphere is now earthed, and a point charge q is placed outside the sphere at a displacement $\mathbf{d}$ from the centre of the sphere. Assuming outside the sphere is a vacuum, find an expression for the potential at a point $\mathbf{s}$ outside the sphere.
[Hint: you may wish to use the method of images.] [8]

7. Write down the *Biot-Savart law*, and use this to derive the magnitude and direction of the $\mathbf{B}$-field produced by a thin wire of length $2a$ carrying a current I, at a point along the perpendicular bisector of the wire, at a distance s from the centre of the wire. Show that this is consistent with Ampère's law for an infinitely long wire. [6]

Consider two parallel very long wires separated by distance d each with a current I flowing in opposite directions. What is the force per unit length between the wires, and will they attract or repel each other? Calculate the self-inductance per unit length of this system, assuming that the wires are thin. [6]

Three parallel infinite wires of negligible width are co-planar, evenly spaced by 1 cm and each carry current of 1 A in the same direction, as in the diagram below. Each wire has a mass per unit length, μ_m, of $1\,\mathrm{g\,m^{-1}}$.



Draw the magnetic field lines around the wires, and calculate the position(s) where the magnetic flux density is zero. [3]

The entire middle wire is displaced a small amount in the y-direction (perpendicular to the initial plane of the three wires), while the other two wires are held fixed. Describe the subsequent motion of the middle wire when it is released, give an expression for the relevant timescale of its motion, and calculate a value for this timescale. [5]

8. Two long hollow metal cylinders are coaxial. The inner cylinder has radius r_1 and the outer has radius r_2, and there is a vacuum between the two cylinders. The outer cylinder is maintained at a constant potential difference V with respect to the inner one. Find expressions for the electric field $\mathbf{E}$ between the two cylinders ($r_1 < r < r_2$), and the capacitance per unit length. [5]

By integrating the energy density of the $\mathbf{E}$-field over the volume between the two cylinders, show that the energy stored in the capacitor is $\frac{1}{2}CV^2$. [4]

Between the cylinders there is a uniform magnetic flux density $\mathbf{B}$ parallel to the axis of the cylinders. Show that protons (mass m and charge $+e$) of a particular speed v can orbit at a radius r between the cylinders if v satisfies a quadratic equation of the form

$$a\,v^2 + b\,v + c = 0$$

and find expressions for a, b and c in terms of the variables given. [You may ignore the effect of gravity and may assume the protons travel with non-relativitistic speeds.] [8]

Calculate the speeds of protons orbiting between the cylinders at a radius of 5 cm in a magnetic flux density of $B = 0.01\,\mathrm{T}$ if the cylinders have radii $r_1 = 0.04\,\mathrm{m}$ and $r_2 = 0.06\,\mathrm{m}$ and a potential difference of 20 V. [3]