A10288W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday 5 September 2017, 9.30 am – 12.00 pm

Time allowed: 2$\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Starting with the Maxwell equations of electromagnetism, show that the **B**-field can obey a wave equation in a vacuum, and derive the speed of this electromagnetic wave in terms of physical constants. For the propagating wave, is the **B**-field transverse or longitudinal? [6]

2. A spherical conducting shell has inner radius r_1 and outer radius r_2, and carries a charge $-3\,q$. A charge q is fixed at the centre of the sphere. Find expressions for the electric field vector **E** for all radii r. What is the charge on the inner surface of the sphere? Calculate the surface charge per unit area on the outer surface. [4]

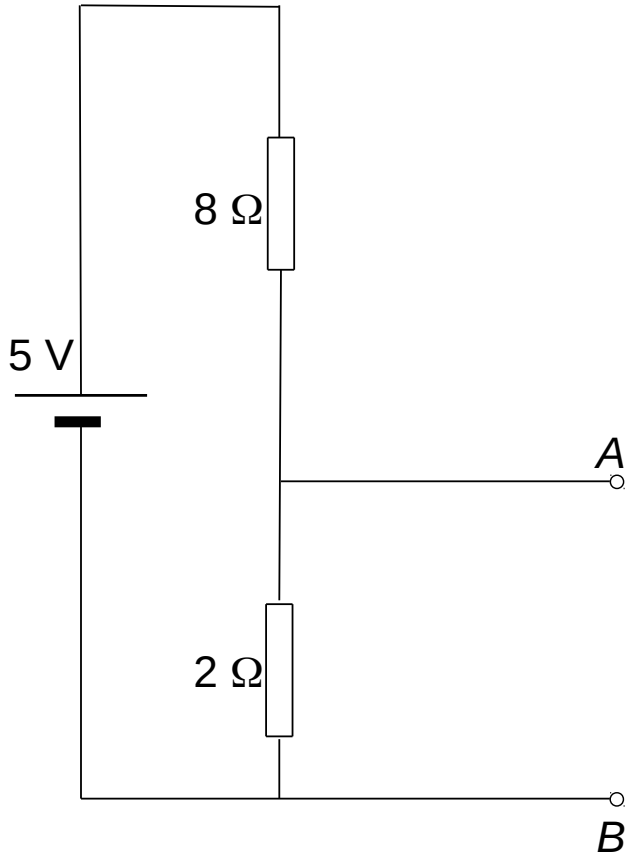
The charge q at the centre is removed, and the interior of the sphere ($r < r_1$) is completely filled with a uniform fixed space charge of density ρ, where ρ is a constant. If the surrounding conducting sphere is now grounded, find the potential at the centre of the sphere, and the total electrostatic energy of the system. [5]

3. A parallel plate capacitor has circular plates of radius $r = 10\,\text{cm}$. The two plates are initially separated by $d = 1\,\text{cm}$, with the gap between them being a vacuum, and a potential difference of 100 V is maintained between the plates by connecting them to a battery. The battery is then disconnected. Find the charge on each plate, and calculate the work done in pulling the plates apart to a separation of $d = 2\,\text{cm}$. What is the potential difference between the plates after this? If an electron started at rest on the plate with the negative charge, what would its speed be when it reaches the positively-charged plate? [8]

4. A uniform beam of particles comprises positive charges q all travelling with velocity $\mathbf{v} = v_0\hat{\mathbf{z}}$ in a vacuum, where v_0 is a constant. By treating the beam as a long cylinder of radius R, with uniform electric charge density ρ at $r < R$ and zero at $r > R$, find the current density **J** for the beam. What are the **E**-field and **B**-field at a distance r from the axis of the cylinder ($r < R$), in terms of ρ, r and v? [6]

5. A diffraction grating has light at normal incidence and is used in third order. After passing through the grating, light of wavelength 589 nm is observed at an angle of 32° to the incident light beam. How many lines per mm are ruled on the grating? What is the maximum order of light that can be observed? [6]

6. In the circuit below, the current and voltage characteristics across the output terminals A and B are the same as that of a battery of e.m.f. $\mathcal{E}_0$ and internal resistance R. Calculate the values of $\mathcal{E}_0$ and R. What is the short-circuit current provided by this battery? [5]



Section B

7. Write down *Gauss' Law* in electrostatics in its integral and differential forms, defining the terms you use in the equations. [3]

A thin disk carries a fixed total charge Q spread with a constant surface charge density between $r = a$ and $r = b$ (with $b > a$) and zero elsewhere, where r is the radius. Calculate the electric field $\mathbf{E}$ and potential V at a point on the axis of symmetry along the normal to the plane of the disk at a distance x from the centre of the disk. Explain why the $\mathbf{E}$-field at the centre of the disk is zero, and find the distance x along the axis where the $\mathbf{E}$-field is maximum. [6]

If a point charge $-Q$ is constrained to slide along this axis, show that it will execute simple harmonic motion for small displacements from the centre of the ring (where the displacement x is much smaller than a and b), and calculate the period of oscillation. [4]

The sliding point charge is removed, and the thin disk is set rotating about the normal to the plane of the disk. State the equation for the *Biot-Savart* law for magnetic flux density $\mathbf{B}$, defining the terms you use, and use this to find the $\mathbf{B}$-field at the centre of the disk. [7]

8. A dipole consists of charges $+q$ and $-q$ separated by $\mathbf{d}$. What is the dipole moment? For such a dipole, derive the electrostatic potential V, and the electric field $\mathbf{E}$ at a point $\mathbf{s}$ (where $|\mathbf{s}| \gg |\mathbf{d}|$) using spherical polar coordinates centred on the dipole (where $\theta = 0$ is aligned along the axis of the dipole). [9]

Explain how the *method of images* can be used to solve problems in electrostatics involving a conducting surface. [3]

If the dipole is placed a perpendicular distance a above a horizontal infinite grounded conducting plane (where $a \gg |\mathbf{d}|$), find expressions for the force, $\mathbf{F}$, and torque, τ, experienced by the dipole. You should express your answers in terms of the angle θ between the dipole and the normal to the conducting plane, and give directions for the force and torque. What would be the total charge on the conducting plane? [8]

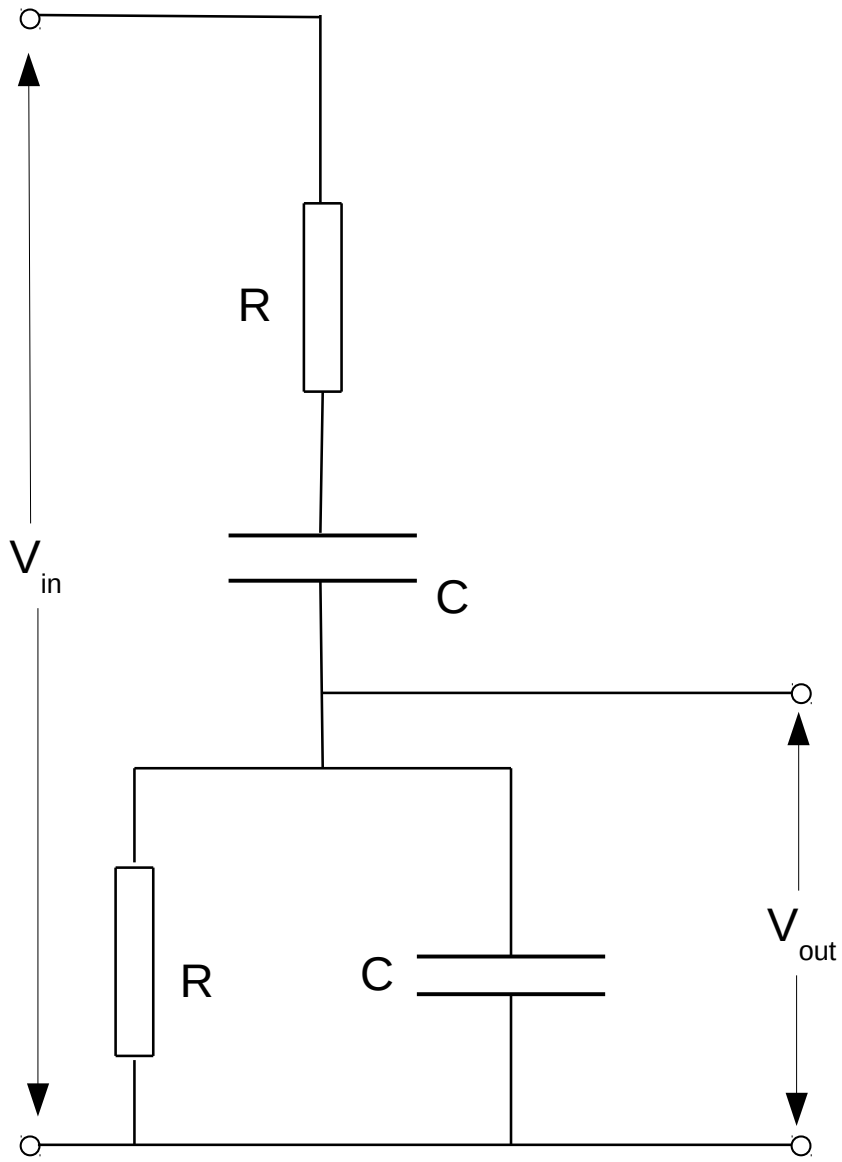
9. Explain *Fermat's principle* in optics, and use it to derive *Snell's law for refraction* at a plane boundary between two media of refractive indices n_1 and n_2 respectively. [6]

What is meant by the *paraxial approximation*? A lens has a refractive index n and thickness $s = s_0 - \alpha y^2$ where y is the distance from the optical axis (the x-axis) in the plane of the lens ($x = 0$), and s_0 and α are positive constants. A ray which crosses the x-axis at $x = -u$, is refracted by the lens at $x = 0$, and crosses the x-axis again on the other side of the lens at $x = v$. Using the paraxial approximation, find expressions for the path length taken by this ray, and for the time taken to travel this path. Using Fermat's principle, derive an expression for the focal length of the lens in terms of n and α. You may assume $n = 1$ in air. [10]

If the refractive index $n = 1.5$ for the lens, and $\alpha = 2$ (where the distances are in units of metres), calculate the focal length of the lens. If an object is placed at a distance $u = 0.7\,\mathrm{m}$, where would the image be formed, and would it be real or virtual? [4]

10. In circuit theory, what is meant by a complex impedance? For a circuit driven by a sinusoidal current of angular frequency ω, derive the complex impedance for each of a resistor, a capacitor and an inductor. [5]

For the circuit below, find an expression for V_{out} if $V_{\text{in}} = V_0 \cos \omega t$, where V_0 is a constant. [5]



Find the angular frequency $\omega = \omega_0$ for which $|V_{\text{out}}|$ is maximum. What is the ratio of $V_{\text{out}}/V_{\text{in}}$ at this frequency? What is the behaviour of V_{out} for $\omega \ll \omega_0$ and for $\omega \gg \omega_0$? [5]

Find the relative phase of V_{in} and V_{out} as a function of ω. What is the relative phase when $\omega = \omega_0$? [5]