A10288W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday 6 June 2017, 2.30 pm – 5.00 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. An old-fashioned television set has a screen 50 cm across and uses an evacuated cathode ray tube: an electron gun behind the screen fires electrons with initial energy 100 eV in the direction of the centre of the screen. Before reaching the screen these electrons pass between a pair of parallel conducting plates located 30 cm behind the screen and at right angles to the screen. If the plates are squares of side 1 cm, and are separated by 1 cm, calculate the range of voltages between the two plates which are required to produce a display across the full width of the television screen. [You may ignore edge effects for the plates and the influence of gravity.] [5]

2. A wire forms a closed circular ring of enclosed area A, resistance R and self-inductance of L. A spatially-uniform external magnetic field is applied normal to the plane of the ring, and the amplitude of the magnetic flux density varies with time as $B = B_0 \cos \omega t$, where B_0 and ω are constants. Find expressions for the e.m.f. and the current induced in the ring due to the applied field. Find an expression for the time-average of the power dissipated in the ring. [6]

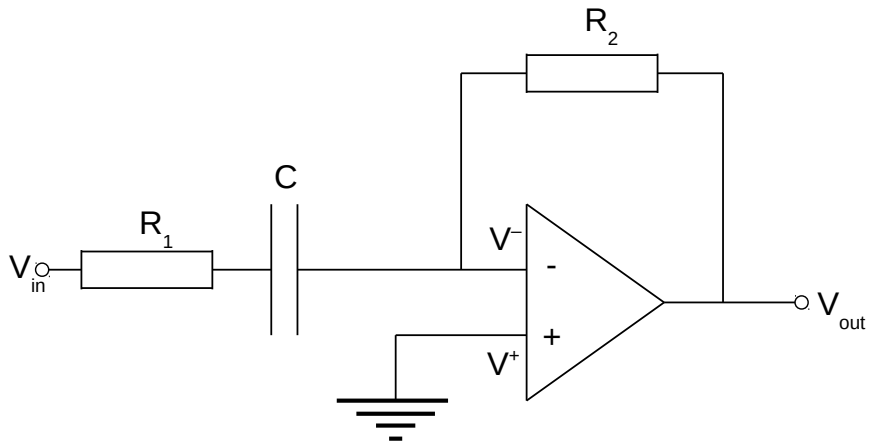
3. Two infinite sheets of electric charge, one with surface charge density $+\sigma$ and the other with surface charge density $-\sigma$, intersect in the centre at right angles. Find the magnitude and direction of the **E**-field, and sketch the **E**-field lines. [5]

4. What are the fluxes of **E**-field ($\iint_{\text{Area}} \mathbf{E} \cdot \mathrm{d}\mathbf{A}$) through each of the faces of a cube of side a in the following cases:
i) if a point charge q is placed at the centre of the cube;
ii) if a point charge q is placed at one of the corners of the cube?
[Hint: you do not need to evaluate the surface integral directly if you consider the symmetries.] [4]

5. Starting from Maxwell's equations in a vacuum, show that the **E**-field of an electromagnetic wave obeys a wave equation, and find the velocity of propagation. If the **E**-field is polarized along the x-axis and varies as $\mathbf{E} = E_0 \cos(k\,z - \omega\,t)\,\hat{\mathbf{x}}$, where $E_0 = 100\,\mathrm{V\,m^{-1}}$, derive an expression for the magnetic flux density vector **B** and calculate the amplitude of the **B**-field. [7]

6. State the *lens maker's formula*, relating the focal length f to the positions of the source (u) and image (v) relative to the centre of the lens along the optical axis. If the human eye can focus on an object as close as 25 cm, what is the maximum magnification of a magnifying glass consisting of a converging lens with focal length 4 cm? Sketch a ray diagram for the case of maximum magnification, and state whether the image produced is real or virtual. [6]

7. The circuit below includes an ideal operational amplifier (i.e. with very large gain). The input voltage is $V_{\text{in}} = V_0 \cos(\omega t)$ where V_0 and ω are positive constants, and the angular frequency ω is in the middle of the amplifier's bandwidth.



Find an expression for the ratio $V_{\text{out}}/V_{\text{in}}$. If $R_1 = R_2$, what is the phase difference between V_{out} and V_{in}? [7]

Section B

8. Two long hollow metal cylinders are coaxial. The inner cylinder has radius r_1 and the outer has radius r_2, and there is a vacuum between the two cylinders. The outer cylinder is maintained at a constant potential difference V with respect to the inner one. Find expressions for the electric field $\mathbf{E}$ between the two cylinders ($r_1 < r < r_2$), and the capacitance per unit length. [5]

By integrating the energy density of the $\mathbf{E}$-field over the volume between the two cylinders, show that the energy stored in the capacitor is $\frac{1}{2}CV^2$. [4]

Between the cylinders there is a uniform magnetic flux density $\mathbf{B}$ parallel to the axis of the cylinders. Show that protons (mass m and charge $+e$) of a particular speed v can orbit at a radius r between the cylinders if v satisfies a quadratic equation of the form

$$a\,v^2 + b\,v + c = 0$$

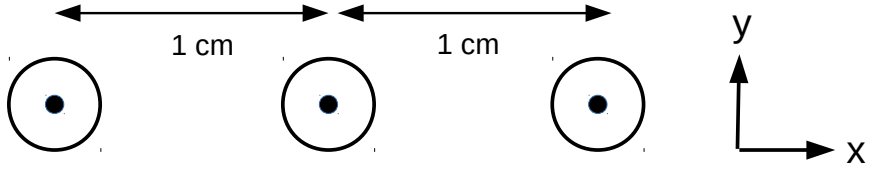
and find expressions for a, b and c in terms of the variables given. [You may ignore the effect of gravity and may assume the protons travel with non-relativitistic speeds.] [8]

Calculate the speeds of protons orbiting between the cylinders at a radius of 5 cm in a magnetic flux density of $B = 0.01\,\text{T}$ if the cylinders have radii $r_1 = 0.04\,\text{m}$ and $r_2 = 0.06\,\text{m}$ and a potential difference of 20 V. [3]

9. Write down the *Biot-Savart law*, and use this to derive the magnitude and direction of the **B**-field produced by a thin wire of length $2a$ carrying a current I, at a point along the perpendicular bisector of the wire, at a distance s from the centre of the wire. Show that this is consistent with Ampère's law for an infinitely long wire. [6]

Consider two parallel very long wires separated by distance d each with a current I flowing in opposite directions. What is the force per unit length between the wires, and will they attract or repel each other? Calculate the self-inductance per unit length of this system, assuming that the wires are thin. [6]

Three parallel infinite wires of negligible width are co-planar, evenly spaced by 1 cm and each carry current of 1 A in the same direction, as in the diagram below. Each wire has a mass per unit length, μ_m, of $1\,\mathrm{g\,m^{-1}}$.



Draw the magnetic field lines around the wires, and calculate the position(s) where the magnetic flux density is zero. [3]

The entire middle wire is displaced a small amount in the y-direction (perpendicular to the initial plane of the three wires), while the other two wires are held fixed. Describe the subsequent motion of the middle wire when it is released, give an expression for the relevant timescale of its motion, and calculate a value for this timescale. [5]

10. Explain what is meant by *Fraunhofer diffraction*. [3]

A slit of width w is illuminated at normal incidence by a collimated beam of coherent light of wavelength λ. Derive an expression for the intensity distribution as a function of angle for the diffraction pattern in the Fraunhofer limit. [6]

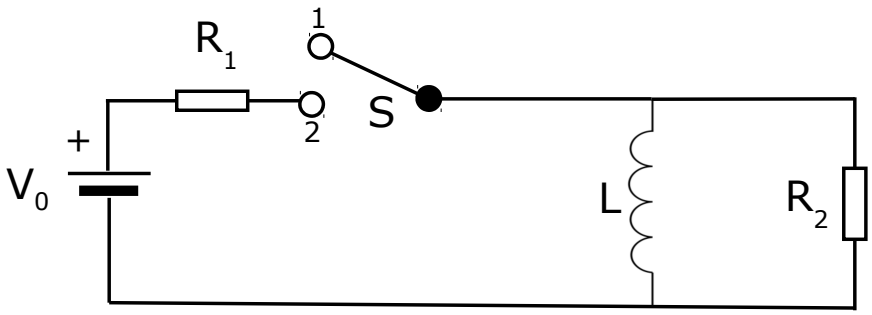
What is meant by the *Rayleigh criterion*? Use this to find an expression for the diffraction-limited angular resolution of a single slit. [3]

A spy satellite orbits 200 km above the surface of the Earth and observes light at visible wavelengths. Estimate the size of the main mirror on the spy satellite if it is able to read the headlines of a tabloid newspaper, with a font thickness of 3 cm. You may assume that only diffraction affects the resolution. [4]

A digital camera is used to record the image at the focus of the spy satellite's main mirror. If the main mirror of the spy satellite has a focal length of 80 m, estimate the maximum distance between the centres of adjacent pixels for the camera to fully resolve the newspaper headlines. Using these size pixels, how large a format would the camera need to be (in terms of the number of pixels across) in order to image all of the High Street in Oxford with one exposure? [Oxford High Street is about 500 m long.] [4]

11. A straight air-cored solenoid consists of N turns of wire, with each turn being a circular coil of radius a. If the solenoid is very long (length $l \gg a$) and carries a current I, find expressions for the magnetic field within the solenoid, and for the self-inductance (L) of the solenoid. If the number of turns per unit length is $2000\,\text{turns}\,\text{m}^{-1}$, and the solenoid has a diameter of 2 cm and a length of 50 cm, calculate the self-inductance, L. [4]

This solenoid (of negligible resistance) is used in the circuit below, along with resistors $R_1 = 10\,\Omega$ and $R_2 = 100\,\Omega$ and a voltage $V_0 = 100\,\text{V}$. The switch S is initially open (position 1) and no current flows. The switch is then closed (moving to position 2) at time $t = 0$. What is the current in resistor R_2 when the switch is just closed? What value does the current in R_2 tend towards when the switch has been closed for a long time, and why? Write down an expression for the current flowing through the inductor as a function of time. [7]



Calculate the total energy that has been dissipated in resistor R_2 since $t = 0$ after the switch has been closed for a long time. [5]

If, after being closed for a long time, the switch is now re-opened (i.e. moved back to position 1), calculate the resulting heat dissipated in resistor R_2 if the switch remains open for a long time. [4]