A10287W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 4 September 2017, 9.30 am – 12:00 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

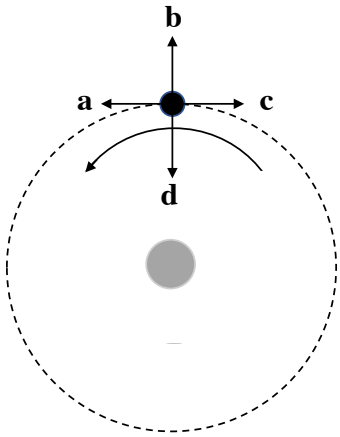
The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. A uniform rod of length L and mass M hangs vertically from a pivot point at its top end. It is free to rotate about the pivot point but initially hangs at rest. A bullet of mass m travels horizontally at velocity v and strikes the lower end of the rod. The bullet is brought to rest and falls to the ground. Assuming that the collision is elastic, what is the mass of the bullet relative to the mass of the rod, and at what angular velocity does the rod initially start to move? [7]

2. Sketch the orbits resulting when a satellite initially in a circular orbit about a central object is given a small impulse in each of the four directions shown in the diagram below. In each case, state whether the energy and angular momentum of the satellite's orbit increases or decreases.



[8]

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3. Estimate the mean distances from the Sun of Mercury and Jupiter, as fractions of the Earth's orbit, given that their orbital periods are 0.24 and 11.9 years respectively. [4]

4. Consider the decay $\pi^+ \to \mu^+ + \nu_\mu$. Show that in the rest frame of the π^+, the resulting muon has total energy

$$E = c^2 \frac{(m_\pi^2 + m_\mu^2)}{2m_\pi}$$

assuming the neutrino ν_μ to be massless. [6]

5. A ladder of length $2l$ and mass m leans against a frictionless wall at an angle θ to the vertical. Both the vertical wall and the horizontal floor on which the ladder stands are frictionless. Assuming that the ladder remains in contact with both the wall and the floor at all times, evaluate the Lagrangian, the conjugate momentum and the equation of motion for the system. [5]

6. A stick is pivoted at one end and rotates at a constant angular speed w in a horizontal plane. A bead of mass m is attached to the stick and is free to slide frictionlessly along the length of the stick. Using r as the radial position of the bead on the stick at any time, and by considering the Lagrangian of the system, find a conserved quantity. Explain why this conserved quantity is not the energy of the bead. [5]

7. A ball of mass m is thrown vertically into the air with an initial speed v_0. If the force due to air resistance is αv, where v is the speed of the ball and α is a constant, find an expression for the time taken for the ball to reach its maximum height. [5]

Section B

8. A space probe is landing vertically on the Moon using a retrorocket to slow its descent. Show that the downward acceleration, $\mathrm{d}v/\mathrm{d}t$, of the vehicle is given by

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - \frac{\alpha u}{(m_0 - \alpha t)},$$

where g is the acceleration due to the Moon's gravity, u is the velocity with which fuel is ejected relative to the rocket, m_0 is the initial mass of the space probe and its fuel and α is the rate at which fuel is ejected. [6]

Find an expression for the downward speed v of the space probe as a function of time t after the retrorocket begins to fire. [3]

If the retrorocket begins to fire when the probe is at a height h_0 above the surface of the Moon, show that the subsequent height h of the space probe as a function of time t is given by

$$h = h_0 - (v_0 - u)t - \frac{gt^2}{2} - \frac{u(m_0 - \alpha t)}{\alpha} \ln \frac{m_0}{(m_0 - \alpha t)}.$$

[7]

If the space probe is travelling at an initial speed $v_0 = 30\,\mathrm{m\,s^{-1}}$ it is able to land on the surface of the Moon with a speed of approximately zero at a time T after firing its retrorockets and using a total mass of fuel $m_0/3$. Using $m_0 = 1.6 \times 10^4\,\mathrm{kg}$, $u = 314\,\mathrm{m\,s^{-1}}$ and the acceleration due to gravity on the Moon $g = 1.62\,\mathrm{m\,s^{-2}}$, calculate the initial height h_0 at which the retrorockets must start to fire and the time T taken to land. [4]

$\left[\text{You may use: } \int \ln x \,\mathrm{d}x = x \ln x - x\right]$

9. Explain, using diagrams, why the use of the centre of mass (or momentum) frame is useful for treating problems of collisions between particles in two dimensions. [4]

Two non-relativistic particles of equal mass m and with velocity vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ collide with each other. How much kinetic energy is available for conversion to other forms of energy? [4]

A particle of mass m travelling with speed V along the $+x$ direction collides elastically with a stationary particle of mass $2m$. After the collision, the particle of mass m is deflected through an angle of $30°$. Draw diagrams to show the motion of the two particles before and after the collision in both the laboratory and centre of momentum frames. [4]

Calculate the final velocity vectors of the two particles in the laboratory frame. [8]

10. State Einstein's postulates of the theory of special relativity. Write down the Lorentz transformations of space and time coordinates in two inertial frames moving at relative speed, v, along the x-axis. [5]

Alison and Belinda are twins and are about to go travelling in space. They each carry a clock to record how much they age during the trip. They leave the Earth at the same time but travel in opposite directions. Alison travels at a steady speed of $5c/13$ to a space station 1 light year away while Belinda travels at a speed of $5c/13$ in the opposite direction. When Alison reaches the space station she immediately turns around and travels at a speed of $12c/13$ towards Belinda, eventually catching up with her. (All speeds and distances are given in the Earth's frame.)

Find the elapsed time between Alison and Belinda leaving Earth and them meeting again in space as measured on:

(a) Earth's clock [5]

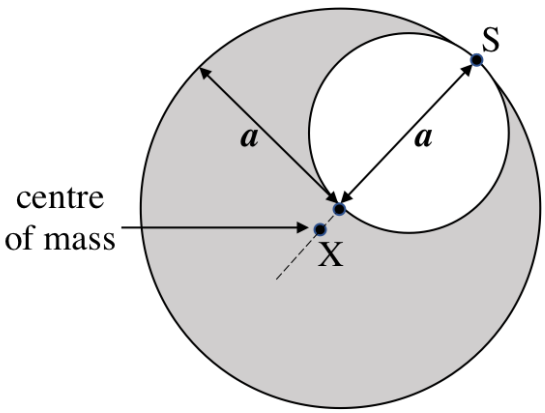
(b) Belinda's clock [5]

(c) Alison's clock. [5]

11. State the perpendicular and parallel axis theorems, explaining all terms and assumptions carefully. [3]

Find the moment of inertia of a disc of radius a and mass m about an axis through its centre and perpendicular to its plane. [4]

An earring consists of a solid disc of radius a with a circular hole in it of radius $a/2$ as shown in the diagram below.



It is suspended at the point S and is free to swing in its own plane. Show that the moment of inertia of the earring about the axis of rotation through the point S is given by

$$I = \frac{45ma^2}{32},$$

where m is the mass of the whole disc before the circular hole was removed. [8]

The centre of mass of the earring is at point X in the diagram above, and is at a distance $a/6$ from the centre of the solid disc. Show that the angular frequency, w, for small oscillations about S in the plane of the earring is given by

$$w = \sqrt{\frac{28g}{45a}}.$$

[5]