A10427W1

SECOND PUBLIC EXAMINATION

Honour School of Physics and Philosophy Part A

A2P: ELECTROMAGNETISM

TRINITY TERM 2016

Tuesday, 7 June, 2.30 pm – 4.10 pm

Time allowed: 1 hour 40 minutes

Answer **all** *of Section A and* **two** *questions from Section B.*

For Section A start the answer to each question on a new page.
For Section B start the answer to each question in a new book.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

The use of approved calculators is permitted.

Do NOT turn over until told that you may do so.

Section A

1. Three identical charges q each of mass m are fixed in an equilateral triangle of side a. Calculate the work done in assembling this charge distribution by bringing the charges from infinite separation. [2]

If the charges are released simultaneously, describe their subsequent motion and find an expression for their final speeds. [2]

2. Give the expression for the force on a charge q in the presence of both a uniform magnetic flux density $\mathbf{B}$ and a uniform electric field $\mathbf{E}$. If $\mathbf{E} = \mathbf{0}$, describe qualitatively the motion of a particle initially moving at velocity $\mathbf{v}$ (a detailed derivation of the equations of motion is not required, but you should give in your answer expressions for any important speeds, timescales and length scales). [4]

3. Two slabs of charge, extending to large distances in the yz plane, are placed touching each other at $x = 0$ in a vacuum. The first slab has a width of a and a uniform fixed charge density of $\rho = -C/a$ for $-a \leq x < 0$, and the second slab has a width of b and a uniform fixed charge density of $\rho = C/b$ for $0 \leq x < b$, where C is a constant. The $\mathbf{E}$-field outside the slabs is zero. Derive expressions for the electric field, $\mathbf{E}$, within each of the slabs. [5]

What is the electric potential difference, ΔV, between the planes $x = -a$ and $x = b$? [3]

4. A closed circular loop of wire of radius r initially lies in the xy plane. It subsequently rotates about the x-axis through its centre with angular velocity ω. A uniform magnetic flux density, $\mathbf{B}$, is applied along the z-axis. If the wire loop has high resistance R, calculate the *emf* induced in the loop, and the instantaneous current. [3]

Find an expression for the mean time-average torque required to rotate the loop. [3]

5. Write down Ampère's law relating the magnetic flux density, $\mathbf{B}$, to the electric current, giving both the integral and the differential forms of the law. Explain why a *displacement current* must be introduced, and show that this modification leads to one of the Maxwell equations. [5]

Section B

6. Explain without formal proof how the method of images can be used to solve electrostatic problems involving conducting surfaces. [4]

An infinite conducting sheet exists in the $z = 0$ plane. A point charge of $+Q$ is located above the sheet at co-ordinates $(0, 0, d)$ and a point charge of $-2Q$ is located at $(0, 0, 2d)$. What is the electric field strength, E, at the point just above the conductor and directly under the charges, i.e., at co-ordinate $(0, 0, 0)$? What is the direction of the E-field at this point? [4]

Derive an expression for the electric field strength at a point just above the conducting sheet and a distance x from the origin. What is the surface charge density of the conducting sheet at this point? Use this to calculate the total charge on the plate. [8]

If $Q = 1\,\mathrm{C}$ and $d = 1\,\mathrm{m}$, at what value of x will the E-field drop to zero? Comment on how the E-field strength just above the conducting sheet will vary with distance x in the plane of the sheet when measured at a large distance ($x \gg d$), and justify your answer. [4]

7. Two concentric thin conducting spheres are arranged such that the inner sphere (of radius a) is maintained at potential V_1, and the outer sphere (of radius b) is earthed. Show that the electric field, $\mathbf{E}$, in the air gap between the spheres (i.e., for $a < r < b$) is given by

$$\mathbf{E} = \frac{V_1}{r^2}\frac{ab}{b-a}\hat{\mathbf{r}}.$$

[5]

If $V_1 = 10^3\,\mathrm{V}$, $a = 0.1\,\mathrm{m}$ and $b = 0.4\,\mathrm{m}$, calculate the maximum electric field strength between the two conducting spheres in this two-sphere system. [2]

A third thin concentric conducting sphere with radius c is maintained at potential V_2 and is inserted between the other two spheres (i.e., $a < c < b$). Derive equations for the electric field strength as a function of radius r in the regions $a < r < c$ and $c < r < b$, expressing your answers as a function of V_1, V_2 and the radii of the spheres (a, b and c). [3]

Let $E_{\max}$ be the maximum electric field strength in the region $a < r < b$. Given that the radius of the middle sphere in the three-sphere system is $c = 0.2\,\mathrm{m}$, calculate the value of V_2 which will minimize $E_{\max}$. For this value of V_2, what is the value of $E_{\max}$? Sketch the variation of V with radius in the region $a < r < b$ for both the two-sphere and three-sphere systems. On another plot, sketch the variation of the $\mathbf{E}$-field, and indicate where the $\mathbf{E}$-field is strongest in each case. [10]

8. Write down Maxwell's equations of electromagnetism in their differential form. What would the equations be for free space? [4]

From these equations in a vacuum, show how a wave equation may be derived for the electric field **E**. What is the speed of propagation of the wave, in terms of the physical constants? [4]

If the electric field of an electromagnetic wave is polarized in the x-direction and is given by $\mathbf{E} = E_0 e^{i(\omega t - kz)}\hat{\mathbf{x}}$, find an expression for the magnetic flux density, **B**. [3]

Write down the expressions for the energy density per unit volume, u, in terms of the magnetic flux density, **B**, and electric field, **E**. What is the ratio of the average energy density of the electric and magnetic fields for electromagnetic radiation in free space? [3]

Give an expression for the Poynting vector, **S**, in terms of the magnetic flux density and electric field vectors, and explain the physical meaning of **S**. A laser has a power of 1 W, and forms a circular beam of parallel light with radius 1 mm. Calculate the amplitudes of the **E**-field and the **B**-fields. [6]